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A Study on Comparison Between Fuzzy Shortest Path With Conventional Method

KEYWORDS: fuzzy set theory, Fuzzy shortest Trapezoidal fuzzy number Fuzzy shortest path problem.

M.Ananthanarayanan
Asst Prof P.G Department of Mathematics A.M JAIN College Chennai (India)

V. Pusparaj
Research Scholar A.M JAIN college Meenambakkam Chennai (India)

ABSTRACT *In this paper Algorithm is presented to perform shortest path method in a fuzzy environment. The trapezoidal fuzzy number given by decision makers or characterized by historical data are utilized to assess the activity time in a project network. The fuzzy shortest path problem thus obtained is compared with conventional method.*

1. Introduction

The shortest path problem deals on finding the path with minimum distance. To find the shortest path from a source node to the other nodes is a fundamental matter in graph theory. In conventional shortest path problems, it is assumed that decision maker is certain about the parameters (distance, time etc.) between different nodes. But in real life situations, there always exists uncertainty about the parameters between different nodes. In such cases, the parameters are represented by fuzzy numbers (Zadeh, 1965).

Klein (1991) presented new models based on fuzzy shortest paths and also gave a general algorithm based on dynamic programming to solve the new models. Lin &Chern (1993) considered the case that the arc lengths are fuzzy numbers and proposed an algorithm for finding the single most vital arc in a network. Okada & Gen (1994) discussed the problem of finding the shortest paths from a fixed origin to a specified node in a network with arcs represented as intervals on real line. Li et al. (1996) introduced the neural networks for solving fuzzy shortest path problems. Gent et al. (1997) investigated the possibility of using genetic algorithms to solve shortest path problems. Shih & Lee (1991) investigated multiple objective and multiple hierarchies minimum cost flow problems with fuzzy costs and fuzzy capacities in the arcs. Okada &Soper (2000) concentrated on a shortest path problem on a network in which a fuzzy number, instead of a real number, is assigned to each arc length. Okada (2001) concentrated on a shortest path problem on a network in which a fuzzy number, instead of a real number, is assigned to each arc length and introduced the concept of "degree of possibility" in which an arc is on the shortest path. Liu & Kao (2004) investigated the network flow problems in that the arc lengths of the network are fuzzy numbers. Seda (2005) dealt with the steiner tree problem on a graph in which a fuzzy number, instead of a real number, is assigned to each edge.

Takahashi &Yamakami (2005) discussed the shortest path problem with fuzzy parameters. He proposed a modification Okada's algorithm (2001), using some properties observed by other authors. He also proposed a genetic algorithm to seek an approximated solution for large scale problems. Chuang & Kung (2005) represented each arc length as a triangular fuzzy set and a new algorithm is proposed to deal with the fuzzy shortest path problems. Nayeem& Pal (2005) considered a network with its arc lengths as imprecise number, instead of a real number, namely, interval number and triangular fuzzy number. Ma & Chen (2005) proposed an algorithm for the on-line fuzzy shortest path problems, based on the traditional shortest path problem in the domain of the operations research and the theory of the on-line algorithms. Kung & Chuang (2005) proposed a new algorithm composed of fuzzy shortest path length procedure and similarity measure to deal with the fuzzy shortest path problem. Gupta & Pal (2006) presented an algorithm for the shortest path problem when the connected arcs in a transportation network are represented as interval numbers.

Moazeni (2006) discussed the shortest path problem from a specified node to every other node on a network in which a positive fuzzy quantity with finite support is assigned to each arc as its arc length. Chuang & Kung (2006) pointed out that there are several methods reported to solve this kind of problem in the open literature. In these methods, they can obtain either the fuzzy shortest length or the shortest path. In their paper, a new algorithm was proposed that can obtain both of them. The discrete fuzzy shortest length method is proposed to find the fuzzy shortest length, and the fuzzy similarity measure is utilized to get the shortest path. Ji et al. (2007) considered the shortest path problem with fuzzy arc lengths. According to different decision criteria, the concepts of expected shortest path, a-shortest path and the shortest path in fuzzy environment are originally proposed, and three types of models are formulated. In order to solve these models, a hybrid intelligent algorithm integrating simulation and genetic algorithm is provided and some numerous examples are given to illustrate its effectiveness.

In existing shortest path problems, it is assumed that decision maker is certain about the parameters (distance, time etc.) between different nodes. But in real life situations, there always exists uncertainty about the parameters between different nodes.

In such cases, the parameters are represented by fuzzy numbers. In this paper the shortcomings of the existing algorithm (Liu & Kao, 2004) are pointed out and to overcome these shortcomings a new algorithm is proposed. By using the proposed algorithm a decision maker can obtain both the fuzzy shortest path and fuzzy shortest distance of each node from source node.

2. Shortcomings of the existing algorithm

In this section existing algorithm Liu & Kao (2004) arepointed out.

By using existing algorithm, to find the fuzzy shortest path and fuzzy shortest distance of each node from source node it is required to formulate and solve the problem several times which is very difficult and time consuming in case of a large network. For example: Let applying the existing algorithm on a network having seven nodes the fuzzy shortest path be 1®2→5→7 then to find the fuzzy shortest path and fuzzy shortest distance of nodes 3, 4 and 6 from source node (say 1) it is required to formulate and solve the problem again and again.

3. fuzzy set theory

In this section some basic definitions, arithmetic operations and ranking function arereviewed.

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3.1. Basic definitions

In this subsection, some basic definitions are reviewed.

Definition 3.1 The characteristic function μ A of a crisp set A⊆ X assigns a valueeither0 or 1 to each member in X . This function can be generalized to a function Amsuch that the value assigned to the element of the universal set X fall within a specified range [0,1] i.e. : [0,1] μA : X ®[0,1] . The assigned values indicate the membership grade of the element in the set A .

The function $\mu_A$is called the membership function and the set `A= {(x,μ_A(x))} defined by $\mu_A$for each xÎXis called a fuzzy set.

Definition 3.2 A fuzzy number A = (a,b,c) is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{(x-a)}{(b-a)}, & a\le x\le b\\ 1 & x=b\\ \dfrac{(x-c)}{(b-c)}, & b\le x\le c\end{cases}$$

where $a,b,c\in R$

Definition 3.3 A fuzzy number ~A= (a b c d)is called a trapezoidal fuzzy number if itsmembership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{(x-a)}{(b-a)}, & a\le x\le b\\ 1 & b\le x\le c\\ \dfrac{(x-d)}{(c-d)}, & c\le x\le d\end{cases}$$

where, $a,b,c,d\in R$

3.2. Arithmetic operations between two triangular fuzzy numbers and trapezoidal fuzzy numbers In this subsection, the arithmetic operations, required for the proposed algorithm, are reviewed (Kaufmann & Gupta, 1985).

Let A=(a_1,b_1,c_1) and B=(a_2,b_2,c_2) be two triangular fuzzy number then

(1) A⊕B = (a_1 + a_2, b_1 + b_2, c_1+c_2)
(2) A ⊖B = (a_1-c_2,b_1-b_2,c_1-a_2)

Let A=(a_1,b_1,c_1,d_1) and B=(a_2,b_2,c_2,d_2) be two trapezoidal fuzzy number then

(1) A⊕B = (a1 + a2, b1 + b2, c1+c2, d1 + d2)
(2) A ⊖B = (a1-d2,b1-c2,c1-b2, d1-a2)

3.3. Ranking function

A convenient method for comparing of fuzzy number is by use of ranking function (Liou, & Wang, 1992).

A ranking function ℜ : F(R) → R, where F(R) is set of all fuzzy numbers defined on set of real numbers, maps each fuzzy number into a real number.

For given two triangular or trapezoidal fuzzy numbers we have:

A >B ifℜ (A) >ℜ(B)
ℜ
A <B ifℜ (A) <ℜ(B)
ℜ
A = B ifℜ (A) =ℜ(B)
ℜ

For a triangular fuzzy number A = (a,b,c) ℜ is given by ℜ (A) =1/4(a+2b+c);

For a trapezoidal fuzzy number A = (a,b,c,d) ℜ is given by ℜ (A) =1/4(a+b+c+d).

3.4. Notations

The notations that will be used throughout the paper are as follow:

N ={1,2,☹,n} : The set of all nodes in a network.

Nd(j) : The set of all predecessor nodes of node j .

đi: The fuzzy distance between node i and first (source) node.

đij The fuzzy distance between node i and j .

Remark:3.4.A node i is said to be predecessor node of node j if
(i) Node i is directly connected to node j .
(ii) The direction of path, connecting node i and j , is from i to j .

4. Proposed algorithm

In this section a new algorithm is proposed for finding the fuzzy shortest path and fuzzy shortest distance of each node from source node.

The steps of the algorithm are summarized as follow:

Step 1 Assumeđ$_1$=(0,0,0,0)and label the source node (say node 1) as [(0,0,0,0), -] .

Step 2 Find d$_{j=}$minimum{ đ$_j$Åđij /i€ Nđ (j)};j≠1, j=2,3,,,,

Step 3 If minimum occurs corresponding to unique value of i i.e., i = r then label nodej as [đ$_{j,}$ r] If minimum occurs corresponding to more than one values of i then it represents that there are more than one fuzzy path between source node and node j but fuzzy distance along all paths is đj so choose any value of i .

Step 4 Let the destination node (node n) be labeled as [,đ$_j$,r] then the fuzzy shortestdistance between source node and destination node is đ$_n$

Step 5 Since destination node is labeled as [đ$_n$,l] So, to find the fuzzy shortest pathbetween source node and destination node, check the label of node l . Let it be[đ$_j$,p]now check the label of node p and so on. Repeat the same procedure until node 1 is obtained.

Step 6 Now the fuzzy shortest path can be obtained by combining all the nodes obtained by the step 5.

Remark 4.1Let A; i=1,2,.. n ,~ = ☺ be a set of fuzzy numbers. If R (A)<R (A)for all i ,then the fuzzy number A_k is the minimum of A i n i ; 1,2, ,..n

5.Description of the model:

Trapezoidal fuzzy numbers are converted into triangular fuzzy numbers and component wise triangular fuzzy numbers are treated as a time between nodes and fuzzy shortest path is calculated by using conventional method algorithms

Conventional method;

Algorithm2
Step1; Construct the network diagram according to Fulkerson rule

Step2: select the component wise fuzzy number (a,b,c)in that order treated as a time between nodes We obtained three stages of shortest path problem.

step3: find the number of possible ways (path from initial vertex to end vertex)

step4: from the possible ways select the minimum value that is required shortest path

5.1 method of converting trapezoidal fuzzy number into triangular fuzzy numbers

Step1: if (a,b,c,d)is the given trapezoidal fuzzy numbers

Step2: taking average of (b+c)/2=b$^{/}$ (I.e average of the sec-

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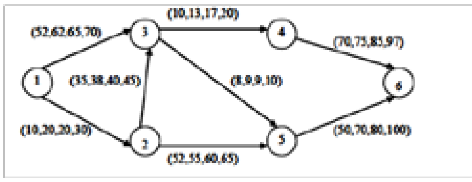
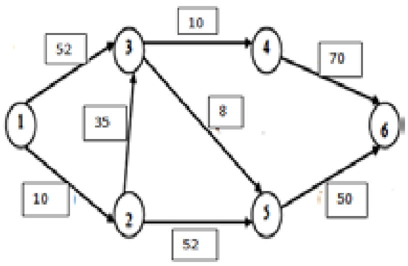
ond and third number) the resulting number substituted in that position. That gives the triangular fuzzy number(a,b/,c)

6.Numerical example1

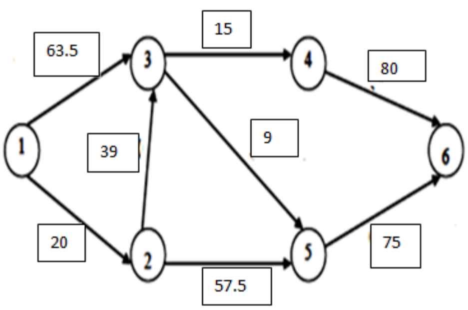
:The problem is to find the shortest path between source node (say node 1) and destination node (say node 6) (Liu & Kao, 2004) on the network with fuzzy distances shown in Figure 1



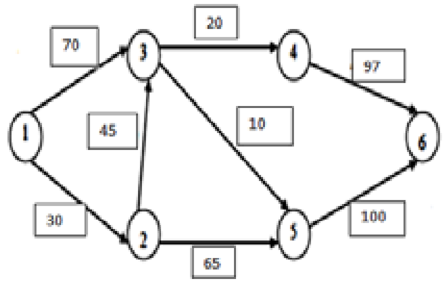
Stage1: component wise the first number applying the conventional method algorithm we obtained he following shortest path 1→2→3→5→6.



Stage2: component wise the second fuzzy number applying the conventional method algorithm we obtained he following shortest path 1→2→3→5→6.



Stage3: component wise the third fuzzy number applying the conventional method algorithm we obtained he following shortest path 1→2→3→5→6.



Conclusion

In the fuzzy shortest path method the shortest path is 1→2→3→5→6. in conventional method we find

the shortest path in three stages we obtain he same unique solution hence he execution of algoritham2 is best one for the implementation by the algoriham1.

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