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Application of Fokker-Planck Equation in a Vector Markov Process

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ABSTRACT *The paper presents a vector Markov process, P(x,t), which form the components of the vector y. The Markov process satisfies the (n+1)- dimensional Fokker-Planck equation partial whose solution under certain initial and boundary conditions of this presentation and the domain of the nonlinear analysis. Specifically we examine the initial and boundary conditions for the aforementioned equation, whose form*

$$\frac{\partial P(y;t)}{dt} + \sum_{k=0}^{N} \frac{\partial}{\partial y_k}\left\{\left[K_k(y,t) - \frac{1}{2}\sum_{l=0}^{N}\frac{\partial}{\partial y_l} K_{lk}(y,t)\right] P(y,t)\right\} = 0 \qquad (1)$$

where the coefficients of intensity given by the following formulas

$$K_k(y,t) = \lim_{\Delta t \to 0} \frac{E\langle \Delta y_k | y\rangle}{\Delta t}, \qquad K_{lk}(y,t) = \lim_{\Delta t \to 0} \frac{E\langle \Delta y_l | \Delta y_k\rangle}{\Delta t} \qquad (2)$$

and $E\langle\cdot|y\rangle$ is the mathematical expectation of the final variable for a given y.

Introducion

Let us now consider a stationary system, where the coefficients

$$K_k(y,t) \quad and \quad K_{lk}(y,t)$$

does not depend explicitly of t. Then the probability density

$$j_k(y,t) = \left\{\left[K_k(y,t) - \frac{1}{2}\sum_{l=0}^{N}\frac{\partial}{\partial y_l} K_{lk}(y,t)\right] P(y,t)\right\}, \quad k = 0,1,\dots,N; \qquad (3)$$

Can be used to calculate the mean number of phase jumps in unit time. Accordingly, the Fokker-Planck equation can be rewritten in the form of the equation of continuity

$$\nabla \cdot j(y,t) + \frac{\partial P(y;t)}{\partial t} = 0 \qquad (4)$$

where

$$(y,t) = [j_0(y,t), \dots, j_N(y,t)]$$

the vector can be interpreted as a probability vector of the current density, and ∇ is the operator of differentiation of a given space.

We will give a geometric interpretation of the Fokker-Planck equation that will be useful and somewhat illustrative example of the application of the Markov process. Specifically, for each given function vector Markov processes, vector trajectory can be thought of as a movement starting point

$$y(t_0)$$

in the (N +1)-dimensional part space

$$y = (\phi, y_1, \dots, y_N).$$

Locant this point at some time t , is

$$[y_0(t), \dots, y_N(t)]$$

can be identified with Braun movement of particles under the influence of diffusion processes in the (N +1) - domenzionom space over time .

Because of the random (Brownian) thermal motion of electrons in the material (conductive middle) electronic components generate random signal , which represents the thermal noise . It appears related to electrical resistance and exists independently of the external electric field , and the source of guidance charge carrier movement. In this case, the power spectral density of thermal noise is independent of frequency.

A set of arbitrary functions of the process is a group of random trajectories . It is shown that the time that particles spend in any part of the space of probability R' is proportional to total probability in that area. The coefficients of (1) can be directly obtained by applying the relation (2) . Differential equations whose solution describes the probability density P (xy; t) is obtained by simply replacing the coefficients

$$K_k \quad and \quad K_{lk}$$

in (1) .

The initial and boundary conditions

In order to get a solution Foker-Planck equation will assume

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the intial and boundary conditions. In our case the initial value of the probability density function

P(x,t) at time $t = t_0$ be

$$\lim_{t\to t_0} P(y;t) = \prod_{k=0}^{N} \delta[y_k - y_k(t_0)] \qquad (5)$$

The boundary conditions are determined from the physical interpretation of the trouble is the probability density function P$[\phi, y_0';t]$, since $K_k(y,t)$ i $K_{lk}(y,t)$ the periodic and by ∅, we have

$$\lim_{t\to\infty} P[\phi, y_0'; t] = \lim_{t\to\infty} P[\phi \pm 2n\pi, y_0'; t] = 0 \qquad (6)$$

in steady state P$[\phi, y_0'; t]$ has unlimited variance. This condition can be directly attributed to the phenomenon of skipping cycle(phase jumps) in the generalized system of monitoring,because they require additional considerations in order to obtain the probability density function with finite variance in the steadey state. For the purpose shall prove the following theorem

Theorem 1

Let's

$$\tilde{p}(\phi, y_0'; t) \triangleq \sum_{n=-\infty}^{\infty} \mathrm{P}[\phi \pm 2n\pi, y_0'; t], \quad \forall \emptyset$$

for every ∅

Note that the function is a periodic solution to the FP equation, by ∅, and that as such it is not, because the probability density function can be represented by an infinite sum of density function on the unit sphere.

Proof: To obtain a solution that has the characteristics of density functions, will define the function

$$p(\phi, y_0'; t) \triangleq \begin{cases} \tilde{p}(\phi, y_0'; t)\ za\ bilo\ koje\ \phi, \phi\epsilon[(2n-1)\pi, (2n+1)\pi] \\ 0 \qquad for\ other\phi \end{cases} \qquad (7)$$

Where $p(y;t) \triangleq p(y;t|y_0, t_0)$.

To justify to validity $\tilde{p}(\phi, y_0'; t)$ of such solutions , we note that P(y;t) is the solution of equations R´ areas where $y_j = \pm\infty$, j=0,...,N. The function $p(y;t)$ is defined in the field of R consisting formed with two hypersurface a distance 2π radians.

Therefore, since every member $\tilde{p}(\phi, y_0'; t)$ of the solution in the field of R´ the sum is also a solution in R. Presenting using cylindrical coordinates, R hypercilidr we thought. Also we assume that the conous and differentiable function and is defines areas nationwide the situation R. Since p(y;t) the transition probability density function defined by(3), the condition of normalization, which is an expression of conservation of probability

$$\tilde{p}(\phi, y_0'; t) \triangleq \sum_{n=-\infty}^{\infty} P[\phi \pm 2n\pi, y_0'; t], \qquad \forall t. \qquad (8)$$

valid for every t,

From this it follows $p(y;t)$ that there must be zero faster than $y_k^{-(1+\varepsilon)}$, $\varepsilon > 0$, when→∞. Now the partial differential equation of second order parabolic type, and its solution is determined with 2N+2 independet boundary conditions. From all this we have the following boundary conditions:

Along each edge of the surface Γ hyperplate R, edging =∞ $y_k = \pm\infty$ for every k=1,2,...,N, we have N boundary conditions

$$y_k p(\phi, y_0'; t)\,|_{y_k=\pm\infty} = 0; \ k = 1,2,\dots,N \qquad (9)$$

Since p(y;t) tends to zero faster than $y_k^{-(1+\varepsilon)}$, $\varepsilon > 0$. $y_k^{-(1+\varepsilon)}$ As a consequence of (8) and also have to be

$$p(\phi, y_0'; t)|_{y_k=\pm\infty} = 0;$$

for every k=1,2,...,N. Since (8) holds for every t we have N other independent boundary conditions,

$$\frac{\partial}{\partial y_k} p(\phi, y_0'; t)\,|_{y_k=\pm\infty} = 0; \ k = 1,2,\dots,N. \qquad (10)$$

Now is $\tilde{p}(\phi, y_0'; t)$ a periodic function of ∅is obtained as the sum of periodic functions. Therefore . the

$$p(-\pi, y_0'; t) = p(\pi, y_0'; t)$$

From(11) it follows that

$$\frac{\partial p(-\pi, y_0'; t)}{\partial y_k} = \frac{\partial p(\pi, y_0'; t)}{\partial y_k}; \quad k = 1,2,\dots,N \qquad (12)$$

What is not independent of conditions (11).

Finally, it the flow of probability must be maintained in all directions of coordinate axes$\tilde{p}(\phi, y_0'; t)$ and having a periodic function, we have

$$\frac{\partial p(\phi, y_0'; t)}{\partial \phi}\,|_{\phi=\pi} = \frac{\partial p(\phi, y_0'; t)}{\partial \phi}\,|_{\phi=-\pi} \qquad (13)$$

Equations (9),(10);(11) and (13) define 2N+2 independent boundary conditions. q.e.d.

Remark: the relations (12) and (13) can be written in vector notation

$$\nabla p(y;t)\,|_{\phi=\pi} = \nabla p(y;t)\,|_{\phi=-\pi}$$

Symmetry suggests that the

$$p(y;t) = p(-y;t)$$

Consequences

It is interesting to note that the Fokker-Planck equation connects the Maxwell field equations with the teory of Markov processes.If we apply Gaussian theorem we can write

$$\oint_R \nabla \cdot j dR = \oint_\Gamma \boldsymbol{n} \cdot j d\Gamma = -\frac{\partial}{\partial t} \oint_R p(y;t) dy = 0$$

Where n is the unit vector normal to the surface Γ and positvely directed outwards. From the Maxwell field equations we know that the divergence of the current density J onliy speed changes the amount of charge density ρ. Also, the fact that the divergence of the flux densities D is equal to

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ρ.Therefore , if ρ is conidered as translitional probability densuty function ρ (x, t) and D as the flux density of probability we can write $\nabla D = p(y; t)$, , ie, the flux desity of probability that highlights thr volume dR at time t is equal to the probability of being in that volume at the moment t.

If we inegrate both sides of equation (12) with respect na i (to the for each j≠k≠0 and we use the boundary conditions (17),(18) and (13), we arrive at the interesting results

$$\frac{\partial p}{\partial t}+\frac{\partial}{\partial \phi}[j_0(\phi, y_k, t)]+\frac{\partial}{\partial y_k}[j_k(\phi, y_k, t)] = 0 \qquad (14)$$

Where is $p = p(\phi, y_k, t)$ and

$$K_0(\phi, y_k, t) = \Omega_0 + \sum_{j\neq k\neq 0}^{n} E(y_j, t \mid \phi, y_k) + y_k - AKF_0 g(\phi)$$

The current probability

$$K_k(\phi, y_k, t) = -\frac{1}{\tau_k}[y_k + (1-F_k)AKg(\phi)]$$

$$K_{00} = \frac{N_0 K^2 F_0^2}{2}; \quad K_{0k} = K_{k0} = \frac{(1-F_k)F_0 N_0 K^2}{2\tau_k}; \quad K_{kk} = \frac{(1-F_k)^2 N_0 K^2}{2\tau_k^{\,2}}; \quad k \neq 0$$

The current probability

$$j_0(\phi, y_k, t) = \left\{\left[K_0(\phi, y_k, t) - \frac{K_{00}}{2}\frac{\partial'}{\partial \phi} - \frac{K_{k0}}{4}\frac{\partial}{\partial y_k}\right]p\right\}$$

$$j_k(\phi, y_k, t) = \left\{\left[K_k(\phi, y_k, t) - \frac{K_{kk}}{2}\frac{\partial}{\partial y_k} - \frac{K_{0k}}{4}\frac{\partial}{\partial \phi}\right]p\right\}$$

REFERENCE 1. Guillemin, E. A., The Mathematics of circuit analysis, Wiley, New York (1949). | 2. S. O. Rice. ¨Mathematical Analysis of Random Noise" Bell System Technical Journal 24 (1945), 46-156. | 3. A. P. Prudnikov, O. I. Marichev, and A. Bryhkov. Integrals and series, Volime 3:More Special Functions (Newark, NJ, Gordon and Breach, 1990), ISBN 2881246826. | 4. R. Wong. Asymptotic Approximations of integrals (SIAM: Society for Industrial and Apllied Mathematics, New edition, 2001). | 5. H. Shin and J. H. Lee. Performance analysis of space-time block codes over over keyhole Nakagami-m fading channels. IEEE Trans. Veh. Technol. 53 (2) (2004), 351-362. | 6. Hald, A., Statistical theory with engineering applications, Willey, New York (1952). | 7. Kolmogorov, A. N., Fundations of the theory of probability, Chelsea, New York (1950). | 8. Laning, J. H. Jr., and Richard, H. B., Random processes in automatic control, McGraw-Hill, New York (1956). |