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Analytical Expression of Concentration of Substrate And Oxygen in Excess Sludge Production Using Adomian Decomposition Method

KEYWORDS: Adomian decomposition method, biological floc, dissolved oxygen, Initial and Boundary value problems, Mathematical modeling and Sludge production.

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ABSTRACT *A mathematical model of excess sludge production from water treatment plants is discussed. The specific growth rate is given by a Monod-type kinetic for two substrates namely carbon source and oxygen. The model is based on system of nonlinear differential equations containing nonlinear terms related to rate equation of substrate and oxygen. Approximate analytical expressions for the two substrates have been derived for all the values of the parameters using the Adomian decomposition method. These analytic results are compared with the numerical results and a good agreement is observed.*

INTRODUCTION

Sludge refers to the residual, semi-solid material left from industrial wastewater or sewage treatment processes. It can also refer to the settled suspension obtained from conventional drinking water treatment and numerous other industrial processes. The term is also sometimes used as a generic term for solids separated from suspension in a liquid. In the industrialized world, cities in particular, have had difficulty in dealing with sewage waste. Seeing sewage contains not only heavy metals and disease pathogens such as Clostridium difficile but nutrients as well. When fresh sewage or wastewater is added to a settling tank, approximately 50% of the suspended solid matter will settle out in an hour and a half. This collection of solids is known as raw sludge or primary solids and is said to be "fresh" before anaerobic processes become active. The sludge will become putrescent in a short time once anaerobic bacteria take over, and must be removed from the sedimentation tank before this happens. Excess solids from biological processes such as activated sludge may still be referred to as "sludge", but "bio solids" or "compost" are public relations terms sometimes used to refer to treated human waste. Industrial wastewater solids are also referred to as sludge, whether generated from biological or physical-chemical processes. Surface water plants also generate sludge made up of solids removed from the raw water.

Component elimination by biological oxidation is one of the main processes in waste water treatment technology [1]. The process involves air or oxygen being introduced into a mixture of screened, and primary treated sewage or industrial wastewater combined with organisms to develop a biological floc which reduces the organic content of the sewage. This material, which in healthy sludge is a brown floc, is largely composed of saprotrophic bacteria. Tyagi et al. [2], discuss the dynamic behavior of activated sludge. Matson and characklis have studied the diffusion coefficients of glucose and oxygen through microbial aggregates and found them to be lower than in pure water [3]. The endogenous respiration in the work of Henze and mladenovski [4] has been defined and categorized in three steps: death, hydrolysis and growth. Lishman and Murphy [5] tried to measure the decay coefficient and demonstrated that the hydrolysis of the organisms is the reaction rate controlling step. Some work has been dedicated to describe the diffusion reaction process within floc [6-10]. In the majority of publications, Monod reactions were employed. Benefield and molz [8, 9] have developed a model that describes the removal of organic substances, oxygen utilization, removal of ammonia-nitrogen and orthophosphate and the product of biomass.

Abbassi et al. [11] developed a mathematical model that describes substrate removal, oxygen utilization and excess sludge production within a microbial floc particle, surrounded by biodegradable substrate. However to the best of author knowledge no general analytical results for the concentration of substrate and oxygen have been published. The purpose of this communication is to derive the analytical expression of the concentration of the substrate and oxygen for all values of the parameters using the Adomian decomposition method.

MATHEMATICAL FORMULATION OF THE PROB LEM:

Oxygen and substrate concentration profiles within a biological floc are given Fig.1.

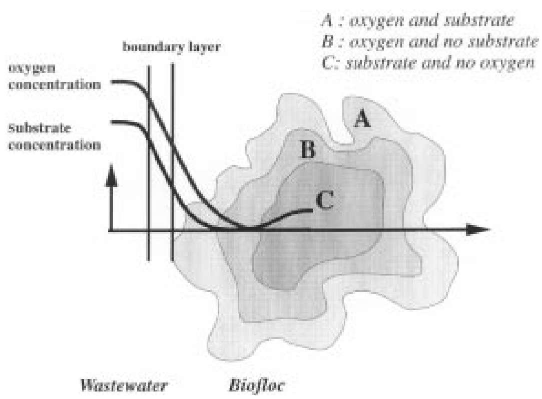

Fig. 1: Oxygen and substrate concentration profiles within a biological floc [11].

The floc particle is spherical in shape and the floc is composed of individual cells distributed uniformly throughout the whole floc volume. Molecular diffusion controls the transfer of the nutrients into the floc particle and the specific growth rate is given by a Monod-type kinetic for two substrates, carbon source and oxygen. With these assumptions the material balance for substrate and oxygen is given by the following nonlinear differential equations [11].

$$\frac{\partial S(r,t)}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(D_S r^2\frac{\partial S(r,t)}{\partial r}\right)-r_S(S,O)+K_{lys}X-r_{res}(S,O) \quad (1)$$

and

$$\frac{\partial O(r,t)}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(D_O r^2\frac{\partial O(r,t)}{\partial r}\right)-r_O(S,O)-\alpha r_{res}(S,O) \quad (2)$$

where and represent the substrate and oxygen. The reac-

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tion rate of substrate due to growth, oxygen due to growth and substrate due to endogenous respiration are given as follows:

$$r_S(S,O) = \left(\frac{\mu_{max}\ X}{Y_S}\right) \frac{S(r,t)\ O(r,t)}{(K_S + S(r,t))\ (K_O + O(r,t))} \quad (3)$$

$$r_O(S,O) = \left(\frac{\mu_{max}\ X}{Y_O}\right) \frac{S(r,t)\ O(r,t)}{(K_S + S(r,t))\ (K_O + O(r,t))} \quad (4)$$

$$r_{res}(S,O) = (K_{res}\ X) \frac{S(r,t)\ O(r,t)}{(K_S' + S(r,t))\ (K_{od} + O(r,t))} \quad (5)$$

At steady state conditions the eqns. (1) and (2) becomes as follows: (6)

and (7)

The boundary conditions are

$$\frac{1}{r^2}\frac{d}{dr}\left(D_S r^2 \frac{dS(r)}{dr}\right) - \left(\frac{\mu_{max} X}{Y_S} - K_{res} X\right)\frac{S(r)O(r)}{(K_S + S(r))(K_O + O(r))} + K_{lys} X = 0$$

$$r = 0,\ \frac{dS}{dr} = 0,\ \frac{dO}{dr} = 0 \quad (8)$$

$$\frac{1}{r^2}\frac{d}{dr}\left(D_O r^2 \frac{dO(r)}{dr}\right) - \left(\frac{\mu_{max} X}{Y_O} - \alpha K_{res} X\right)\frac{S(r)O(r)}{(K_S + S(r))(K_O + O(r))} = 0$$

$$r = R,\ S = S_F,\ O = O_F \quad (9)$$

The nonlinear differential eqns. (6) and (7) is made dimensionless by defining the following dimensionless parameters:

$$\rho = \frac{r}{R}, U = \frac{S}{S_F}, V = \frac{O}{O_F}, l = \frac{K_S}{S_F},\ m = \frac{K_O}{O_F},\ l' = \frac{K_S'}{S_F},\ m' = \frac{K_{od}}{O_F} \quad (10)$$

Therefore, the eqn. (6) and (7) reduces to the following dimensionless form:

$$\frac{d^2U(\rho)}{d\rho^2} + \frac{2}{\rho}\frac{dU(\rho)}{d\rho} - \frac{\alpha_1 U(\rho)V(\rho)}{(l+U(\rho))(m+V(\rho))} + \alpha_2 - \alpha_3\left(\frac{U(\rho)V(\rho)}{(l'+U(\rho))(m'+V(\rho))}\right) = 0 \quad (11)$$

$$\frac{d^2V(\rho)}{d\rho^2} + \frac{2}{\rho}\frac{dV(\rho)}{d\rho} - \frac{\alpha_4 U(\rho)V(\rho)}{(l+U(\rho))(m+V(\rho))} - \left(\frac{\alpha\alpha_5 U(\rho)V(\rho)}{(l'+U(\rho))(m'+V(\rho))}\right) = 0 \quad (12)$$

with the boundary conditions:

$$\rho = 0, \frac{dU}{d\rho} = 0, \frac{dV}{d\rho} = 0 \text{ and } \rho = 1, U = 1, V = 1 \quad (13)$$

where

$$\alpha_1 = \frac{\mu_{max} R^2 X}{Y_S D_S S_F},\ \alpha_2 = \frac{K_{lys} R^2 X}{D_S S_F},\ \alpha_3 = \frac{K_{res} R^2 X}{D_S S_F}, \quad (14)$$
$$\alpha_4 = \frac{\mu_{max} R^2 X}{Y_O D_O O_F},\ \alpha_5 = \frac{K_{res} R^2 X}{D_O O_F}$$

ANALYTICAL EXPRESSION FOR THE CONCENTRATION OF SUBSTRATE AND OXYGEN USING THE ADOMIAN DECOMPOSITION METHOD

Recently the Adomian decomposition method is applied for many different problems like boundary value problems, algebraic equations and partial differential equations. The Adomian decomposition method accurately determines the series solution, which is of great interest in applied sciences. The method provides convergent series with the parameters that can be easily computed. The main advantage of the method is that it can be applied directly for all types of differential and integral equations, linear or nonlinear, homogeneous or inhomogeneous, with constant coefficients or with variable coefficients. Another important advantage is that the method is capable of greatly reducing the size of computation work while still maintaining high accuracy of the numerical solution. However, some modifications are proposed by several authors [12]. The basic concept of ADM is given in Appendix A. Using this Adomian decomposition method (see Appendix B), the solution of the equations (11) and (12) becomes as follows:

$$U(\rho) = 1 + \frac{1}{6}\left[\frac{\alpha_1}{(l+1)(m+1)} - \alpha_2 + \frac{\alpha_3}{(l'+1)(m'+1)}\right](\rho^2 - 1) + \left(\frac{L+M}{12}\right)(\rho^3 - 1) \quad (15)$$

$$V(\rho) = 1 + \frac{1}{6}\left[\frac{\alpha_4}{(l+1)(m+1)} + \frac{\alpha\alpha_5}{(l'+1)(m'+1)}\right](\rho^2 - 1) + \left(\frac{L'+M'}{12}\right)(\rho^3 - 1) \quad (16)$$

Where

$$L = \alpha_1 \frac{[(l+1)(m+1)K_2 + K_1 - K_2(l+1) - K_1(m+1)]}{(l+1)^2(m+1)^2}, L' = \frac{\alpha_4 L}{\alpha_1},$$

$$M' = \frac{\alpha\alpha_5 M}{\alpha_3}, K_1 = \frac{\alpha_1}{6(l+1)(m+1)} - \frac{\alpha_2}{6} + \frac{\alpha_3}{6(l'+1)(m'+1)},$$

$$K_2 = \frac{\alpha_4}{6(l+1)(m+1)} + \frac{\alpha\alpha_5}{6(l'+1)(m'+1)},$$

$$M = \alpha_3 \frac{[(l'+1)(m'+1)K_2 + K_1 - K_2(l'+1) - K_1(m'+1)]}{(l'+1)^2(m'+1)^2} \quad (17)$$

RESULTS AND DISCUSSION

Eqns. (13) and (14) represents the analytical expression for the concentration of substrate and oxygen in activated sludge flocs. Fig.2 represents the dimensionless concentration of carbon source U verses radius ρ for various values parameter. The dimensionless concentration of carbon source U steadily increases when α_1 decreases and it becomes almost constant when $\rho \le 0.5$. In Fig.3-5, the dimensionless concentrations of carbon source U verses radius ρ are plotted. From these figures, it is inferred that the concentration U increases and reaches the steady-state value when parameter decreases and it attains constant when $\rho \le 0.4$.

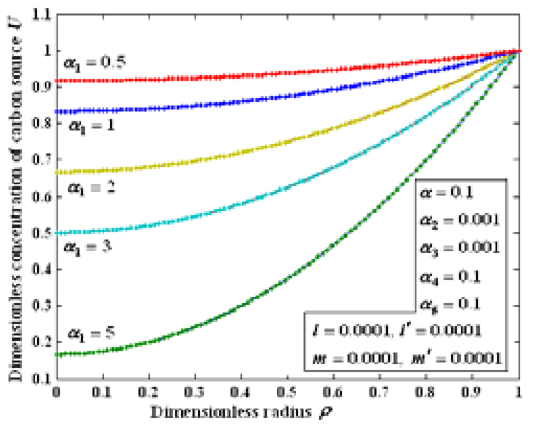

Fig.2: Dimensionless concentration of carbon source U is plotted from Eqn. (15) for various values of dimensionless parameters α1. Solid lines represent the analytical solution obtained in this work whereas the dotted lines for the numerical solution.

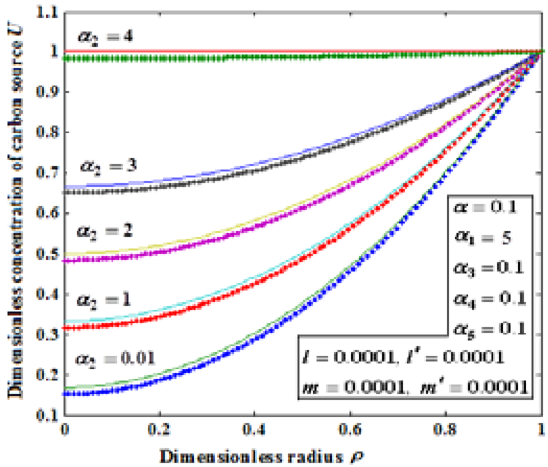

Fig.3: Dimensionless concentration of carbon source U is plotted from Eqn. (15) for various values of dimensionless parameters α_2. Solid lines represent the analytical solution obtained in this work whereas the dotted lines for the numerical solution.

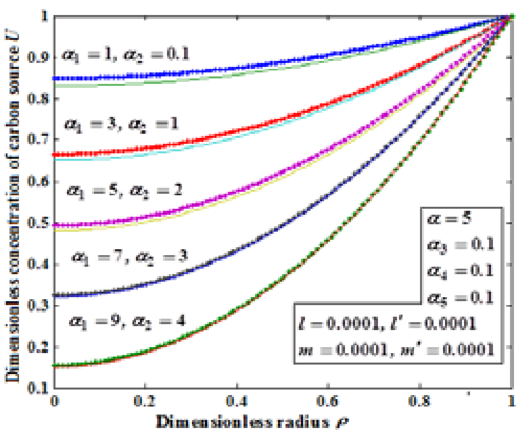

Fig.4: Dimensionless concentration of carbon source U is plotted from Eqn. (15) for various values of dimensionless parameters $\alpha 1$ and $\alpha 2$. Solid lines represent the analytical solution obtained in this work whereas the dotted lines for the numerical solution.

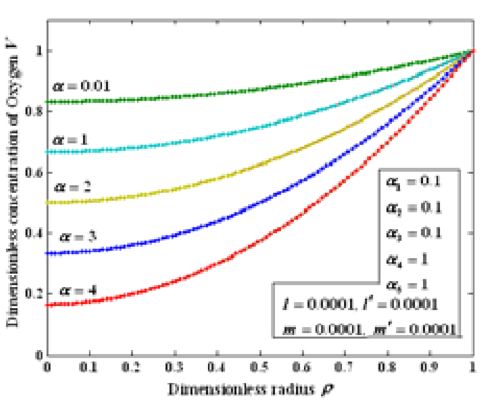

Fig.5: Dimensionless concentration of carbon source V is plotted from Eqn. (16) for various values of dimensionless parameters α. Solid lines represent the analytical solution obtained in this work whereas the dotted lines for the numerical solution.

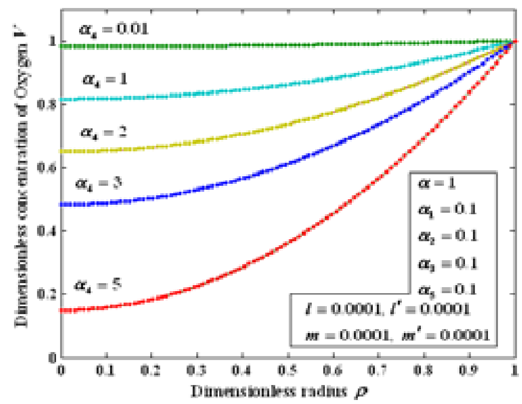

Fig.6: Dimensionless concentration of carbon source V is plotted from Eqn. (16) for various values of dimensionless parameters $\alpha 4$. Solid lines represent the analytical solution obtained in this work whereas the dotted lines for the numerical solution.

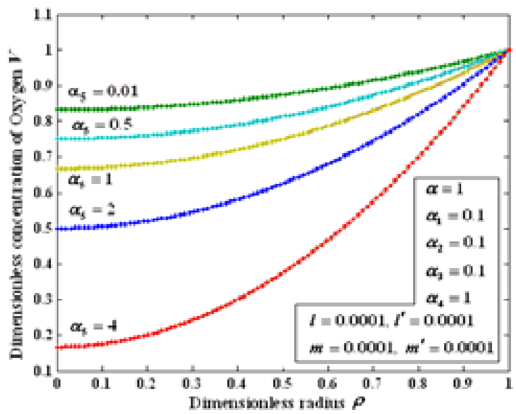

Fig.7: Dimensionless concentration of carbon source V is plotted from Eqn. (16) for various values of dimensionless parameters $\alpha 5$. Solid lines represent the analytical solution obtained in this work whereas the dotted lines for the numerical solution.

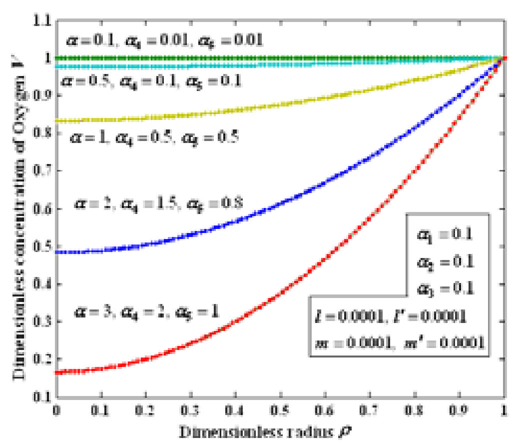

Fig.8: Dimensionless concentration of carbon source V is plotted from Eqn. (16) for various values of dimensionless parameters α, $\alpha 4$ and $\alpha 5$. Solid lines represent the analytical solution obtained in this work whereas the dotted lines for the numerical solution.

Fig.6-8, represents the dimensionless concentration of Oxygen verses radius ρ and from these figure it is inferred that

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the concentration V increases and attains the uniform value when $\rho \leq 0.5$.

NUMERICAL SIMULATION

The non-linear differentials Eqns. (11)-(12) are solved by numerical methods. The function bvp4c in SCILAB/MATLAB software which is a function of solving the initial-boundary value problems for ordinary differential equation is used to solve this equation. Its numerical solution is compared with Adomian decomposition method and it gives a satisfactory result. The SCILAB/MATLAB program is also given in Appendix C.

CONCLUSION

The analytical expressions for substrate and oxygen are obtained by solving the system of nonlinear differential equations using Adomian decomposition method. These analytical results will be useful in the process of minimizing the excess of sludge production by increasing the oxygen concentration in activated sludge flocs.

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APPENDIX A: Basic concept of the Adomian Decomposition Method (ADM)

The Adomian decomposition method decomposes the non-linear differential equation..

$$F[x, y(x)] = 0 \tag{A.1}$$

in to two components

$$L[y(x)] + N[y(x)] = 0 \tag{A.2}$$

where L and N is the linear and the nonlinear part of F respectively. The operator L is assumed to be an invertible operator. Solving for $L(y)$ leads to

$$L[y(x)] = -N[y(x)] \tag{A.3}$$

Applying the inverse operator L^{-1} to both sides of Eqn. A.3 yields

$$y(x) = -L^{-1}[N(y)] + \phi(x) \tag{A.4}$$

where $\phi(x)$ is the function that satisfies the condition $L(\phi)$ =0. Now suppose that the solution y can be represented as an infinite series of the form

$$y(x) = \sum_{n=0}^{\infty} y_n \tag{A.5}$$

The modified Adomian decomposition method assumes that the nonlinear term $N(y)$ can be written as an infinite series in terms of the Adomian polynomials A_n:

$$N(y) = \sum_{n=0}^{\infty} A_n \tag{A.6}$$

where the Adomian polynomials A_n of $N(y)$ are evaluated using the formula

$$A_n(x) = \frac{1}{n!}\frac{d^n}{d\lambda^n} N\left(\sum_{n=0}^{\infty} \lambda^n y_n\right)\Bigg|_{\lambda=0} \tag{A.7}$$

where $\lambda \in [0, 1]$ is a hypothetical parameter. Substituting, Eqns. (A.5) and (A.6) in A.4 gives

$$\sum_{n=0}^{\infty} y_n(x) = \phi(x) - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right) \tag{A.8}$$

By equating the terms in the linear system of Eqn. (A.8) one obtains the recurrence formula:

$$y_0(x) = \phi(x), \quad y_{n+1}(x) = -L^{-1}(A_n) \qquad n \geq 0 \tag{A.9}$$

However, in practice all terms of the series (A.6) cannot be determined, and the solution is approximated by the truncated series $\sum_{n=0}^{N} y_n$.

APPENDIX B: Analytic solution of nonlinear differential equation using ADM

In this appendix we present how equations (12) and (13), in this paper are derived. Consider the equations

$$\frac{d^2U}{d\rho^2} + \frac{2}{\rho}\frac{dU}{d\rho} - \frac{\alpha_1 UV}{(l+U)(M+V)} + \alpha_2 - \alpha_3\left(\frac{UV}{(l'+U)(M'+V)}\right) = 0 \tag{B.1}$$

$$\frac{d^2V}{d\rho^2} + \frac{2}{\rho}\frac{dV}{d\rho} - \frac{\alpha_4 UV}{(l+U)(M+V)} - \left(\frac{\alpha\alpha_5 UV}{(l'+U)(M'+V)}\right) = 0 \tag{B.2}$$

$$\rho = 0, \ \frac{dU}{d\rho} = 0, \ \frac{dV}{d\rho} = 0 \text{ and } \rho = 1, \ U = 1, \ V = 1 \tag{B.3}$$

Considering only the nonlinear term, we get

$$\frac{d^2U}{d\rho^2} + \frac{2}{\rho}\frac{dU}{d\rho} = 0$$

So $L(U) = 0$,therefore $L^{-1}(L(U)) = A\rho + B$ .Solving this with the boundary conditions (B.3), we get

$$U_0 = 1 \tag{B.4}$$

Similarly,

$$V_0 = 1 \tag{B.5}$$

$$U_1 = L^{-1}(A_0)$$

where

$$A_n(x) = \frac{1}{n!}\frac{d^n}{d\lambda^n} N\left(\sum_{n=0}^{\infty} \lambda^n y_n\right)\Bigg|_{\lambda=0} \tag{B.6}$$

Thus $A_0 = N[U_0, V_0]$. (B.7)

$$U_1 = L^{-1}(A_0) = \rho^{-1}\int_0^{\rho}\int_0^{\rho} \rho\left(L^{-1}(A_0)\right) d\rho\, d\rho \tag{B.8}$$

Integrating the above and applying the boundary conditions, we get

$$U_1 = \left[\frac{\alpha_1}{6(l+1)(m+1)} - \frac{\alpha_2}{6} + \frac{\alpha_3}{6(l'+1)(m'+1)}\right](\rho^2 - 1) \tag{B.9}$$

Similarly we get

$$V_1 = \left[\frac{\alpha_4}{6(l+1)(m+1)} + \frac{\alpha\alpha_5}{6(l'+1)(m'+1)}\right](\rho^2 - 1) \tag{B.10}$$

Using the above technique we get next approximations as

$$U_2 = \left(\frac{L+M}{2}\right)(\rho^3 - 1) \tag{B.11}$$

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$$V_2 = \left(\frac{L' + M'}{12}\right)\left(\rho^3 - 1\right) \quad \text{(B.12)}$$

$$U = U_0 + U_1 + U_2 \quad \text{(B.13)}$$

and $V = V_0 + V_1 + V_2$ (B.14)
Using the above system we get eqn. (15) and (16) in the paper.

APPENDIX C
Matlab program to find the numerical solution of the eqn. (11) - (12)

```
function pdex4
m = 2;
x = linspace(0,1);
t=linspace(0,1000000000);
sol = pdepe(m,@pdex4pde,@pdex4ic,@pdex4bc,x,t);
u1 = sol(:,:,1);
u2 = sol(:,:,2);
%--------------------------------------------------------------
figure
plot(x,u1(end,:))
title('u1(x,t)')
xlabel('Distance x')
ylabel('u1(x,1)')
%--------------------------------------------------------------
%figure
%plot(x,u2(end,:))
%title('u2(x,t)')
%xlabel('Distance x')
%ylabel('u2(x,2)')
%--------------------------------------------------------------
function [c,f,s] = pdex4pde(x,t,u,DuDx)
c = [0; 0];
f = [1; 1] .* DuDx;
a=0.1; a1=0.01; a2=0.1; a3=0.1; a4=0.1; a5=0.1; l=0.0001;
m=0.0001; ld=0.0001; md=0.0001;
F1=-a1*u(1)*u(2)/((l+u(1))*(m+u(2)))+a2-a3*u(1)*u(2)/
((ld+u(1))*(md+u(2)));
F2=-a4*u(1)*u(2)/((l+u(1))*(m+u(2)))-a*a5*u(1)*u(2)/
((ld+u(1))*(md+u(2)));
s=[F1; F2];
%--------------------------------------------------------------
function u0 = pdex4ic(x)        %create a initial conditions
u0 = [1; 1];
%--------------------------------------------------------------
function [pl,ql,pr,qr]=pdex4bc(xl,ul,xr,ur,t)
pl = [0; 0];
ql = [1; 1];
pr = [ur(1)-1; ur(2)-1];
qr = [0; 0];
```

APPENDIX D
Nomenclature

D_S Effective substrate diffusion coefficient in the floc particle $(m^2 d^{-1})$

D_O Effective Oxygen diffusion coefficient in the floc particle $(m^2 d^{-1})$

K_{lys} Maximum hydrolysis rate of micro-organisms $(mg\,BOD_5\,mg\,MLSS^{-1} d^{-1})$

K_O Half-velocity constant of oxygen $(mg\,O l^{-1})$

K_{od} Half-velocity constant of oxygen in endogenous respiration $(mgOmg\,MLSS^{-1} d^{-1})$

K_{res} Maximum reaction rate of endogenous respiration $(mgBOD_5\,mgMLSS^{-1} d^{-1})$

K_S Half-velocity constant of substrate $(mgBOD_5\,l^{-1})$

K_S' Half-velocity constant of substrate in the endogenous respiration term $(mgBOD_5\,l^{-1})$

O Oxygen concentration $(mgOl^{-1})$

O_F Oxygen concentration in the bulk liquid $(mgOl^{-1})$

r Position along floc radius (m)

R Radius of floc particle (m)

r_O Reaction rate of oxygen due to growth $(mgOl^{-1})$

r_S Reaction rate of substrate due to growth $(mg\,BOD_5\,l^{-1} d^{-1})$

S Substrate concentration $(mg\,BOD_5\,l^{-1})$

S_F Substrate concentration in the bulk liquid $(mg\,BOD_5\,l^{-1})$)

T Temperature (^0C)

U Dimensionless Substrate concentration

V Dimensionless Oxygen concentration

X Micro-organism concentration $(mgMLSS^{-1} l^{-1})$

Y_S Substrate yield coefficient $(mg\,MLSSmg\,BOD^{-1})$

Y_O Oxygen yield coefficient $(mg\,MLSSmg\,O^{-1})$

α Proportionality constant corresponds to oxygen yield coefficient due to the endogenous respiration in the core of the floc particle.

μ_{max} Maximum specific growth rate (d^{-1})

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