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Production Inventory Model for Weibull Deteriorating Items with Price and Quantity Dependent Demand, Time Varying Holding Cost and Shortages

KEYWORDS: Inventory, Deterioration, Production, Time varying holding cost, Shortages, Demand

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ABSTRACT *A production inventory model for deteriorating items with price and quantity dependent demand is developed. Holding cost is linear function of time. Shortages are allowed and completely backlogged. Deterioration rate increases with time. Sensitivity with respect to parameters have been carried out. Numerical example is taken to support the model.*

INTRODUCTION:

The economic order quantity (EOQ) is extended as the economic production quantity (EPQ), replaced the assumption of instantaneous replenishment by the assumption that the replenishment order is received at a constant finite rate over time (Pentico et al., [11]). The lead time and demand are known and constant are the main assumptions of the basic EPQ and EOQ models. This means that an order will be placed when the inventory available is exactly sufficient to cover the demand under the conditions of demand certainty (Sana, [15]).

Deterioration is another important factor for controlling the inventory in a production system. Mishra [8] developed the first production lot size model in which both constant and variable rate of deterioration were considered. Choi and Hwang [2] developed a model determining the production rate for deteriorating items to minimize the total cost function over a finite planning period. Panda et al. [9] developed a single item economic production quantity model with ramp type quadratic demand. Manna and Chiang [7] developed an economic production quantity model for deteriorating item with ramp type demand.

In classical inventory models the demand rate is assumed to be a constant. But in reality demand of physical goods may be price and/ or stock dependent. Gupta and Vrat [3] presented an inventory model for stock dependent consumption rate. However, their expressions, for average total system cost was based on the demand rate which was based on initial stock level rather than instantaneous inventory. Mandal and Phaujdar [6] suggested corrections to the model developed by Gupta and Vrat [3]. Burewell [1] developed economic lot size model for price dependent demand under quantity and freight discounts. A model for a deteriorating item with a stock dependent demand rate was developed by Sana and Chaudhury [14] in which the production rate of the item in stock was partly constant and partly dependent on instantaneous stock and demand. You [17] developed an inventory model when demand for products is price and time dependent. Teng and Chang [16] considered the economic production quantity model for deteriorating items with stock level and selling price dependent demand. Jain et al. [4] developed an economic production quantity model with shortages by incorporating the deterioration effect and stock dependent demand rate. Roy and Chaudhury [12] developed two production inventory models (Model I and Model II) for deteriorating items when the demand rate depends on the instantaneous inventory level. The production rate was assumed to be stock and demand dependent. Sahoo et al. [13] developed an inventory model for constant deteriorating items with price dependent demand and time varying holding cost. Patra et al. [10] considered a deterministic inventory model with price dependent quadratic demand rate. Kontantaras and Skouri [5] modified the model developed by Roy and Chaudhury [12] using the classical method of Lagrange's multiplier and thereby they converted the constrained problem as unconstrained problem.

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In the present paper, we have developed a production inventory model for deteriorating items with demand as a function of price and quantity. Holding cost is linear function of time. Shortages are allowed and are completely backlogged. Numerical example is taken and sensitivity analysis is also carried out.

NOTATIONS AND ASSUMPTIONS:

The following notations and assumptions are used here:

NOTATIONS:

k : rate of production

$D(p, I(t))$: $\left[(a - p) + \rho I(t)\right]$, demand is linear function of price and quantity, where $a > 0$, $\rho > 0$,

A : Ordering cost per order

p : Selling price per unit

c : Unit purchasing cost per item

c_2 : Shortage cost per unit

$h(t)$: $x+yt$, inventory variable holding cost per unit excluding interest charges

T : Duration of production cycle

$I(t)$: Inventory level at any instant of time t, $0 \le t \le T$

Q_1 : Inventory level initially at time t_1

Q_2 : Shortage of inventory

Q : Order quantity

α : Scale parameter ($0 < \alpha < 1$)

β : Shape parameter ($\beta > 0$)

$\alpha\beta t^{\beta-1}$: the two parameter Weibull deterioration rate.

ASSUMPTIONS:

The following assumptions are used in the development of the model:

- The demand of the product is declining as a function of price and quantity.
- Rate of production is k.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and completely backlogged.
- The deteriorated units can neither be repaired nor replaced during the cycle time.

Here we consider a single commodity deterministic production inventory model with a time dependent decreasing demand rate $D(p,I(t)) = \left[(a - p) + \rho I(t)\right]$. The production of the item is started at time 0, the production is stopped when the inventory level reaches Q_1 and the inventory is depleted at a rate $D(p,I(t))$. Shortages start of size Q_2 units upto time period t_3 and then the production is started at rate k to clear the backlog up to time T. Then the next production cycle starts at the rate of k.

THE MATHEMATICAL MODEL AND ANALYSIS:

Let $I(t)$ be the inventory at time t ($0 \le t \le T$) as shown in figure.

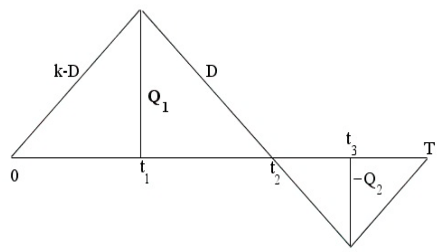

Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = k - \left[(a - p) + \rho I(t)\right], \quad 0 \le t \le t_1, \qquad (1)$$

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -\left[(a - p) + \rho I(t)\right], \quad t_1 \le t \le t_2, \qquad (2)$$

$$\frac{dI(t)}{dt} = -\left[a - p\right], \quad t_2 \le t \le t_3, \qquad (3)$$

$$\frac{dI(t)}{dt} = k - \left[a - p\right], \quad t_3 \le t \le T, \qquad (4)$$

with the boundary conditions at $I(0) = 0$, $I(t_1) = Q_1$, $I(t_2)=0$, $I(t_3)= - Q_2$ and $I(T) =0$.

The solutions of equations (1) tod (4) using boundary conditions are:

$$I(t) = (k - a + p)\left[t - \frac{\alpha\beta t^{\beta+1}}{\beta+1} - \frac{\rho t^2}{2}\right] \qquad (5)$$

$$I(t) = (a - p)\begin{bmatrix}(t_1 - t) + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - t^{\beta+1}\right) + \frac{\rho}{2}\left(t_1^2 - t^2\right) \\ - \alpha t^{\beta}(t_1 - t) - \rho t(t_1 - t)\end{bmatrix}, \qquad (6)$$
$$+ Q_1\left[1 - \alpha t^{\beta} - \rho t + \alpha t_1^{\beta} + \rho t_1\right]$$

$$I(t) = (a - p)(t_2 - t), \qquad (7)$$

$$I(t) = (k - a + p)(t - T). \qquad (8)$$

At $t= t_2$, $I(t_2) = 0$ and from equation (6), we get

$$Q_1 = -(a - p)\begin{bmatrix}(t_1 - t_2) + \frac{\alpha\left(t_1^{\beta+1} - t_2^{\beta+1}\right)}{\beta+1} + \frac{1}{2}\rho\left(t_1^2 - t_2^2\right) \\ - \alpha t_1^{\beta}(t_1 - t_2) - \rho t_1(t_1 - t_2)\end{bmatrix} \qquad (9)$$

putting $t =t_3$ and $I(t_3) = - Q_2$ in (7) we get the value of Q_2

$$Q_2 = (a - p)(t_3 - t_2). \qquad (10)$$

(by neglecting the higher power of α and ρ.)

The associated costs are:

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(i) Ordering cost (OC) = A (11)

(ii) Holding cost:

$$HC=\int_0^{t_1} h(t)I(t)\,dt+\int_{t_1}^{t_2} h(t)I(t)\,dt$$

$$=\int_0^{t_1}(x+yt)\,(k-a+p)\left[t-\frac{\alpha\beta t^{\beta+1}}{\beta+1}-\frac{\rho t^2}{2}\right]dt$$

$$+\int_{t_1}^{t_2}(x+yt)\left[\begin{array}{l}(a-p)\left[\begin{array}{l}(t_1-t)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-t^{\beta+1}\right)+\frac{\rho}{2}\left(t_1^2-t^2\right)\\ -\alpha t^{\beta}\left(t_1-t\right)-\rho t\left(t_1-t\right)\end{array}\right]\\ +Q_1\left(1-\alpha t^{\beta}-\rho t+\alpha t_1^{\beta}+\rho t_1\right)\end{array}\right]dt$$

$$=\frac{1}{2(\beta+1)(\beta+2)(\beta+3)}\left[\begin{array}{l}2\alpha(-k+a-p)t_1\beta\left(\begin{array}{l}(x+t_1y)\beta\\+2t_1y+3x\end{array}\right)t_1^{\beta+1}\\ -2\alpha\left(\begin{array}{l}(t_1-t_2)(x\beta+t_1y)^{2}\\+\left((-4t_1y-5x)t_2+2t_1^{2}y+3xt_1\right)\beta\\-6\left(x+\frac{1}{2}t_1y\right)t_2\end{array}\right)t_1(a-p)t_1\\ -4\alpha t_2^{2\beta}\left((x+yt_2)\beta+\frac{3}{2}yt_2+3x\right)(a-p)t_2\\ +(\beta+2)(\beta+3)\\ \left(\begin{array}{l}-2\alpha\left(\frac{1}{2}t_1y+\frac{1}{2}yt_2+x\right)t_2(a-p)(t_1-t_2)t_2^{\beta}\\ +(\beta+1)\left(\begin{array}{l}\frac{1}{12}y\rho(a-p)t_2^4+\frac{1}{3}(y+x\rho)(a-p)t_2^3\\-(a-p)\left(\rho t_1x-x+\frac{1}{2}y\rho t_1 t_1^2\right)t_2^2\\+\left(\frac{2}{3}\rho t_1^2 y x\rho-(y t_1-2x t_1 a)-\rho(t_1)\right)t_2\\-\frac{1}{3}\left(\frac{3}{4}\rho t_1 y t_1^2 2y+x\rho t_1-3x_1 t\gamma\right)t_1^2\end{array}\right)\end{array}\right)\end{array}\right]$$

(12)

(iii) Deterioration cost:

$$DC=c\left[\int_0^{t_1}\left(k-(a-p)\right)dt-\int_0^{T}(a-p)\,dt\right]$$

$$=c\left[(k-a+p)t_1-(a-p)T\right] \qquad (13)$$

(iv) Shortage cost:

$$SC=-c_2\left[\int_{t_2}^{t_3}I(t)\,dt+\int_{t_3}^{T}I(t)\,dt\right]$$

$$=-c_2\left[\begin{array}{l}\frac{1}{2}(p-a)\left(t_3^2-t_2^2\right)+(a-p)t_2\left(t_3-t_2\right)\\+\frac{1}{2}(k-a+p)\left(T^2-t_3^2\right)-(k-a+p)T\left(T-t_3\right)\end{array}\right]$$

(14)

The total profit per unit during a cycle $\pi(t_1,t_2,t_3,T,p)$ is consisted of the following:

$$\pi(t_1,t_2,t_3,T,p)=p(a-p)-\left(\frac{OC+HC+DC+SC}{T}\right)$$

$$=p(a-p)-\frac{1}{T}(A)$$

$$-\frac{1}{T}\left[\begin{array}{l}\frac{1}{2(\beta+1)(\beta+2)(\beta+3)}\\ \left[\begin{array}{l}2\alpha(-k+a-p)t_1\beta\left(\begin{array}{l}(x+t_1y)\beta\\+2t_1y+3x\end{array}\right)t_1^{\beta+1}\\ -2\alpha\left(\begin{array}{l}(t_1-t_2)(x\beta+t_1y)^{2}\\+\left((-4t_1y-5x)t_2+2t_1^{2}y+3xt_1\right)\beta\\-6\left(x+\frac{1}{2}t_1y\right)t_2\end{array}\right)t_1(a-p)t_1\\ -4\alpha t_2^2\left((x+yt_2)\beta+\frac{3}{2}yt_2+3x\right)(a-p)t_2^{\beta}\\ +(\beta+2)(\beta+3)\\ \left(\begin{array}{l}-2\alpha\left(\frac{1}{2}t_1y+\frac{1}{2}yt_2+x\right)t_2(a-p)(t_1-t_2)t_2^{\beta}\\ +(\beta+1)\left(\begin{array}{l}\frac{1}{12}y\rho(a-p)t_2^4+\frac{1}{3}(y+x\rho)(a-p)t_2^3\\-(a-p)\left(\rho t_1x-x+\frac{1}{2}y\rho t_1 t_1^2\right)t_2^2\\+\left(\frac{2}{3}\rho t_1^2 y x\rho-(y t_1-2x t_1 a)-\rho(t_1)\right)t_2\\-\frac{1}{3}\left(\frac{3}{4}\rho t_1 y t_1^2 2y+x\rho t_1-3x\right)t_1^2\end{array}\right)\end{array}\right)\end{array}\right]\end{array}\right]$$

$$-\frac{c}{T}\left[(k-a+p)t_1-(a-p)T\right]$$

$$+\frac{c_2}{T}\left[\begin{array}{l}\frac{1}{2}(p-a)\left(t_3^2-t_2^2\right)+(a-p)t_2\left(t_3-t_2\right)\\+\frac{1}{2}(k-a+p)\left(T^2-t_3^2\right)-(k-a+p)T\left(T-t_3\right)\end{array}\right].$$

(15)

Putting $t_1=vt_2$, $(0<v<1)$ in (15), the profit function becomes

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$$\pi_1(t_2,t_3,T,p) = p(a-p) - \frac{1}{T}(A)$$

$$-\frac{1}{T}\left[\frac{1}{2(\beta+1)(\beta+2)(\beta+3)}\left[\begin{array}{l} 2\alpha(-k+a-p)vt_2\beta\left((x+vt_2y)\beta+2t_1y+3x\right)v^{\beta+1}t_2^{\beta+1} \\ -2\alpha\left(\begin{array}{l}(vt_2-t_2)(x+\beta vt_2y)^2 \\ +\left(\begin{array}{l}(-4vt_2y-5x)vt_2 \\ +2v^2t_2^2y+3xvt_2\end{array}\right)\beta \\ -6\left(x+\frac{1}{2}vt_2y\right)t_2\end{array}\right)vt_2(a-p)v^{\beta}t_2^{\beta} \\ -4\alpha t_2^{2\beta}\left((x+yt_2)\beta+\frac{3}{2}yt_2+3x\right)(a-p)t_2 \\ +(\beta+1)(\beta+2)(\beta+3) \\ \left(\begin{array}{l}-2\alpha\left(\frac{1}{2}vt_2y+\frac{1}{2}yt_2+x\right)t_2(a-p)(vt_2-t_2)t_2^{\beta} \\ +\left(\begin{array}{l}\frac{1}{12}y\rho(a-p)t_2^4+\frac{1}{3}(y+x\rho)(a-p)t_2^3 \\ -(a-p)\rho\left(vt_2x-xt_2+\frac{1}{2}y\rho v^2t_2^2\right)t_2^2 \\ +\left(\frac{2}{3}\rho v^2t_2^2y\,x\rho-(y\,vt_2-)2x_2\right)vt_2(a-p)t_2 \\ -\frac{1}{3}\rho v\left(\frac{3}{4}ty+^2{}_2^2\,2y(+x\rho\,vt-)3x_2\,v\,t\,y\right)^2{}_2^2\end{array}\right)\end{array}\right)\end{array}\right]\right]$$

$$-\frac{c}{T}\left[(k-a+p)vt_2-(a-p)T\right]$$

$$+\frac{c_2}{T}\left[\begin{array}{l}\frac{1}{2}(p-a)(t_3^2-t_2^2)+(a-p)t_2(t_3-t_2) \\ +\frac{1}{2}(k-a+p)(T^2-t_3^2)-(k-a+p)T(T-t_3)\end{array}\right] \quad (16)$$

The optimum values of t_2*, t_3*, T* and p* which maximize the profit function $\pi(t_2,t_3,T,p)$ are the solutions of the equations

$$\frac{\partial\pi}{\partial t_2}=0,\ \frac{\partial\pi}{\partial t_3}=0,\ \frac{\partial\pi}{\partial T}=0,\ \frac{\partial\pi}{\partial p}=0, \quad (17)$$

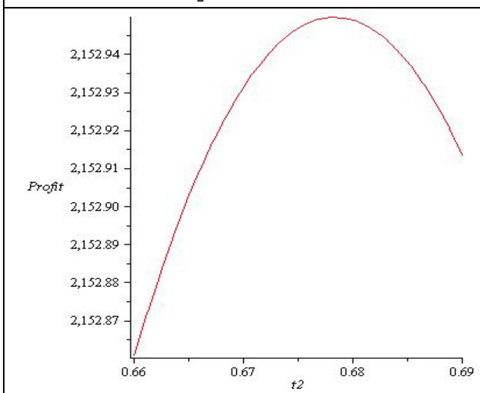
provided that these values of t_2*, t_3*, T* and p* satisfy the conditions $\pi > 0$, where π is the Hessian determinant is given by

$$\pi = \begin{vmatrix} \frac{\partial^2\pi}{\partial^2 t_2} & \frac{\partial^2\pi}{\partial t_2\partial t_3} & \frac{\partial^2\pi}{\partial t_2\partial T} & \frac{\partial^2\pi}{\partial t_2\partial p} \\ & \frac{\partial^2\pi}{\partial^2 t_3} & \frac{\partial^2\pi}{\partial t_3\partial T} & \frac{\partial^2\pi}{\partial t_3\partial p} \\ & & \frac{\partial^2\pi}{\partial^2 T} & \frac{\partial^2\pi}{\partial T\partial p} \\ & & & \frac{\partial^2\pi}{\partial^2 p} \end{vmatrix} > 0. \quad (18)$$

NUMERICAL EXAMPLES:

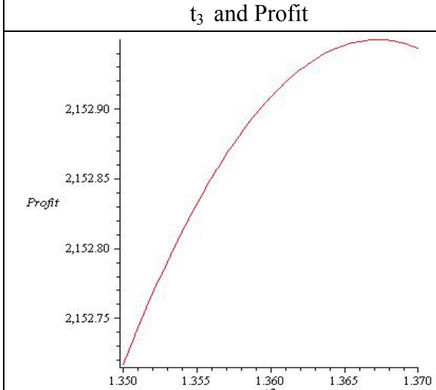
Considering A = 200, a = 100, b = 0.05, k= 300, α = 0.9, β=2, c= 25, ρ = 0.1, x=5, y=0.05, c_2 = 8 and v = 0.25 in appropriate units. Then we obtained the optimal value of t_1* = 0.2713, t_2* = 0.6783, t_3*=1.3672, T*=1.5258, p* = 43.8634 and the optimal total profit π*= Rs. 2152.9498. The second order condition given in equation (18) is also satisfied. The graphical representation of the concavity of the profit function is also given.

t_2 and Profit



Graph 1

t_3 and Profit



Graph 2

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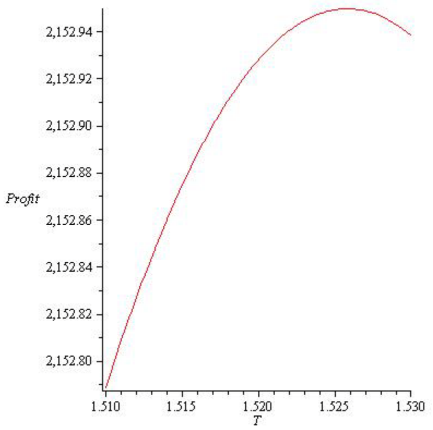
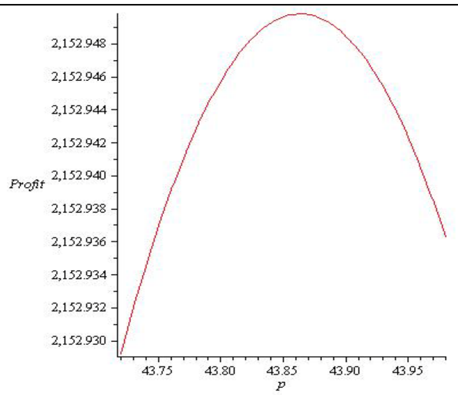
T and Profit



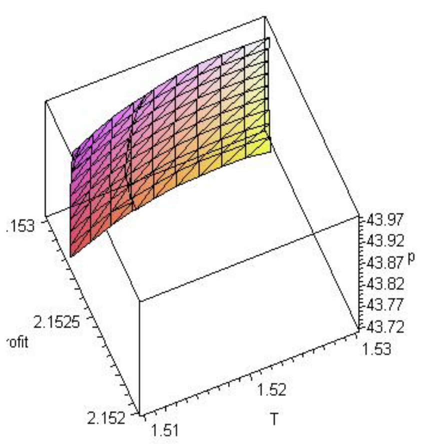
Graph 3

p and Profit



Graph 4

P, T and Profit



Graph 5

SENSITIVITY ANALYSIS:

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Sensitivity Analysis

Para-meter	%	t_2	t_3	T	p	Profit
α	+15%	0.663	1.358	1.517	43.989	2152.520
	+10%	0.668	1.361	1.520	43.948	2152.658
	-10%	0.689	1.374	1.532	43.773	2153.255
	-15%	0.695	1.377	1.535	43.725	2153.420
ρ	+15%	0.599	1.322	1.485	44.557	2150.065
	+10%	0.677	1.366	1.525	43.871	2152.915
	-10%	0.679	1.367	1.526	43.855	2152.983
	-15%	0.679	1.368	1.526	43.852	2153.001

From the table we observe that as parameter α increases/ decreases average total profit decreases/ increases. Similarly for change in parameter ρ, there is slight change in total profit.

For remaining parameters the range for the time interval is not satisfied.

CONCLUSION:

In the present paper, we have developed a production inventory model for deteriorating items with demand as a function of price and quantity. Holding cost is linear function of time. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding decrease/ increase in the value of profit.

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