



## Study of Superdeformed Bands in $^{81}\text{Sr}$ by Using Two-Parameter Formulae

### KEYWORDS

Superdeformed bands, Transition energies, Kinematic moment of inertia, RTEOS.

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**ABSTRACT** Superdeformed (SD) bands in  $^{81}\text{Sr}$  have been studied by using two-parameter phenomenological formulae, namely *ab* formula and power expression. Transition energies calculations have been performed on two SD bands of  $^{81}\text{Sr}$  and theoretical results are compared with the available experimental data. Kinematic moment of inertia have also been extracted for these SD bands and are plotted against rotational frequencies. Finally, the applicability of the two phenomenological models to the SD bands has been adjudged from the plots of ratio of transition energy over spin (RTEOS) against spin and their comparison with the available experimental data. An overall satisfactory agreement with the observed results was shown by the theoretical calculations.

### 1. Introduction

Superdeformation is one of the interesting and challenging topics in nuclear structure physics. Superdeformed (SD) bands were first observed in fission isomers in the actinide region [1]. Vast experimental and theoretical studies have been undertaken for the superdeformed nuclei over the past two decades. Experimental investigations have revealed many SD bands in various mass regions  $A=30, 60, 80, 130, 150$  and  $190$  [2-3]. SD bands display many interesting features which differ from normal deformed (ND) bands. A superdeformed nucleus is highly deformed with axes ratios around 2:1:1 and thus it acquires ellipsoidal shape. An SD nucleus possesses collective rotation because of higher degree of deformation and is therefore, a good quantum rotor. The rotational energy spectrum of SD nucleus is characterized by evenly spaced highly collective electric quadrupole (E2) transitions. Another amazing feature of SD bands is that the transition energies and dynamic moment of inertia of different SD cascades are nearly equal. It is called identical band phenomenon [4].

Despite numerous attempts, the searches for SD shapes in the lighter region ( $A \sim 80$ ) have been hampered for many years primarily because of experimental difficulties. SD bands have now been established in  $^{80-83}\text{Sr}$ ,  $^{82-84}\text{Y}$ ,  $^{83,84}\text{Zr}$  and  $^{86}\text{Zr}$  [5-9]. Multiple bands are known in many of these nuclei, and most of these bands have been shown to be highly deformed by lifetime measurements. Most of the SD bands in this region have been interpreted as prolate shaped due to their highly collective nature and large moments of inertia. A detailed understanding of the structure of SD bands in this mass region is still emerging. The excitation energy and the spin-parity of most of the SD bands are not known. Several empirical formulae with two or more parameters were formulated to characterize the SD bands. The transition energies, spins and identical band phenomenon for SD bands in the mass 150 and 190 regions have been predicted by various parameterizations [10-14]. In the present work, study of the evolution of the SD bands of  $^{81}\text{Sr}$  nucleus has been undertaken by employing variable moment of inertia models using three types of two-parameter formulae namely the *ab* formula and the power law formula. The two phenomenological models have been used to calculate the transition energies and kinematic moment of inertia for the two SD bands of  $^{81}\text{Sr}$  and compared it with the experimental data. Finally, the ratio of transition energy over spin has been plotted against spin to verify the applicability of the two models to the SD bands. The present paper has been organised as follows. Section 2 of the paper presents a brief theory of the empirical methods used in the present work. In section 3, a comparative study of the theoretical and experimental results on transition ener-

gies and kinematic moment of inertia is presented. Finally, the paper is summarised in Section 4.

### 2. Phenomenological Formulae

Holmberg and Lipas [15] have formulated a simple two-parameter *ab* formula to fit transition energies of SD bands.

$$E = a \left( \sqrt{1 + bI(I+1)} - 1 \right) \quad (1)$$

C.S. Wu et al. [16] have also derived theoretically this type of phenomenological formula for rotational spectrum of well-deformed nucleus with axial symmetry. The centrifugal stretching effect is taken into account by this *ab* formula as the two parameters include the variable moment of inertia effect. The transition energies of SD bands have been reproduced in many nuclei by applying this empirical formula [17-18].

Since the exact level spins and energies are not known for SD bands, transition energy from spin *I* to spin *I*-2 can be obtained from *ab* formula as

$$E(I) = E(I) - E(I-2) = a \left( \sqrt{1 + bI(I+1)} - \sqrt{1 + b(I-1)(I-2)} \right) \quad (2)$$

The observed transition energies of SD band can be used to fit the equation (3) and thus, the value of the parameters are easily determined and transition energy spectrum can be calculated.

Gupta et al. [19] replaced the concept of the arithmetic mean of the two terms used in the Bohr-Mottelson expression by the geometric mean and introduced a two-parameter formula called the power expression

$$E = aI^b \quad (3)$$

The index *b* is the measure of the deformation of the nuclear core, being 1.0 for a spherical nucleus and about 2.0 for a deformed rotor. The coefficient *a* plays the role of inverse of moment of inertia. The parameters *a*, *b* can be determined by fitting the E2 transitions of SD band.

$$E(I) = E(I) - E(I-2) = a \left( I^b - (I-2)^b \right) \quad (4)$$

From the nuclear transition energy spectra, kinematic moment of inertia  $\mathfrak{I}^{(1)}$  for SD bands can be extracted by employing the equation.

$$\mathfrak{S}^{(I)}(I) = \frac{(2I+1)\hbar^2}{E_\gamma(I \rightarrow I-2)} \quad (5)$$

Its value is nearly constant with nuclear spin for SD bands. In order to check the reliability of the phenomenological expressions to SD bands, the transition energies over spins (RTEOS) i.e.  $\frac{E_\gamma(I)}{I-\frac{1}{2}}$  are also calculated.

**3. Results and Discussion**

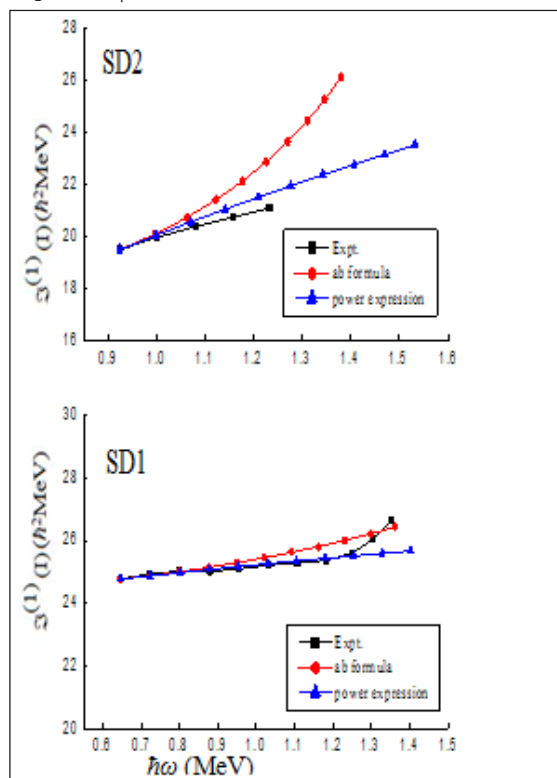
In this work, study of two SD bands of <sup>81</sup>Sr, viz., SD1 and SD2 has been undertaken. The E(31/2→27/2) and E<sub>γ</sub>(35/2→31/2) transitions have been fitted to determine the values of the parameters of both ab formula and power expression for SD1 band. The experimental data for transition energies and the spin assignment of levels has been adopted from ref [2]. The transition energies for both these SD bands are calculated up to spin 75/2h. For SD2 band, experimental data for transition energies is available only upto spin 55/2h but the theoretical values for this band have been obtained upto spin 75/2h. For this SD band, the values of parameters for both the phenomenological have been determined by fitting the experimental transition energies for E<sub>γ</sub>(35/2→31/2) and E<sub>γ</sub>(39/2→35/2) transitions. A comparison of the theoretical energies so obtained for SD1 and SD2 bands by using the two phenomenological formulae with the corresponding experimental values has been presented in Table 1.

**Table 1. Comparative study of theoretical and experimental results on transition energies of SD1 and SD2 bands.**

SD1				SD2			
Spin (I)	E <sub>γ</sub> (I) (MeV)			Spin (I)	E <sub>γ</sub> (I) (MeV)		
	Expt.	power	ab		Expt.	Power	ab
31/2	1.214	1.214	1.214	35/2	35/2	1.773	1.773
35/2	1.370	1.370	1.370	39/2	39/2	1.924	1.924
39/2	1.518	1.525	1.524	43/2	43/2	2.083	2.070
43/2	1.678	1.678	1.676	47/2	47/2	2.238	2.213
47/2	1.839	1.831	1.825	51/2	51/2	2.395	2.352
51/2	1.986	1.983	1.971	55/2	55/2	2.537	2.487
55/2	2.138	2.134	2.115	59/2	59/2	-	2.620
59/2	2.292	2.284	2.256	63/2	63/2	-	2.750
63/2	2.439	2.434	2.394	67/2	67/2	-	2.877
67/2	2.562	2.583	2.528	71/2	71/2	-	3.003
71/2	2.658	2.732	2.660	75/2	75/2	-	3.1261
75/2	2.748	2.879	2.788	-	-	-	-

It is clear from Table 1 the experimental transition energies for SD1 band increase as the spin increases. Similar trend is displayed by the transition energies obtained theoretically by using the power expression and ab formula. Theoretical values of the transition energies for SD1 band show satisfactory agreement with the corresponding experimental ones except for higher spins. It may further be noted from Table 1 that the transition energies obtained theoretically agree reasonably with the available experimental results. Although the deviations from experimental data are more at higher spins yet the agreement is quite good.

The variation of kinematic moment of inertia with angular frequency for the two SD bands of the <sup>81</sup>Sr nucleus has been studied by using the ab formula as well as power expression. The theoretical results so obtained have been compared with those computed from experimental data. Figure 1 illustrates that for both the SD bands, theoretical results show satisfactory agreement with experimental data except at higher angular frequency. However, the overall agreement is excellent for power expression and quite reasonable for ab formula. For SD2 band, the theoretical values exceed the experimental ones in higher spin region. It is worth to mention here that the parameters for both the empirical formulae were derived for the lowest spins, therefore, discrepancy results at higher angular frequencies.



**Figure 1. Plots showing variation of Kinematic moment of Inertia with angular frequency.**

Experimental transition energies over spin (RTEOS) versus spin for SD1 and SD2 bands of <sup>81</sup>Sr are shown in Figure 1. Plots of RTEOS against spin obtained from theoretical models have also been presented in the same figure for the sake of comparison.

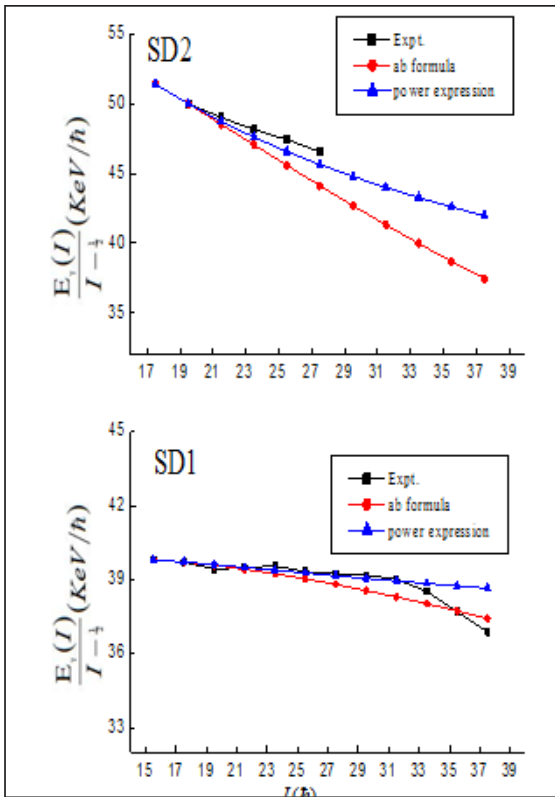


Figure 2. Plots of RTEOS versus spin.

Figure 2 shows that the theoretical values of RTEOS obtained by using power expression are in reasonably good agreement with experimental results except for higher spins. Similar results have been obtained by the ab formula for these bands except that the deviation from experimental plot is more as compared to power expression. For SD2 band, deviation from experimental results is more for the ab formula than power expression in higher spin domain in all the SD bands, however, the overall agreement is quite satisfactory. The discrepancy between experimental and theoretical results at higher spins arises owing to the fact that the values of the parameters have been determined for the transition energies closer to the band head.

#### 4. Conclusions

Superdeformed bands in  $^{81}\text{Sr}$  nucleus have been studied within the context of two different phenomenological approaches. The nuclear properties like transition energies, variation of kinematic moment of inertia with rotational frequency and ratios of transition energy over spin have been obtained for SD1 and SD2 bands and compared with the corresponding experimental results. An overall satisfactory agreement obtained for both the theoretical approaches shows the reliability of these methods in explaining SD bands. The deviation of theoretical results from experimental ones at higher spin domain could be removed by fitting the whole transition energy spectra into the empirical formulae.

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