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Magnetohydrodynamic Oscillatory Flow of Viscoelastic Stratified Fluid Through Porous Medium Between Parallel Vertical Plates

KEYWORDS: Stratified flow, Similarity transformation, Magnetohydrodynamics, Permeability parameter, Brunt – Vaisala frequency

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ABSTRACT *Magnetohydrodynamic viscous oscillatory flow of stratified fluid through porous medium placed between two vertical parallel plates with injection through one plate and suction through another plate is considered in this paper. Similarity transformation is applied to transform the partial differential equation into an ordinary differential equation and solved. The results obtained for transverse velocity and axial velocity are prsented through graph using MATLAB. The outcome of the investigation reveals that stratification as well as electromagnetic induction inhibit fluid motion in vertical direction.*

INTRODUCTION

The study of flow of an electrically conducting fluid has many applications in engineering problems such as magnetohydrodynamics (MHD) generators, plasma studies, nuclear reactors, geothermal energy extraction, and the boundary layer control in the field of aerodynamics. Fluid motion influenced by the density and viscosity variations in the fluid is characterized as stratified flow. The applications of stratified fluid in the field of ocean and atmosphere necessitate the study of fluid flow with density stratification and viscosity. Berman [1] was the first researcher who studied the problem of steady flow of an incompressible viscous fluid through a porous channel with rectangular cross-section, when the Reynolds number is low. He obtained a perturbation solution assuming normal wall velocities to be equal. Then Sellars [2] extended the problem studied by Berman when the Reynolds number is very high. Afterwards Yuan [3] and Terill [4] studied the problem for various values of suction and injection Reynolds numbers. Terill and Shrestha [5] have analysed the same problem, assuming different normal velocities at the walls. Drake [6] has considered the flow of an incompressible viscous fluid in a long channel of rectangular section due to a periodic pressure gradient. Bagchi [7] has studied the unsteady flow of visco-elastic Maxwell fluid with transient pressure gradient through a rectangular channel. Forced Oscillation in an inviscid stratified fluid have been considered by many authors Krishna & Sharma [8], Hendershott [9], Rao and Rao [10], Sharma and Naidu [11], K.B Naidu [12] and PrasannaVenkatesh [13]. The flow of a viscous stratified fluid with variable viscosity past a porous bed was studied by Channabasappa and Ranganna [14] to identify a technique for studying the pore size in a porous medium. Unsteady stratified flow has been considered by Gupta and Goyal [15] using Laplace transform technique. Kumar et al. [16] have discussed the problems of viscous stratified MHD flow along an infinite flat plate. Das et al. [17] have studied the effect of heat source and magnetic field on a viscous stratified fluid past a porous moving plate. Khandelwal and Jain [18] have considered unsteady MHD heat transfer problems of stratified fluid. In this paper we consider the magnetohydrodynamic oscillatory flow of a viscoelastic stratified fluid between two vertical porous plates through porous medium, induced by suction and injection at each of the porous walls within the frame work of Boussinesq approximation for density. Analytical solution for the problem is obtained by introducing stream function and similarity transformation.

MATHEMATICAL FORMULATION

We consider viscous oscillatory fully developed flow through porous medium betweeen two vertical parallel porous plates. One of the plates is placed on y – axis and the other parallel to it and at a distance of 'h'. The fluid is viscoelastic and stratified with vertical layering of density. In the unperturbed state, it is assumed to be linearly distributed while in perturbed state it is a function of x, y and t. Electromagnetic induction is applied in both axial as well as transerse direction. The walls are of variable permeability. Fluid is injected periodically with a velocity of $u_1 e^{i\omega t}$ through the plate placed on y – axis and suction takes place with the velocity $u_2 e^{i\omega t}$ through the other parallel plate. The mathematical model of the problem is given by

Equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Equation of Incompressibility for stratified incompressible flow

$$\frac{d\rho}{dt}\equiv\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+v\frac{\partial \rho}{\partial y}=0 \qquad (2)$$

Equation of Motion

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial x^2}-\sigma_e B_0{}^2 u-\frac{\mu}{\kappa}u \qquad (3)$$

$$\rho\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y}+\mu\frac{\partial^2 v}{\partial x^2}-\mu_1\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right)-\sigma_e B_0{}^2 v-\frac{\mu}{\kappa}v-\rho g \qquad (4)$$

Here μ and μ_1 represents the coefficient of viscosity and viscoelasticity, ρ the density of the fluid. The density distribution in the undisturbed state is taken as

$$\rho=\rho_0(y)+\rho'(x,y,t) \qquad (5)$$

$$\rho_0(y)=\rho_0'(1-\beta y) \qquad (6)$$

ρ_0' is a constant, density, $\rho_0(y)$ is linearly distributed, $\rho'(y,t)$ is perturbation density, stratification parameter (a constant) so that Brunt – Vaisala frequency N becomes $N^2=\beta g$

From (2), (5) and (6) we get (neglecting second order terms)

$$\frac{\partial \rho'}{\partial t}=\rho_0'\beta v \qquad (7)$$

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We use Bousenesq's approximation in the inertial terms which means

$$\rho_0'\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2} - \sigma_e B_0^{\,2}\, u - \frac{\mu}{\kappa}u \quad (8)$$

$$\rho_0'\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu\frac{\partial^2 v}{\partial x^2} - \mu_1\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) - \sigma_e B_0^{\,2}\, v - \frac{\mu}{\kappa}v - \rho' g \quad (9)$$

Differentiating (8) and (9) with respect to t

$$\rho_0'\frac{\partial^2 u}{\partial t^2} = -\frac{\partial^2 p}{\partial t \partial x} + \mu\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}\right) - \left(\sigma_e B_0^{\,2} + \frac{\mu}{\kappa}\right)\frac{\partial u}{\partial t} \quad (10)$$

$$\rho_0'\frac{\partial^2 v}{\partial t^2} = -\frac{\partial^2 p}{\partial t \partial y} + \mu\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) - \mu_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 v}{\partial x^2}\right) - \left(\sigma_e B_0^{\,2} + \frac{\mu}{\kappa}\right)\frac{\partial v}{\partial t} - g\frac{\partial \rho'}{\partial t} \quad (11)$$

Using (7) in (11)

$$\rho_0'\frac{\partial^2 v}{\partial t^2} = -\frac{\partial^2 p}{\partial t \partial y} + \mu\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2}\right) - \mu_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 v}{\partial x^2}\right) - \left(\sigma_e B_0^{\,2} + \frac{\mu}{\kappa}\right)\frac{\partial v}{\partial t} - \rho_0'\beta g v \quad (12)$$

The boundary conditions of the problem are

$u(0, y) = u_1$, $u(h, y) = u_2$

$v(0, y) = 0$, $v(h, y) = 0$ (13)

Eliminating pressure from equation (10) and (12) we have

$$\rho_0'\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = \mu\frac{\partial}{\partial t}\frac{\partial^2}{\partial x^2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) - \left(\sigma_e B_0^{\,2} + \frac{\mu}{\kappa}\right)\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + \mu_1\frac{\partial^2}{\partial t^2}\left(\frac{\partial^3 v}{\partial x^3}\right) + \beta g\, \rho_0'\frac{\partial v}{\partial x} \quad (14)$$

Put $\psi(x,y,t) = \Psi(x,y)\, e^{i\omega t}$,

$u = u(x,y)\, e^{i\omega t}$, $v = v(x, y)e^{i\omega t}$, and $p = p(x,y)e^{i\omega t}$

And we define Stream Function ψ and introducing $f(\eta)$ as follows

$$u = \frac{\partial \psi}{\partial y} \quad and \quad v = -\frac{\partial \psi}{\partial x} \quad (15)$$

$$\Psi = h\left(\frac{v_0}{a} + \frac{u_2 y}{h}\right) f(\eta) \quad (16)$$

Where $\eta = \frac{x}{h}$, $a = 1 - \frac{u_1}{u_2}$, $0 \le u_1 \le u_2$ and

v_o is the average entrance velocity.

Substituting the above in equation (15)

$$\left[D^4 - h^2\left(\frac{\mu i\omega\left(\sigma_e B_0^{\,2} + \frac{\mu}{\kappa}\right) + \rho_0'(N^2 - \omega^2)}{\mu i\omega + \mu_1\omega^2}\right) D^2\right] f(\eta) = 0 \quad (17)$$

where $D^2 = \frac{d^2}{d\eta^2}$.

Therefore the General Solution of equation (22) is given by

$$f(\eta) = c_1 + c_2\eta + c_3 e^{\alpha h\eta} + c_4 e^{-\alpha h\eta} \quad (18)$$

The Boundary Conditions are transformed in terms of $f(\eta)$ are as follows

$f(0) = 1 - a$, $f(1) = 1$ and $f'(0) = f'(1) = 0$ (19)

The result obtained by applying the boundary conditions on $f(\eta)$, are

$$f(0) = 1 - a = c_1 + c_3 + c_4 \quad (20)$$

$$f(1) = 1 = c_1 + c_2 + c_3\, e^{\alpha h} + c_4\, e^{-\alpha h} \quad (21)$$

$$f'(0) = 0 = c_2 + \alpha h\, c_3 - \alpha h\, c_4 \quad (22)$$

$$f'(1) = 0 = c_2 + \alpha h c_3 e^{\alpha h} - \alpha h c_4 e^{-\alpha h} \quad (23)$$

Solving the above equations, the values of the constants are as follows.

$$c_1 = 1 - a - \frac{a(e^{\alpha h} + e^{-\alpha h} - 2)}{4 + e^{\alpha h}(\alpha h - 2) - e^{-\alpha h}(2 + \alpha h)} \quad (24)$$

$$c_2 = \frac{\alpha h a(e^{\alpha h} - e^{-\alpha h})}{4 + e^{\alpha h}(\alpha h - 2) - e^{-\alpha h}(\alpha h + 2)} \quad (25)$$

$$c_3 = \frac{a(e^{-\alpha h} - 1)}{4 + e^{\alpha h}(\alpha h - 2) - e^{-\alpha h}(\alpha h + 2)} \quad (26)$$

$$c_4 = \frac{a(e^{\alpha h} - 1)}{4 + e^{\alpha h}(\alpha h - 2) - e^{-\alpha h}(\alpha h + 2)} \quad (27)$$

$$u = \left(\frac{v_0}{a} - \frac{u_2 y}{h}\right)\frac{\left(e^{\alpha h} - e^{-\alpha(x-h)} - e^{-\alpha h} + e^{\alpha(x-h)} + e^{-\alpha x} - e^{\alpha x}\right)}{4 + e^{\alpha h}(\alpha h - 2) - e^{-\alpha h}(\alpha h + 2)}\alpha h a e^{i\omega t} \quad (28)$$

$$v = v_2 e^{i\omega t}\left[\left\{\frac{(1-a)(4 + e^{\alpha h}(\alpha h - 2) - e^{-\alpha h}(\alpha h + 2) - a(e^{\alpha h} + e^{-\alpha h} - 2)}{4 + e^{\alpha h}(\alpha h - 2) - e^{-\alpha h}(\alpha h + 2)}\right\} + \left\{\frac{a\alpha(e^{\alpha h} - e^{-\alpha h})x + a(e^{\alpha(x-h)} + e^{-\alpha(x-h)}) - a(e^{\alpha x} + e^{-\alpha x})}{4 + e^{\alpha h}(\alpha h - 2) - e^{-\alpha h}(\alpha h + 2)}\right\}\right] \quad (29)$$

From equation (5), (6) and (7) we get the density distribution as follows

RESULTS AND DISCUSSIONS

For all figures the range of values of x and y is assumed to be 0 to 1. Figure 1 depicts the transverse velocity profile for different values of ωt varying from 0 to π. As time increases transverse velocity decreases. The velocity profile is observed to be parabolic in nature and for ωt = π/2, the transverse velocity is non linear. Figure 2 explains that the transverse velocity increases with increase in height. Figure 3 represents the effect of stratification on velocity in vertical direction along the flow. It is very clear from the graph that as the stratification increases the velocity also increases which is observed to be due to the suction velocity and the gravitational pull acting along the flow direction. Figure 4 shows that the electromagnetic induction decreases the velocity in flow direction. Similar effect is observed from Figure 5 by permeability parameter. Figure 6 represents axial velocity at different time. Axial velocity decreases with increase in time. The fact that the increase in electromagnetic induction results in decrease in fluid flow velocity in axial direction is

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clearly shown by Figure 7. Figure 8 and Figure 9 represents effects of stratification and permeability on axial velocity. It can be easily seen that stratification inhibits flow in axial direction where as permeability exhibits flow in the same direction. Figure 10 show that as viscoelasticity increases the transverse velocity decreases. For N=0 the results reduces to that of S.Ganesh [10] and for B0 = 0 it reduces to that of L.Prasanna Venkatesh [11]. For both N=0, B0 = 0 and for constant density the paper reduces to viscous oscillatory flow between parallel plates and the results thus obtained add one more class of exact solution to S.Ganesh [13], Wang [14] and to that of a few presently available in literature.

CONCLUSIONS

In this paper magnetohydrodynamic viscous oscillatory flow of stratified fluid through porous medium due to injection through one plate and suction through another plate is considered. The governing partial differential equations are solved by introducing stream function and the results were presented graphically. The pictorial representations of the solution signify the following conclusions.

- As time increases velocity in both axial as well as transverse direction decreases.
- Transverse velocity increases with height and stratification whereas it decreases with electromagnetic parameter and permeability parameter.
- Axial velocity decreases with stratification and electromagnetic induction.
- Stratification inhibits flow in axial direction while permeability exhibits the flow.
- Viscoelastic viscosity inhibits motion of the fluid in vertical direction.

Figure 1: Transverse Velocity for different values of ωt

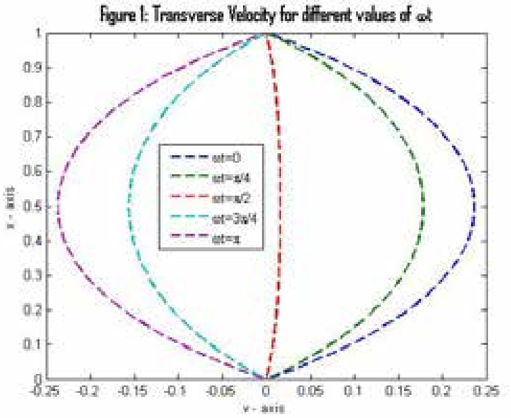


Figure 2: Transverse Velocity for different y values

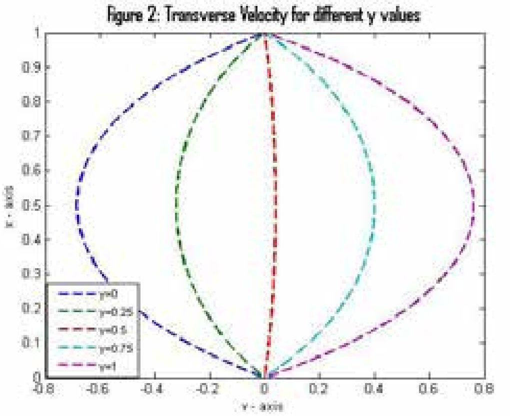


Figure 3: Transverse Velocity for different values of N

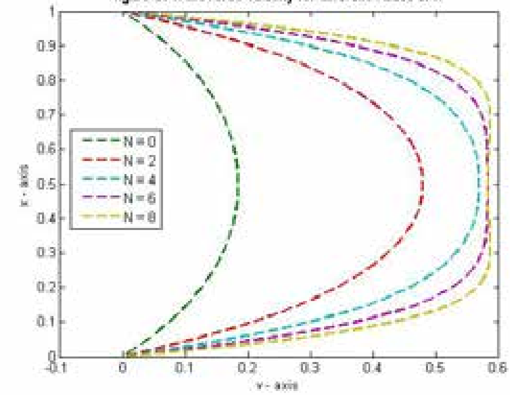


Figure 4: Transverse Velocity for different values of B0

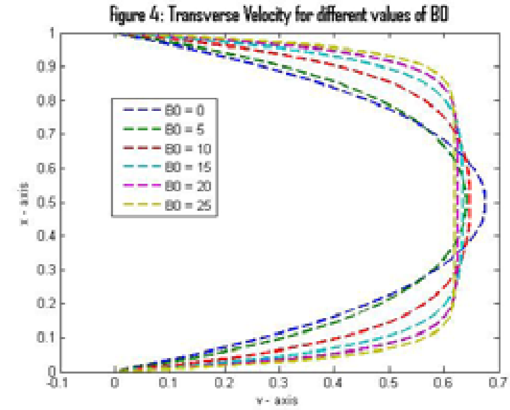


Figure 5: Transverse Velocity for different permeability values

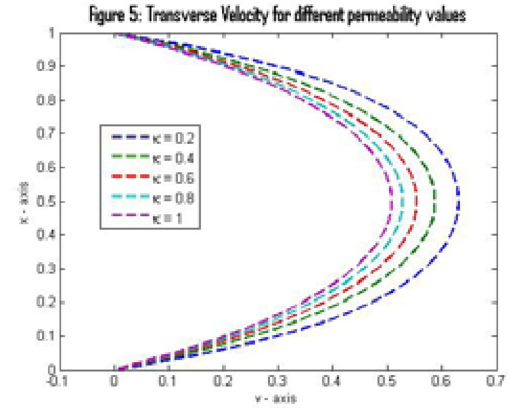
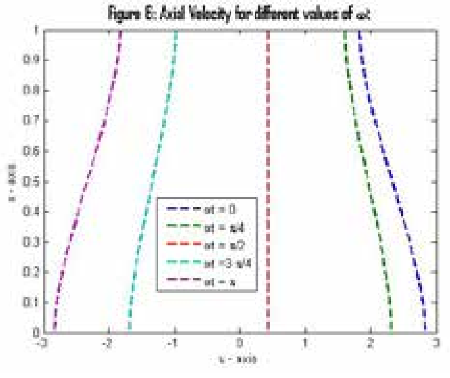


Figure 6: Axial Velocity for different values of ωt



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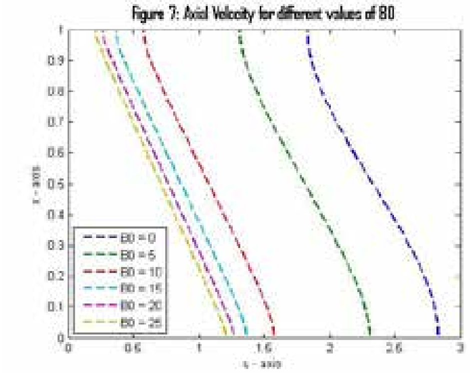

Figure 7: Axial Velocity for different values of B0

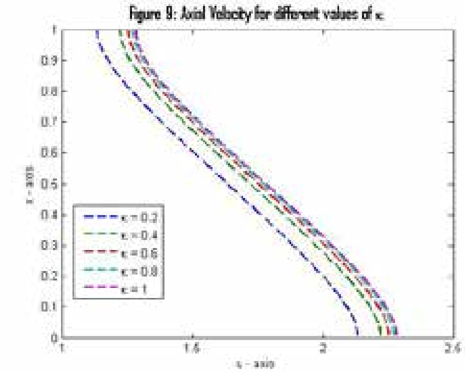

Figure 9: Axial Velocity for different values of ε

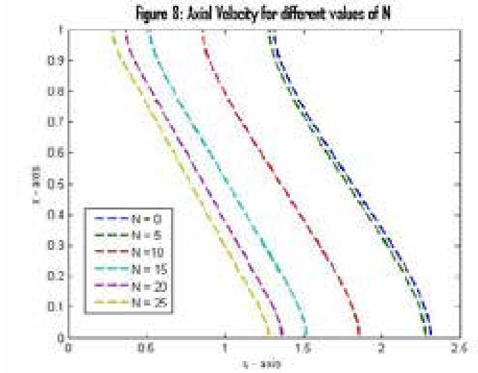

Figure 8: Axial Velocity for different values of N

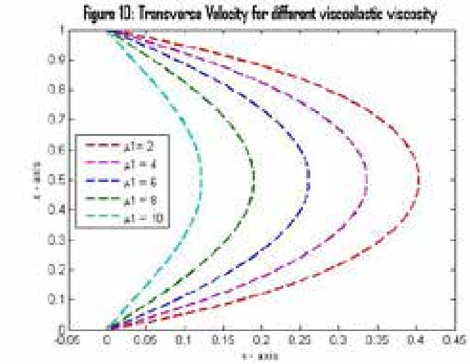

Figure 10: Transverse Velocity for different viscoelastic viscosity

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