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Construction of PBIB Designs using Modified Magnificent (Mn) Matrices of Type- III

KEYWORDS: Mn- matrices, partially balanced incomplete block (PBIB) design, regular bipartite and Mn-graphs

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ABSTRACT *Partially balanced incomplete block designs with four and five associate classes have been constructed by using modified (Mn) magnificent matrices which are otherwise known as Mn-matrices. A modified structure of the magnificent (Mn) matrices of type- III originally introduced by Mohan [9] is proposed in this paper. The present work is the continuation of our earlier research work in this direction. Here, we have also studied the properties and applications of the modified magnificent (Mn) matrix of type III. Illustrations of the construction of some symmetric PBIB designs, corresponding M-graphs that can be constructed with the help of such designs have also been discussed in detail.*

1. Introduction: Block designs are extensively used in almost all fields of human activities including agriculture, industry, animal husbandry etc. Among the class of block designs, incomplete block designs and specifically partially balanced incomplete block (PBIB) designs are most widely used type of designs. In view of this, there is a need to construct such type of designs. Various authors Bose and Nair [1], Bose [2], Bose and Shimamoto [3] Garg and Syed [6], Kageyama [7], Mohan [9], Mohan.*et.al* [2006a], Mohan.*et.al* [2006b], Cohn [4], Liu[8] and Raghavarao [12], have constructed two and higher associate class PBIB designs.

Recently, some type of designs which are useful in signal and image processing, in the construction of codes, designs, and graphs have been constructed by Colbourn [5]. In this paper, we have attempted to modify the magnificent (M_n) matrix of type III earlier introduced by Mohan [9] and also tried to construct PBIB designs with four and five associate classes along with their association schemes.

2. Some definitions

Definition 2.1 when n is a prime takes values 7,11 only. M_n-matrices (a_{ij}) is defined as a matrix obtained from a_{ij}=(i+1)(j+1)mod(n) i,j= 1,2,3,…, n. This is a symmetric matrix . In the resulting matrix retain 1 as it is and substitute 0 for even numbers and +1 for odd numbers .let the resulting matrix be called as M_n-matrix of type I.

Definition 2.2 M_n-matrix of second type is obtained by the equation a_{ij}= (i+1)(j-1)mod(n) where n is a prime takes values 7,11 only. In this matrix since each row or column has n elements whereas n is a prime, 1 to n elements do come in all the columns and rows , and each element comes once in each row and each column . In the resulting matrix substitute 1 for odd numbers and -1 for even numbers keeping 1 as it is. Then this resulting matrix is called the M_n- matrix of type II, each row (column) consists of an equal number of +1's and -1's numbering to (n-1)/2. This is also n×n symmetric matrix. We discuss these two types of matrices while giving examples for their construction and applications of these matrices in the later sections.

The third possible ways by taking by (a_{ij}) =($d_i d_h d_j$) mod (n) by suitably defining d_i, d_h, d_j , in different ways. Where is a Kronecker product.

a_{ij} =2(i+j)mod(n) where (n>3) is a positive odd integer

Definition 2.3 The M_n-matrix of type III is obtained by the equation a_{ij} = 2(i+j) mod(n) where (n>3) is a positive odd integer and i,j=1,2,3 …. n. In this matrix since each row or column has n elements .When n is odd 1 to n elements do come in all columns and rows, and each element comes once in each row and each column. In the resulting matrix delete n^{th} element from all rows and n^{th} row, Substitute 1 for odd numbers and 0 for even numbers, keeping 1 in the matrix as 1 itself. Then this resulting matrix is called M_n-matrix of type III .In the resulting matrix, each row and each column the number of +1's is (n-1)/2 and the number of -1's is (n-1)/2.

Proposition 2.4 M_n matrices of type III is obtained by using a_{ij} = 2(i+j) mod(n), where (n>3) is a positive odd integer .In the resulting matrix substitute 1 for odd numbers and 0 for even numbers keeping 1 in the matrix as it is. Then this resulting matrix is called M_n-matrix of type III. In the resulting M_n-matrix of type III in each of its rows and columns, the number of +1's are (n-1)/2 and the number of -1's are (n-1)/2.

Proof: Since n is odd, In the resulting matrix in each row (column) each element of 1,2,3… n-1 occurs exactly once .And among these (n-1)/2 elements are odd numbers (n-1)/2 are even numbers. Consequently as we replace even number by 0's and odd numbers by +1's retaining 1 as it is we get the (n-1)/2 elements are +1's and (n-1)/2 elements are -1's.

As these two types of M_n-matrices are non-orthogonal, we define the orthogonal number for them as follows:

Definition 2.5 The orthogonal number of a given matrix with entries $\pm$ defined as sum of products of the corresponding numbers in the two given two rows of the matrix (called inner product of the rows).Consider any two rows $R_l=(r_1,r_2,\dots,r_n)$ and $R_m=(s_1,s_2,\dots,s_m)$ and then the orthogonal number denoted by 'g' can be defined as $g=(R_l R_m)=\sum r_i s_i$

3. New association scheme

When we consider after converting the M_n- matrix of type III as an incidence matrix then it becomes a PBIB design in which there are 'v' rows and 'v' columns .We consider v columns as blocks of the new designs. The association scheme of the newly constructed designs will be as follows:

If two treatments occurs together 0, 1, 2, ... , n-1 times in the blocks of the new design, then they are respectively I^{st}, 2^{st}, 3^{rd},. . ., n^{th} associates and n_1=1, n_2=2,. . ., n_n=2.

4. Illustrations: In this section, we have given two illustrations as below:

4.1 Take n=9, from definition 2.3, M_n = (a_{ij}) = 2 (i+j) mod (n) for i,j=1,2,3. . . n. Magnificent (M_n) matrix is given by

$$M_n=\begin{bmatrix} 4&6&8&1&3&5&7&9&2\\ 6&8&1&3&5&7&9&2&4\\ 8&1&3&5&7&9&2&4&6\\ 1&3&5&7&9&2&4&6&8\\ 3&5&7&9&2&4&6&8&1\\ 5&7&9&2&4&6&8&1&3\\ 7&9&2&4&6&8&1&3&5\\ 9&2&4&6&8&1&3&5&7\\ 2&4&6&8&1&3&5&7&9 \end{bmatrix}$$

Delete n^{th} element from all rows and leaving n^{th} row, we get

$$M_n=\begin{bmatrix} 4&6&8&1&3&5&7&2\\ 6&8&1&3&5&7&2&4\\ 8&1&3&5&7&2&4&6\\ 1&3&5&7&2&4&6&8\\ 3&5&7&2&4&6&8&1\\ 5&7&2&4&6&8&1&3\\ 7&2&4&6&8&1&3&5\\ 2&4&6&8&1&3&5&7 \end{bmatrix}$$

Now ,substituting 1 for odd numbers and -1 for even numbers and keeping 1 as it is and further considering -1's as zeros, the above M_n matrix reduces to

$$M_n=\begin{bmatrix} 0&0&0&1&1&1&1&0\\ 0&0&1&1&1&1&0&0\\ 0&1&1&1&1&0&0&0\\ 1&1&1&1&0&0&0&0\\ 1&1&1&0&0&0&0&1\\ 1&1&0&0&0&0&1&1\\ 1&0&0&0&0&1&1&1\\ 0&0&0&0&1&1&1&1 \end{bmatrix}$$

Consider the above M_n matrix as the incidence matrix, then it will generate a four associate class PBIB design with parameters

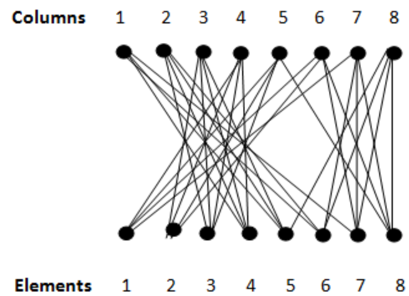
v=8=b, r=4=k, $\lambda_1=0$, $\lambda_2=1$, $\lambda_3=2$, $\lambda_4=3$, $n_1=1, n_2=2, n_3=2, n_4=2$ which follows the association scheme 3.

P-matrices of the association scheme are given by

$$P_1=\begin{bmatrix} 0&0&0&0\\ 0&0&0&2\\ 0&0&2&0\\ 0&2&0&0 \end{bmatrix} \quad P_2=\begin{bmatrix} 0&0&0&1\\ 0&0&1&0\\ 0&1&0&1\\ 1&0&1&0 \end{bmatrix}$$

$$P_3=\begin{bmatrix} 0&0&1&0\\ 0&1&0&1\\ 1&0&0&0\\ 0&1&0&1 \end{bmatrix} \quad P_4=\begin{bmatrix} 0&1&0&0\\ 1&0&1&0\\ 0&1&0&1\\ 0&0&1&0 \end{bmatrix}$$

Also, by considering the above M_n matrix as an adjacency matrix of a graph, which is called the M_n-graph, can be drawn as follows:



4.2 Take n=11, from definition 2.3 $M_n=(a_{ij}) = 2(i+j) \bmod (n)$ for i,j=1,2,3. . . n. Magnificent (M_n) matrix is given by

$$M_n=\begin{bmatrix} 4&6&8&10&1&3&5&7&9&11&2\\ 6&8&10&1&3&5&7&9&11&2&4\\ 8&10&1&3&5&7&9&11&2&4&6\\ 10&1&3&5&7&9&11&2&4&6&8\\ 1&3&5&7&9&11&2&4&6&8&10\\ 3&5&7&9&11&2&4&6&8&10&1\\ 5&7&9&11&2&4&6&8&10&1&3\\ 7&9&11&2&4&6&8&10&1&3&5\\ 9&11&2&4&6&8&10&1&3&5&7\\ 11&2&4&6&8&10&1&3&5&7&9 \end{bmatrix}$$

Now, delete n^{th} element from all rows and leaving n^{th} row, we get

$$M_n=\begin{bmatrix} 4&6&8&10&1&3&5&7&9&2\\ 6&8&10&1&3&5&7&9&2&4\\ 8&10&1&3&5&7&9&2&4&6\\ 10&1&3&5&7&9&2&4&6&8\\ 1&3&5&7&9&2&4&6&8&10\\ 3&5&7&9&2&4&6&8&10&1\\ 5&7&9&2&4&6&8&10&1&3\\ 7&9&2&4&6&8&10&1&3&5\\ 9&2&4&6&8&10&1&3&5&7\\ 2&4&6&8&10&1&3&5&7&9 \end{bmatrix}$$

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Now, substituting 1 for odd numbers and -1 for even numbers keeping 1 as it is, and further considering -1's as zeros, the above M_n matrix reduces to

$$M_n = \begin{bmatrix} 0&0&0&0&1&1&1&1&1&0\\ 0&0&0&1&1&1&1&1&0&0\\ 0&0&1&1&1&1&1&0&0&0\\ 0&1&1&1&1&1&0&0&0&0\\ 1&1&1&1&1&0&0&0&0&0\\ 1&1&1&1&0&0&0&0&0&1\\ 1&1&1&0&0&0&0&0&1&1\\ 1&1&0&0&0&0&0&1&1&1\\ 1&0&0&0&0&0&1&1&1&1\\ 0&0&0&0&0&1&1&1&1&1 \end{bmatrix}$$

Consider the above M_n matrix as the incidence matrix then it will generate a five associate class PBIB design with parameters

$v=10=b$, $r=5=k$, $\lambda_1=0$, $\lambda_2=1$, $\lambda_3=2$, $\lambda_4=3$, $\lambda_5=4$, $n_1=1, n_2=2, n_3=2$, $n_4=2, n_5=2$ and follows the association scheme 3

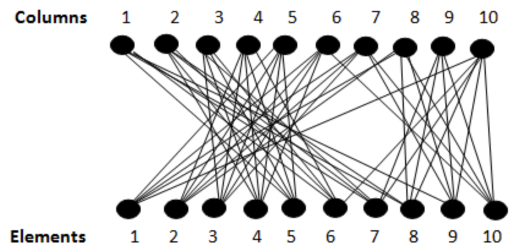
P-matrices of the new association scheme are given by

$$P_1 = \begin{bmatrix} 0&0&0&0&0\\ 0&0&0&0&2\\ 0&0&0&2&0\\ 0&0&2&0&0\\ 0&2&0&0&0 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 0&0&0&0&1\\ 0&0&0&1&0\\ 0&0&1&0&1\\ 0&1&0&1&0\\ 1&0&1&0&0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0&0&0&1&0\\ 0&0&1&0&1\\ 0&1&0&0&0\\ 1&0&0&0&1\\ 0&1&0&1&0 \end{bmatrix} \qquad P_4 = \begin{bmatrix} 0&0&1&0&0\\ 0&1&0&1&0\\ 1&0&0&0&1\\ 0&1&0&0&0\\ 0&0&1&0&1 \end{bmatrix}$$

$$P_5 = \begin{bmatrix} 0&1&0&0&0\\ 1&0&1&0&0\\ 0&1&0&1&0\\ 0&0&1&0&1\\ 0&0&0&1&0 \end{bmatrix}$$

Also, by considering the above M_n matrix as an adjacency matrix of a graph, which is called the M_n-graph, can be drawn as follows



Conclusion: In this paper, we have proposed the modified M_n-matrices of Type III, which are structurally different but serve the same purpose as defined by Mohan [9]. Here, we have substituted 1 for odd numbers, 0 for even numbers and keeping 1 in the matrix as it is, as a result we get M_n-matrix of type III that generates new higher associate class PBIB designs.

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