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Wgrα-I-Closed Sets in Ideal Topological Spaces

KEYWORDS: wgrα-I-closed sets, wgrα-I-open sets.

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ABSTRACT *In this paper, we introduce and study the properties of wgrα-I-closed sets in ideal topological spaces. Their relationships with other existing generalized closed sets in topological and ideal topological spaces are established.*

1. Introduction

In 1990, Jankovic and Hamlett investigated the applications of topological ideals[5].In 1999,Dontchev et al. studied the notion of generalized closed sets in ideal topological spaces called I_g-closed sets [3]. Navaneethakrishnan and joseph [10] further investigated and characterized I_g-closed sets and I_g-open sets by the use of local functions. In this paper, we define and characterize wgrα-I-closed sets and wgrα-I-open sets.

2. Preliminaries

An ideal I on a topological space (X,τ) is non-empty collection of subsets of X which satisfies the following properties. (1) A∈I and B⊆A implies B∈I, (2) A∈I and B∈I implies A∪B∈I. An ideal topological spaces is a topological space(X,τ) with an ideal I on X and is denoted by (X,τ,I).For a subset A⊆X,$A^*(I,\tau)$={x∈X : A∩U∉I for every U∈τ(X,x) is called the local function of A with respect to I and τ } [7]. We simply write A^* in case there is no chance for confusion. A kuratowski closure operator $cl^*(I,\tau)$ called the *-topology, finer than τ is defined by $cl^*(A) = A\cup A^*$ [14]. If A⊆X, cl(A), int(A) will respectively, denote the closure and interior of A in (X,τ).

Definition 2.1.

A subset A of a space (X,τ) is called

1. regular open [13] if A= int(cl(A)).
2. regular α-open [13] if there is a regular open set U such that U⊂A⊂αcl(U).
3. α-open[4] if A⊆int(cl(int(A)).
4. semi-open[13] if A⊆cl(int(A)).

Definition: 2.2

A subset A of (X,τ) is said to be

1. g-closed [9], if cl(A)⊆U, whenever A⊆U and U is open in (X,τ).
2. wgrα-closed[6],if cl(int(A))⊆U, whenever A⊆U and U is regular α-open in (X,τ).
3. ω-closed[13], if cl(A)⊆U, whenever A⊆U and U is semi-open in (X,τ).
4. rgα-closed[13], if αcl(A)⊆U, whenever A⊆U and U is regular α-open in (X,τ).
5. swg-closed[2], if cl(int(A))⊆U, whenever A⊆U and U is semi-open in (X,τ).

Definition: 2.3

A subset A of (X,τ,I) is said to be

1. α-I-closed [1], if $cl(int^*(cl(A)))$⊆A.
3. I_{rg}-closed [11], if A^*⊆U, whenever A⊆U and U is regular-open in (X,τ).
4. $I_{\hat{\omega}}$-closed [8], if A^*⊆U, whenever A⊆U and U is $\hat{\omega}$-open in (X,τ).
5. *-closed[5],if A^*⊆A.
6. I-open[5], if A⊆$int(A^*)$.
7. I-R closed[1],if A=$cl^*(int(A))$.
8. rps-I-closed[12], if spIcl(A)⊆U, whenever A⊆U and U is a rg-I-open in (X,τ).

3. wgrα-I-closed sets.

Definition: 3.1

A subset A of an ideal space (X,τ,I) is said to be wgrα-I-closed if cl*(int(A))⊆U whenever A⊆U and U is regular α-open.

Definition: 3.2

A subset A of an ideal space (X,τ,I) is said to be wgrα-I-open if X–A is wgrα-I-closed.

Theorem: 3.3

1. Every closed set is wgrα-I-closed.
2. Every α-closed set is wgrα-I-closed.

3. Every ω-closed set is wgrα-I-closed.

4. Every rgα-closed set is wgrα-I-closed.

5. Every swg-closed set is wgrα-I-closed.

6. Every wgrα-closed set is wgrα-I-closed.

7. Every *-closed set is wgrα-I-closed.

8. Every α-I-closed set is wgrα-I-closed.

Proof

Straight forward.

Remark: 3.4

Converse of the above theorem need not be true as shown in the following examples.

Example: 3.5

Let X = {a, b, c, d},τ ={φ, X,{a},{b},{a ,b},{b, c},{a, b, c}}, I = {φ,{a}},then τ^c ={ φ, X,{d}, {a ,d}, {c, d},{a, c ,d},{b, c, d},τ*-open ={φ,X,{a},{b}, {a,b}, {b,c},{a,b,c},{b,c,d}} and τ*-closed = {φ,X,{a},{d},{a,d},{c,d},{b,c,d}, {a,c ,d}}. wgrα-I-closed = { φ, X,{a},{c},{d}, {a,b},{a,c}, {a,d}, {b,d}, {c,d}, {a,b,c}, {a,b,d},{a,c,d},{b,c,d}}.

(i).{a} is wgrα-I-closed, but not closed.

(ii).{a,b} is wgrα-I-closed, but not α-closed.

(iii).{c} is wgrα-I-closed, but not ω-closed.

(iv).{a} is wgrα-I-closed, but not rgα-closed.

(v).{a ,c} is wgrα-I-closed, but not swg-closed.

(vi).{a} is wgrα-I-closed, but not wgrα-closed.

(vii).{b,d} is wgrα-I-closed, but not *-closed.

Example: 3.6

Let X={a,b,c,d},τ = {ϕ, X ,{a},{c,d},{a,c,d}}, τ^c = {ϕ,X,{b},{a,b},b,c,d} }, I= {φ,{a}}, τ*-open={ϕ,X, {a},{c,d},{a,c,d},{b,c,d}},τ*-closed= {ϕ,X,{a} ,{b}, {a,b},{b,c,d}}and wgrα-I-closed= {ϕ,X,{a},{b}, {c},{d},{a,b}, {a,c},{a,d}, {b,c}, {b,d},{a,b,c},{a,b,d}, {a,c,d},{b,c,d}}.Here {a,b,c} is wgrα-I-closed ,but not α-I-closed.

Remark:3.7

The concepts semi-closed, I_ω closed, rps-I-closed and wgrα-I-closed are independent.

Example: 3.8

Let X = {a, b, c, d},τ ={φ, X,{a},{b},{a ,b},{b, c},{a, b, c}}, I = {φ,{a}},then τ^c = { φ,X,{d}, {a ,d}, {c, d},{a, c ,d},{b, c, d},τ*-open = { φ, X,{a},{b},{a, b},{b , c}, {a, b, c}, {b, c, d}} and τ*-closed = { φ,X,{a},{d},{a ,d},{c,d},{b,c,d},{a, c ,d}}. wgrα-I-closed = { φ,X, {a}, {c}, {d}, {a,b},{a,c},{a,d}, {b,d},{c,d}, {a,b,c} , {a,b,d}, {a,c,d},{b,c,d}}.

(i).{b} is semi-closed, but not wgrα-I-closed.{a,b} is wgrα-I-closed, but not semi-closed.

(ii).{b,c} is I_ω-closed, but not wgrα-I-closed. {c} is wgrα-I-closed, but not I_ω-closed.

(iii).{b,c} is rps-I-closed, but not wgrα-I-closed.{a,b,c} is wgrα-I-closed, but not rps-I-closed.

Remark:3.9

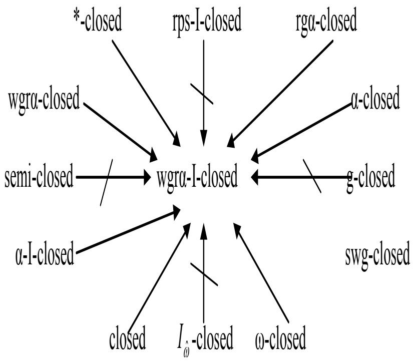
The concepts wgrα-I-closed and g-closed are independent of each other.

Example:3.10

Let X={a,b,c,d},τ = {ϕ, X ,{a},{c,d},{a,c,d}}, τ^c = {ϕ,X,{b},{a,b},{b,c,d}}, I={φ,{a}}, τ*-open ={ϕ,X, {a},{c,d},{a,c,d},{b,c,d}}, τ*-closed ={ϕ,X,{a},{b}, {a,b},{b,c,d}}and wgrα-I-closed= {ϕ,X,{a},{b},{c},{d},{a,b}, {a,c},{a,d},{b,c}, {b,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}.{a} is wgrα-I-closed, but not g-closed. {c,d} is g-closed, but not wgrα-I-closed.

Remark:3.11

From theorem:3.3, remark :3.7 and remark:3.9,following diagram holds.



Remark:3.12

Union of two wgrα-I-closed set is not wgrα-I-closed.

Example:3.13

Let X={a,b,c,d},τ = {ϕ,X,{a},{c,d},{a,c,d}}and I={ϕ,{a}}.{c} and {d} are wgrα-I-closed, but {c,d} is not wgrα-I-closed.

Remark:3.14

Intersection of two wgrα-I-closed set is not wgrα-I-closed.

Example:3.15

Let X={a,b,c,d},τ = {ϕ,X,{a},{b},{a,b},{b,c},{a,b,c}}and I={ϕ,{a}}.{a,b} and {b,d} are wgrα-I-closed, but {b} is not wgrα-I-closed.

Theorem: 3.16

Let (X, τ, I) be an ideal space and A⊆ X. If A is wgrα-I-closed, then cl*(int(A)) – A contains no non-empty regular α-open set.

Proof

Let A be a wgrα-I-closed set in X and U be a regular-α-open subset of cl*(int(A)) – A. Then A ⊆ X–U and X–U is regular α-open. Since A is wgrα-I-closed, cl*(int(A)) ⊆ X–U .Which implies that U ⊆ X–cl*(int(A)).Thus U⊆(cl*(int(A))∩(X–cl*(int(A))=φ. Hence cl*(int(A))–A contains no non-empty regular α-open set.

Theorem: 3.17

Let (X, τ, I) be an ideal space and A⊆ X. If A is wgrα-I-closed, then cl*(int(A)) – A contains no non-empty regular α-closed set.

Proof Follows from theorem: 3.16.

Theorem: 3.18

Let (X, τ, I) be an ideal space and A⊆ X. If A is wgrα-I-closed, then cl*(int(A)) – A contains no non-empty regular open set.

Proof Follows from theorem: 3.16 and the fact that every regular open set is regular α-open.

Theorem: 3.19

Let (X, τ, I) be an ideal space and A⊆X. If A is wgrα-I-closed, then (intA)* – A contains no non-empty regular α-open set.

Proof

Let A be a wgrα-I-closed set in X. Suppose that U is a regular α-open set such that cl*(int(A)) ⊆ X –U, which implies that (int(A))*⊆ X–U, thus ,(int(A))* – A contains no non-empty regular α-open set.

Theorem: 3.20

Let A be a wgrα-I-closed set of an ideal topological space X. Then the following are equivalent.

(i) A is I-R-closed.

(ii) cl*(int(A)) – A is a regular-α-closed set.

(iii) (int(A))* –A is a regular-α-closed set.

Proof

(i)⇒(ii) Let A be I-R-closed.We have cl*(int(A))=A, then cl*(int(A)) – A = φ Thus, cl*(int(A)) – A is a regular-α-closed set.

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(ii)⇒(iii) Let cl*(int(A)) − A be regular-α-closed.Cl*(int(A))− A=(int(A))* − A. Therefore (int(A))* − A is a regular-α-closed set.

(iii)⇒(i) Let (int(A))* − A be a regular-α-closed set, cl*(int(A))−A= (int(A))*− A =φ.Thus cl*(int(A))=A. Hence A is I-R-closed.

Theorem: 3.21

Let (X, τ, I) be an ideal space and A⊆ X. If A is wgrα-I-closed, and whenever A⊆ U and U is a regular-open set in X , then A is weakly I_{rg}-closed set.

Proof

Let A be a wgrα-I-closed set, we have, cl*(int(A))⊆U. Then (int(A))*⊆U. By hypothesis, A is weakly I_{rg}-closed set.

Theorem: 3.22

Let (X, τ, I) be an ideal space and A⊆ X. If A is regular-open and wgrα-I-closed, then A is *-closed set.

Proof

Let A⊆A and A be regular open .Since A is wgrα-I-closed in X, cl*(int(A))⊆A, which implies that, cl*(A)=cl*(int(A)⊆A. Therefore A is *-closed set in X.

Theorem: 3.23

Let (X, τ, I) be an ideal space .Then either {x} is regular closed (or) X−{x} is wgrα-I-closed for every x∊X.

Proof

Suppose {x} is not regular-closed, then X−{x} is not regular-open and the only regular-open set containing X −{x} is X and cl*(int(X−{x}))⊆X. Hence X−{x} is wgrα-I-closed set in X.

Theorem: 3.24

Let (X, τ, I) be an ideal space, A is regular-open and A⊆ X. Then the following properties are equivalent.

(i) A is *-closed.

(ii) A is I-R-closed.

(iii) A is wgrα-I-closed

Proof

(i)⇒(ii)Let A be *-closed and regular-open, cl*(int(A))=cl*(A)=A. Thus, A is I-R-closed.

(ii)⇒(iii)Let A⊆A and A be regular open. Since A is I-R-closed and every regular-open set is regular-α-open, cl*(int(A))⊆A. Thus A is wgrα-I-closed.

(iii)⇒(i) follows from theorem 3.22.

Theorem: 3.25

Let A be a wgrα-I-closed set in an ideal space X such that A⊆B⊆cl*(int(A)), then B is also an wgrα-I-closed set.

Proof

Let U be an regular α-open set of X, such that B⊆U. Then A⊆B⊆U. Since A is wgrα-I-closed, cl*(int(A))⊆U. Now cl*(int(B))⊆cl*(int(cl*(int(A)))) =cl*(int(A))⊆U. Therefore B is wgrα-I-closed.

Theorem: 3.26

Let A be a wgrα-I-closed set in an ideal space X. Then A∪(X−cl*(int(A))) is wgrα-I-closed if and only if (int(A))* −A is wgrα-I-open.

Proof

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Let (int(A))*− A be wgrα-I-open in X ⇔ X−((int(A))*−A) is wgrα-I-closed.

X−((int(A))*−A) ⇔ X∩((int(A))*∩A^C)^C

⇔ A∪(X−cl*(int(A))).

Hence the proof.

Theorem: 3.27

Let (X, τ, I) be an ideal space and A⊆X. Th ¤ A is wgrα-I-open if and only if U⊆int*(cl(A)),whenever U is regular α-open and U⊆A.

Proof

Let A be wgrα-I-open and U be a regular α-open set in X contained in A.X−U is also regular α-open set containing X−A and X−A is wgrα-I-closed,we have cl*(int(X−A))⊆X−U.Which implies that X−int*(cl (A))⊆X−U.Thus U⊆int*(cl(A)).

Conversely, U⊆int*(cl(A)),whenever U is regular α-open and U⊆A. We have X−A⊆ U. Then X−U⊆A and so X−U⊆int*(cl(A)).Therefore cl*(int(X−A))⊆U. Hence A is wgrα-open.

Theorem: 3.28

Let (X, τ, I) be an ideal space and A⊆ X. If A is wgrα-I-closed, then cl*(int(A)) − A is a wgrα-I-open set in X.

Proof

Let A be a wgrα-I-closed set in X. Suppose that U is a regular-α-open set such that U⊆cl*(int(A))−A. Since A is wgrα-I-closed, it follows from theorem 3.16 that U=φ. Thus, we have U ⊆ int*(cl(cl*(int(A))−A)),by theorem 3.27,cl*(int(A))−A is wgrα-I-open in X.

Theorem: 3.29

Let A be a wgrα-I-open set in an ideal space X and A⊆X such that int*(cl(A))⊆B⊆A, then B is also an wgrα-I-open.

Proof

Since A is wgrα-I-open, then X−A is wgrα-I-closed. cl*(int(X−A))−(X−A) contains no non-empty regular α-open set. Since int*(cl(A))⊆int*(cl(B)),we have cl*(int(X−B))⊆cl*(int(X−A)),which implies that cl*(int(X−B))−(X−B)⊆cl*(int(X−A))−(X−A).Thus B is wgrα-open.

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