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Complete Dominating Number of Graphs

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ABSTRACT *In this paper, we introduce the complete domination number in graphs. Some interesting relationships are known between domination and degree equitable domination and independent domination. It is also shown that any positive integers with $\gamma(G)=\gamma_n=\gamma_i$ are realizable as the domination number , complete domination number and independent domination number of a graph.*

1 Introduction:

The concept of domination in graphs evolved from a chess board problem known as the Queen problem- to find the minimum number of queens needed on an 8x8 chess board such that each square is either occupied or attacked by a queen. C.Berge[3] in 1958 and 1962 and O.Ore[8] in 1962 started the formal study on the theory of dominating sets. Thereafter several studies have been dedicated in obtaining variations of the concept. The authors in [7] listed over 1200 papers related to domination in graphs in over 75 variation.

Throughout this paper , G(V , E) a finite , simple , connected and undirected graph where V denotes its vertex set and E its edge set. Unless otherwise stated the graph G has n vertices and m edges. Degree of a vertex v is denoted by d(v), the *maximum degree* of a graph G is denoted by Δ(G). Let C_n a cycle on n vertices , P_n a path on n vertices and a complete graph on n vertices by K_n. A graph is *connected* if any two vertices are connected by a path . A maximal connected subgraph of a graph G is called a *component* of G .The *number of components* of G is denoted by ω(G). The *complement* $\bar{G}$ of G is the graph with vertex set V in which two vertices are adjacent iff they are not adjacent in G. A tree is a connected acyclic graph. A *bipartite graph* is a graph whose vertex set can be divided into two disjoint sets V_1 and another in V_2 . A *complete bipartite graph* is a bipartite graph with partitions of order $|V_1|$ =m and $|V_2|$=n, is denoted by $K_{m,n}$. A star denoted by $K_{1,n-1}$ is a *tree* with one root vertex and n-1 pendant vertices. A bistar , denoted by B(m,n) is the graph obtained by joining the root vertices of the stars denoted by F_n can be constructed by identifying n copies of the cycle C_3 at a common vertex. A *wheel* graph denoted by W_n is a graph with n vertices formed by connecting a single vertex to all vertices of C_{n-1}. A *Helm* graph denoted by H_n is a graph obtained from the wheel W_n by attaching a pendant vertex to each vertex in the outer cycle of W_n.

The *chromatic number* of a graph G denoted byχ(G) is the smallest number of colors needed to colour all the vertices of a graph G in which adjacent vertices receive different colours . For any real number x , $\lceil x \rceil$ denotes the largest integer greater than or equal to x and $\lfloor x \rfloor$ the smallest integer less than or equal to x. A Nordhaus - Gaddum – type result is a lower or upper bound on the sum or product of a parameter of a graph and its complement. Throughout this paper, we only consider undirected graphs with no loops .The basic definitions and concepts used in this study are adopted from[11].

Given a graph G = (V(G) , E(G)) , the cardinality $|V(G)|$ of the vertex set V(G) is the order of G is n. The distance d_G(u ,v) between two vertices u and v of G is the length of the *shortest path* joining u and v .If d_G(u ,v) = 1, u and v are said to be adjacent.

For a given vertex v of a graph G , The open neighbourhood of v in G is the set N_G(v) of all vertices of G that are adjacent to v.

The degree deg_G(v) of v refers to$|N_G(v)|$, and $\Delta(G) = \max\{deg_G(v): \quad v \in V(G)\}$. The closed neighbourhood of v is the set $N_G[v]$ = $N_G(v)$ ∪ v for S ⊆ $V(G)$, $N_G(S)$ =$\cup_{v\epsilon s} N_G(v)$ and $N_G[v]$= $N_G(S) \cup S$. If $N_G[v]$=V(G), then S is a dominating set in G. The minimum cardinality among dominating sets in G is called the *domination number* of G and is denoted by $\gamma(G)$.

A dominating set S in a graph G is an independent dominating set if for every pair of distinct vertices u and v in S, u and v are non adjacent in G . The minimum cardinality $\gamma_i(G)$ of an independent dominating set in G is called the *independent domination number* of G. Given a graph G, choose $v_1 \in V(G)$

and put $S_1 = \{v_1\}$. If $N_G[S_1] \neq V(G)$,choose $v_2 \in V(G)$, $|N_G(v_2)| = |N_G(S_1)|$ and put if $S_2 = \{v_1, v_2\} \neq V(G)$ where possible $k \geq 3$, choose $v_k \in V(G)$; $|N_G(v_k)| = |N_G(S_{k-1})|$and put $S_k = \{v_1, v_2 \ldots\ldots v_k\}$, there exists a positive integer k such that $N_G[S_k] = V(G)$. A dominating set obtained in this way above is called *a degree equitable dominating set* .The minimum cardinality of a degree equitable dominating set is called the *degree equitable domination number* denoted by γ_N [14] .

2. Relationships between domination and Complete domination numbers

Definition:2.0

Given a graph G , choose $v \epsilon V(G)$ and put S ={v}; For every v we have $N_G(S)$ = V- S denoted by S' is the complete dominating set .The smallest cardinality of a *complete dominating set* is called the *complete domination number of G* is denoted by $\gamma_n(G)$. The following results are observed.

Result 2.1: The regular graph of $n \leq 4$ is a complete dominating set $\gamma_n(G) = \left\lfloor \frac{n}{2} \right\rfloor$

Result 2.2: Any star graph of $K_{1,n}$ is a Complete dominating set $\gamma_n(G) = 1$

Result 2.3: Any Complete Bipartite graph $K_{m,n}$ is a complete dominating set $\gamma_n(G)$=m if $m \leq n$.

Result 2.4: For any Bipartite graph $K_{m,n}$ is not a complete dominating set .

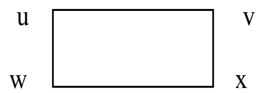
Result 2.5: Any regular graph of $n \geq 5$ is a degree equitable but not complete dominating set.

Result 2.6: Any wheel graph W_n, $\gamma_n(G)$= 1.

Lemma 2.7: Let G be any graph , Let S and S' be any two dominating set where $S' = N_G(S) = V - S$ then $S \cap S' = \emptyset$.

Lemma 2.8: Let G be any graph . Let S and S' be any two dominating set where $S' = N_G(S) = V - S$ then $S \cup S' = V(G)$.

Example 2.9: For the regular graph n=4 ,we have S ={u,x) and S'={v ,w}



Result 2.10: For any graph G, If $N_G(S)$ and S' = V-S are not complete then$\bigcup_{v \epsilon S} N_G(S) = V - S$.

Proof: Given $N_G(S)$ and V-S are not complete, There exists some S such that $N_G(S) \neq$ V-S . Hence $\bigcup_{v \epsilon S} N_G(S) = V - S$.

Theorem2.11: Let S and S' be any two complete dominating sets in a graph G then their domination numbers are

(i)$\gamma_n(G) = 1$ if G is $K_{1,n}$ (ii) $\gamma_n(G) = \left\lfloor \frac{n}{2} \right\rfloor$if G is regular with $n \leq 4$, (iii) $\gamma_n(G) = 1$ if G $=W_n$ (iv) $\gamma_n(G) = m$ if G = $K_{m,n}$ where $m \leq n$, (v) $\gamma_n(G) = 1$ if G= K_n , $n \geq 1$

Proof : Given S and S' be any two complete dominating sets in a graph G.

(i) Given a star graph G= $K_{1,n}$, Since $S' = N_G(S) = V - S$, Here $S \cap S' = \emptyset$ and there exists a vertex $v \epsilon S$ such that $N_G(S) = S'$ we have $\gamma_n(G) = 1$ if G is $K_{1,n}$.

(ii) Given G is regular graph with $n \leq 4$,Thre exists a vertex $v \epsilon S$ and $N_G(S) = S'$, Since G is a regular graph every vertex is of even degree we have $\gamma_n(G) = \left\lfloor \frac{n}{2} \right\rfloor$

(iii) Given G is a wheel graph W_n, Since by definition. There exists $v \epsilon S$ such that N(S) = S' which implies $\gamma_n(G) = 1$ if G = W_n.

(iv) Given a graph G is complete bipartite with two set of vertices $m \leq n$,since by the definition $N_G(S) = S'$. where S = m and S'= n implies $\gamma_n(G) = m$ if G = $K_{m,n}$ where $m \leq n$.

(v) A graph G is a complete graph K_n, since by definition there exists $v \epsilon S$ such that $N_G(S) = S'$ implies $\gamma_n(G) = 1$ if G = K_n , $n \geq 1$.

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Lemma 2.12: For any graph G. If $N_G(S)$ and V-S are not complete then S is not a degree equitable dominating set.

Proof: Given $N_G(S)$ and V-S are not complete. There exists a $v\epsilon S_1$ such that $N_G(v_1) \neq V - S$

Hence $\bigcup_{v\epsilon S} N_G(S) = V - S$. Since $N_G(S)$ and V-S are not complete, then S is not degree equitable dominating set.

Theorem2.13: For any regular graph G with $n \leq 4$. If $N_G(S)$ and V-S are complete then S is a degree equitable dominating set in G.

Proof: Given $N_G(S)$ and V-S are complete then S is dominating set in G , There exists a vertex $v_1 \epsilon S$ such that $N_G(v_1)$ = V-S, choose another vertex $v_2 \epsilon S$ such that $N_G(v_2)$ = V-S, There exists any positive integer k such that $v_k \epsilon S$ such that $N_G(v_k)$ = V-S , Since $\cap N_G(S)$ = V- S and complete then S is a degree equitable dominating set.

Lemma2.14: Suppose S and S' are not complete dominating sets of G then there exists at least one $v \epsilon G$ such that $N_G(S) \cap N_G(S') = \emptyset$

Proof: Obviously , by the definition of the complete domination set.

Lemma 2.15: Suppose S and S' are complete dominating subsets of G such that $N_G(S) \cap N_G(S') \neq \emptyset$

Proof: Obviously , by the definition of the complete domination set.

Lemma 2.16: Suppose S and S' be the dominating sets of V(G∘H) such that S and S' are not complete

Proof: Given S and S' be the dominating set of V(G∘H), By the definition of Corona graph V(G∘H) is either V(G) or V(H) is dominating set, There exists some $v_1 \epsilon S$ such that $N_S(v_1) = H_{v_1}$, similarly there exists some $v_2 \epsilon S$ such that $N_S(v_2) = H_{v_2}$, For every positive integer k such that $v_k \epsilon S$ and $N_S(v_k) = H_{v_k}$ where S' = $\{H_{v_1}, H_{v_2} \ldots\ldots H_{v_k}\}$, hence $\cap N_S(v_k) = \emptyset$, where k = 1,2,…. Which implies S and S' are not complete dominating set.

Theorem 2. 17: Every complete dominating set is a degree equitable dominating set

Proof: Given G be a graph with complete dominating sets S and S' ,By the definition of complete dominating sets $N_G(S) = S'$, for every vertex $v\epsilon S$ we have $N_G(S) = S'$ since every vertex $v\epsilon S$ is a neighborhood of S' , We have $|N_G(S)| = |N_G(S')|$ which implies the complete dominating set S is a degree equitable dominating set in G.

3.CONCLUSION

In this paper we found an upper bound for the complete domination number and Relationships between Complete domination numbers and Complete domination numbers of graphs and characterized the corresponding extremal graphs. Similarly Complete domination numbers with other graph theoretical parameters can be considered.

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