RESEARCH PAPER | Mathematics | Volume : 4 | Issue : 5 | May 2014 | ISSN - 2249-555X

Further New Skolem Difference Mean Graphs

KEYWORDS: Path, Star, Skolem difference mean labeling

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ABSTRACT *A graph G =(V,E) with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices x ϵ V with distinct elements f (x) from {1,2,…,p+q} in such a way that the edge e =uv is labeled with |f(u)-f(v) |/2 if |f(u)-f(v) | is even and (|f(u)-f(v) |+1)/2 if |f(u)-f(v) | is odd and the resulting labels of the edges are distinct and are from{1,2,…,q}. A graph that admits skolem difference mean labeling is called a skolem difference mean graph. In this paper, some new classes of graphs are defined and their skolem difference mean labeling is studied.*

1. INTRODUCTION

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary[1].The symbols V(G) and E(G) denote respectively the vertex set and edge set of a graph. Labeled graphs serve as useful models for a broad range of applications.

Many studies in graph labeling refer to Rosa's research in 1967[5]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (edges) then the labeling is called a vertex (edge) labeling. There are several types of graph labeling and a detailed survey is found in [2].

The concept of skolem difference mean labeling was introduced in [3] and some new skolem difference mean graphs were studied in [4].In this paper, some other new skolem difference mean graphs are studied.

The following definitions are necessary for the present investigations.

1.1 Definition: A path of length n in a graph G is a sequence of distinct vertices $\{v_0, v_1, \dots, v_n\}$ where $e_i = v_i v_{i+1}$ for $i = 0,1,\dots,n-1$. A path on n vertices is denoted by P_n.

1.2 Definition: The complete bipartite graph $K_{1,n}$ or $K_{n,1}$ is called a star.

1.3 Definition: A graph $G = (V, E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $\{1,2,\dots,p+q\}$ in such a way that the edge $e = uv$ is labeled with $\frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting labels of the edges are distinct and are $1,2,\dots,q$. A graph that admits skolem difference mean labeling is called a skolem difference mean graph.

2. RESULTS

In this section, some new graphs are introduced and their skolem difference mean labeling is studied

2.1 Definition: Consider two copies of a graph G (wheel, star, fan & friendship) namely G_1 and G_2. Then the graph G' $= < G_1 \blacktriangle G_2 >$ is the graph obtained by joining the apex vertices of G_1 and G_2 by an edge as well as to a new vertex v'.

2.2 Theorem: The graph $< K_{1,n} \blacktriangle K_{1,m} >$ is skolem difference mean for all $n, m \geq 1$.

Proof: Let G be the graph $< K_{1,n} \blacktriangle K_{1,m} >$
Let $V(G) = \{u, u_i, v, v_j, w; 1 \leq i \leq n, 1 \leq j \leq m\}$
$E(G) = \{uu_i, vv_j, uv, uw, vw; 1 \leq i \leq n, 1 \leq j \leq m\}$. Then $|V(G)| = |E(G)| = n + m + 3$.
Let $f: V(G) \rightarrow \{1,2,\dots,2n+2m+6\}$ be defined as follows.

Case (i) when $n = m$
$f(u) = 1$; $f(u_i) = 2i; 1 \leq i \leq n$
$f(v) = 2n + 2m + 6$;
$f(v_j) = 2j + 1; 1 \leq j \leq$ m and
$f(w) = 2n + 2$
Let f^* be the induced edge labeling of f. Then we have
$f^*(uu_i) = i; 1 \leq i \leq n$
$f^*(uv) = m + n + 3$
$f^*(vv_j) = m + n + 3 - j; 1 \leq j \leq m$
$f^*(uw) = n + 1$
$f^*(vw) = n + 2$

Case (ii) when $n < m$
$f(u) = 1$
$f(u_i) = 2i; 1 \leq i \leq n$
$f(v) = 2n + 2m + 6$
$f(v_j) = 2j + 1; 1 \leq j \leq$ n
$\quad = 2j + 3; n + 1 \leq j \leq$ m

$f(w) = 2n + 2$

Let f^* be the induced edge labeling of *f*. Then we have

$f^*(uu_i) = i; 1 \leq i \leq n$

$f^*(uv) = m + n + 3$

$f^*(vv_j) = m + n + 3 - j; 1 \leq j \leq n$

$= m + n + 2 - j; n + 1 \leq j \leq \mathrm{m}$

$f^*(uw) = n + 1$

$f^*(vw) = n + 2$

Case (*iii*) when $n > m$

$f(u) = 1$

$f(u_i) = 2i; 1 \leq i \leq m$

$= 2i + 2; m + 1 \leq i \leq \mathrm{n}$

$f(v) = 2n + 2m + 6$

$f(v_j) = 2j + 1; 1 \leq j \leq \mathrm{m}$

$f(w) = 2m + 2$

Let f^* be the induced edge labeling of *f*. Then we have

$f^*(uu_i) = i; 1 \leq i \leq m$

$= 1 + i; m + 1 \leq i \leq n$

$f^*(uv) = m + n + 3$

$f^*(vv_j) = m + n + 3 - j; 1 \leq j \leq m$

$f^*(uw) = m + 1$

$f^*(vw) = n + 2$

In all the cases, the induced edge labels are $1,2, \dots, n + m + 3$ which are distinct. Hence the theorem. ■

2.3 Definition [7]: Consider m copies of the star$K_{1,n}$.Then G $= \left[< K_{1,n}^{(1)} : K_{1,n}^{(2)} : \dots : K_{1,n}^{(m)} >\right]$ is the graph obtained by joining the apex vertices of the stars ${K_{1,n}}^{(p-1)}$ and ${K_{1,n}}^{(p)}$ to a new vertex x_{p-1} where $1 \leq p \leq m$.

2.4 Theorem: $\left[< K_{1,n}^{(1)} : K_{1,n}^{(2)} : \dots : K_{1,n}^{(m)} >\right]$ is skolem difference mean for all $n, m \geq 1$.

Proof: Let $G = \left[< K_{1,n}^{(1)} : K_{1,n}^{(2)} : \dots : K_{1,n}^{(m)} >\right]$

Let $V(G) = \{v_i, v_{ij}, x_k; 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq m - 1\}$and

E(G) $= \{v_i v_{ij}; 1 \leq i \leq m, 1 \leq j \leq n,$ $v_k x_k; x_k v_{k+1}; 1 \leq k \leq m - 1\}$

Then $|V(G)| = m(n + 2) - 1$ and $|E(G)| = m(n + 2) - 2$

Let $f: V(G) \to \{1,2, \dots, 2m(n + 2) - 3\}$ be defined as follows.

Case(*i*) When *m* is odd

$f(v_i) = 2i - 1; 1 \leq i \leq m$

$f(x_k) = 2m(n + 2) - 1 - 2k; 1 \leq k \leq m - 1$

$f(v_{2i+1\,j}) = 2mn + 3 - 4(n - 1)i - 2j; 0 \leq i < \frac{m+1}{2}; 1 \leq j \leq n$

$f(v_{2i\,j}) = 2mn - 2n + 4 - 4(n - 1)(i - 1) - 2j; 1 \leq i < \frac{m+1}{2}; 1 \leq j \leq n$

Case(*ii*) When *m* is even

$f(v_i) = 2i - 1; 1 \leq i \leq m$

$f(x_k) = 2m(n + 2) - 1 - 2k; 1 \leq k \leq m - 1$

$f(v_{2i+1\,j}) = 2mn + 3 - 4(n - 1)i - 2j; 0 \leq i < \frac{m}{2}; 1 \leq j \leq n$

$f(v_{2i\,j}) = 2mn - 2n + 4 - 4(n - 1)(i - 1) - 2j; 1 \leq i \leq \frac{m}{2}; 1 \leq j \leq n$

In both the cases, let f^* be the induced edge labeling of *f*. Then

$f^*(v_k x_k) = m(n + 2) - 2k; 1 \leq k \leq m - 1$

$f^*(x_k v_{k+1}) = m(n + 2) - 1 - 2k; 1 \leq k \leq m - 1$

$f^*(v_i v_{ij}) = mn + 1 - n(i - 1) - j; 1 \leq i \leq m; 1 \leq j \leq n$

Then the induced edge labels are distinct and are $1,2, \dots, m(n + 2) - 2$. Hence the theorem.

■

2.5 Theorem: The graph$\left[< K_{1,n}^{(1)} : K_{1,m}^{(1)} >\right] \odot K_1$ is skolem difference mean for all *n*, $m \geq 1$.

Proof: Let G be the given graph.Let $V(G) = \{v, u, w, v_i, u_j, x, y, z, v_{ii}, u_{jj}; 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E(G) = \{wv, wu, vv_i, uu_j, wx, vy, uz, v_i v_{ii}, u_j u_{jj}; 1 \leq i \leq n, 1 \leq j \leq m\}$.Then $|V(G)| = 2n + 2m + 6$ and $|E(G)| = 2n + 2m + 5$

Let f: $V(G) \to \{1,2,\dots,4n+4m+11\}$be defined as follows.
$f(y) = 4n+4m+11$
$f(z) = 4n+4m+10$
$f(w) = 4n+4m+7$; $f(v) = 1$
$f(u) = 3$; $f(x) = 4n+4m+6$
$f(v_i) = 4n+4m+5-2i; 1 \le i \le n$
$f^*(wv) = 2n+2m+3$
$f^*(wu) = 2n+2m+2$; $f^*(wx) = 1$
$f^*(vv_i) = 2n+2m+2-i; 1 \le i \le n$
$f^*(uu_j) = 2m+2-j; 1 \le j \le m$
$f^*(v_iv_{ii}) = 2m+1+i; 1 \le i \le n$
$f^*(u_ju_{jj}) = 1+j; 1 \le j \le m$
The induced edge labels are distinct and are *1,2,…,* $2n+2m+5$.Hence the theorem. ■

2.6 Definition[6]: Suppose integers $m \ge 2$ *and* $n \ge 1$. $C_m \odot N_n$ is called an actinia graph, denoted by $A(m,n)$, where N_n is the null graph of order *n*. Similarly the graph $A(m,n_1,n_2,\dots,n_m)$can be defined.

2.7Theorem: The graph $A(3,n,m,r)$ is skolem difference mean for all *n, m ,r* ≥ 1.

Proof: Let *G* be the graph $A\ (3,n,m,r)$. Let $V(G) = \{v_1,v_2,v_3,v_{1i},v_{2j},v_{3k}; 1 \le i \le n, 1 \le j \le m, 1 \le k \le r\}$ and $E(G) = \{v_1v_2, v_2v_3, v_3v_1, v_1v_{1i}, v_2v_{2j}, v_3v_{3k}; 1 \le i \le n, 1 \le j \le m, 1 \le k \le r\}$Then $|V(G)| = |E(G)| = n+m+r+3$. Let f: $V(G) \to \{1,2,\dots,2(n+m+r+3)\}$ be

defined as follows.
$f(v_1) = 1$
$f(v_2) = 2(n+m+r+3)$
$f(v_3) = 2(n+m+r+2)$
$f(v_{1i}) = 2(n+m+r)+5-2i; 1 \le i \le n$
$f(v_{2j}) = 2(n+m+r)+4-2j; 1 \le j \le m$
$f(v_{3k}) = 2(n+r)+2-2k; 1 \le k \le r$
Let f^* be the induced edge labeling of f. Then
$f^*(v_1v_2) = n+m+r+3$
$f(u_j) = 4m+7-2j; 1 \le j \le m$
$f(v_{ii}) = 4n+4-4i; 1 \le i \le n$
$f(u_{jj}) = 4m+6-4j; 1 \le j \le m$
Let f^* be the induced edge labeling of *f*. Then$f^*(vy) = 2n+2m+5$
$f^*(uz) = 2n+2m+4$

$f^*(v_3v_1) = n+m+r+2$
$f^*(v_1v_{1i}) = n+m+r+2-i; 1 \le i \le n$
$f^*(v_2v_{2j}) = 1+j; 1 \le j \le m$
$f^*(v_3v_{3k}) = m+1+k; 1 \le k \le r$
The induced edge labels are distinct and are *1,2,…,* $n+m+r+3$. Hence the theorem■

2.8 Definition: The tensor product of two graphs G_1 and G_2 denoted by $G_1(T_p)G_2$ has the vertex set $V(G_1(T_p)G_2) = V(G_1) \times V(G_2)$ and the edge set $E(G_1(T_p)G_2) = \{(u_1,v_1),(u_2,v_2); u_1u_2 \in (G_1)$ and $v_1v_2 \in E(G_2)\}$

2.9 Theorem: The graph $K_{1,n}(T_p)P_2$ is skolem difference mean for all *n* ≥ 1.

Proof: Let $u_1,u_2,\dots,u_n,u_{n+1}$ be the vertices of $K_{1,n}$ with u_1 as the apex vertex and let v_1,v_2 be the vertices of the path P_2. Let G be the graph $K_{1,n}(T_p)P_2$
We divide the vertex set of G into two disjoint sets $T_1 = \{(u_i,v_1); i = 1,2,\dots,n+1\}$ and $T_2 = \{(u_i,v_2); i = 1,2,\dots,n+1\}$. Then $|V(G)| = 2n+2$ and $|E(G)| = 2n$. Let f: $V(G) \to \{1,2,\dots,4n+2\}$ be defined as follows.
$f(u_1,v_1) = 1$
$f(u_i,v_2) = 4n+3-2i; 2 \le i \le n$
$f(u_1,v_2) = 2$
$f(u_i,v_1) = 2n+4-2i; 2 \le i \le n$
Let f^* be the induced edge labeling of f. Then we have
$f^*(u_iv_2) = 2n+1-i; 1 \le i \le n$
$f^*(u_iv_1) = n+1-i; 1 \le i \le n$
The induced edge labels are *1,2,…,2n* which are distinct. Hence the theorem. ■

REFERENCE

1.Frank Harary, Graph Theory, Narosa Publishing House, New Delhi,2011.2.Joseph A. Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, 15(2008), #DS6.3.K. Murugan and A. Subramanian, Skolem difference mean labeling of H-graphs, International Journal of Mathematics and Soft Computing, Vol.1, No.1,2011, p115-129.4.K.Murugan and A.Subramanian, Some new skolem difference mean graphs, Proceedings of the 3rd National Conference on Recent Advancements in Science and Humanities, 2014,p112-119.ISBN978-81-920799-4-35.A.Rosa, On certain valuations of the vertices of a graph, Theory of Graphs (International symposium, Rome, July1966),Gorden and Breach, N.Y and Dunod Paris 1967,p349-3556.Wai Chee Shiu and Peter Che Bor LAM, Super edge graceful labeling of multi level wheel graphs, fan graphs and actinia graphs 7.S K Vaidya and Lekha Bijukumar, Some new perspectives on odd sequential graphs, International Journal of Mathematics and Soft Computing, Vol.4, No.1, 2014, p93 – 101 8.S K Vaidya and N.B.Vyas, Product cordial labeling in the context of Tensor product of graphs, Journal of Mathematics Research,Vol.3,No.3, 2011,p83-88