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An Innovative Method for Solving Fuzzy Transportation Problem

KEYWORDS: Fuzzy sets (normal and convex), Membership Functions, Trapezoidal fuzzy number, Triangular fuzzy number, Robust ranking technique, Fuzzy Transportation problem.

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ABSTRACT *This paper deals about the Fuzzy transportation problem, where the transportation cost, supply and demand quantities are fuzzy quantities. The objective of Fuzzy transportation problem is to determine the shipping schedule that minimizes the total transportation cost while satisfying fuzzy supply and fuzzy demand limits. Using Robust ranking method fuzzy quantities are transformed into crisp quantities. To obtain the optimal solution a new algorithm is proposed. Finally a numerical illustration is given to check the validity of the proposal.*

1.INTRODUCTION

Transportation problem is a particular class of linear programming, which is associated with routine activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from one place to another. The goods are transported from a set of sources (e.g., factory) to a set of destinations (e.g., warehouse) to meet the specific requirements. In other words, transportation problems deal with the transportation of a single product manufactured at different plants (supply origins) to a number of different warehouses (demand destinations). The objective is to satisfy the demand at destinations from the supply constraints at the minimum possible transportation cost. To reach this objective, we must know the quantity of available supplies and demand. In addition, we must know the location, to find the cost of transporting single homogeneous commodity from the place of origin to the destination. The model is useful for making strategic decisions involved in selecting optimum transportation routes so as to allocate the production of various plants to several warehouses or distribution centres. The objective of the transportation model is to determine the amount to be shipped from each source to each destination to maintain the supply and demand requirements at the lowest transportation cost.

In this paper we shall study fuzzy transportation problem, and weintroduce an approach for solving a wide range of such problem byusing a method which apply it for ranking of the fuzzy numbers. Some of the quantities in a fuzzy transportation problem may be fuzzy orcrisp quantities. In many fuzzy decision problems, the quantities arerepresented in terms of fuzzy numbers. Fuzzy numbers may be normalor abnormal, triangular or trapezoidal or any LR fuzzy number. Thus,some fuzzy numbers are not directly comparable. First, we transformthe fuzzy quantities as the cost, coefficients, supply and demands, into crisp quantities by Robust ranking method [14] which satisfies the properties of compensation, linearity and additivity, and then by using the classicalalgorithms we solve and obtain the solution of the problem. The newmethod is a systematic procedure, easy to apply and can be utilized for all types of transportation problem whether maximize or minimizeobjective function.

2.PRELIMINARIES

Zadeh [13] in 1965 first introduced Fuzzy set as a mathematical way ofrepresenting impreciseness or indistinctness in everyday life.

2.1. Definition

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or the universe of discourse X to the unit interval [0, 1] (i.e.) $A = \{x, \mu_A(x), x\epsilon X\}$. Here $\mu_A: X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

2.2. Definition

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one $x \in X$ such that $\mu_A(x) = 1$

2.3. Definition

A fuzzy set A is convex if and only if, for any $x_1, x_2 \epsilon X$, the membership function of A satisfies the inequality $\mu_A\{\lambda x_1 + (1-\lambda)x_2\} \geq \min(\mu_A(x_1), \mu_A(x_2)), 0 \leq \lambda \leq 1$

2.4. Definition

For a triangular fuzzy number A(x), it can be represented by A (a,b,c;1) with membership function μ(x) is given by

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x = b \\ \dfrac{c-x}{c-b}, & b \leq x \leq c \\ 0, & Otherwise \end{cases}$$

2.5. Definition

For a trapezoidal fuzzy number A(x), it can be represented by A (a, b, c, d; 1) with membership function μ(x) is given by

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \dfrac{d-x}{d-c}, & c \leq x \leq d \\ 0, & Otherwise \end{cases}$$

2.6. Definition

The α-cut of a fuzzy number is defined as ,$A(\alpha) = \{x: \mu(x) \geq \alpha, \alpha\epsilon[0,1]\}$

2.7. Definition

Addition and Subtraction of two triangular fuzzy numbers can be performed as

$$(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$
$$(a_1, b_1, c_1) - (a_2, b_2, c_2) = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$$

Addition and Subtraction of two trapezoidal fuzzy numbers can be performed as

$$(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$$
$$(a_1, b_1, c_1, d_1) - (a_2, b_2, c_2, d_2) = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$$

3. Robust Ranking Technique – Algorithm

Using robust ranking technique fuzzy numbers can be converted into crisp ones. Robust ranking technique whichsatisfies compensation, linearity, and additive properties and provides results which are consistent with human intuition.Give a convex fuzzy number ã , the Robust Ranking Index is defined by

R (ã) =$\int_0^1 (0.5)(a_\alpha^L, a_\alpha^U)$dα

Where $(a_\alpha^L, a_\propto^U)$ is the ∝ level cut of the fuzzy number ã and$(a_\propto^L, a_\propto^U)$= $\{(b-a)\alpha + a), (c - (c-b)\,\alpha)\}$

In this paper the above mentioned method is used for ranking the fuzzy numbers. The Robustranking index R(ã) gives the representative value of the fuzzy number ã . It satisfies thelinearity and additive property.

4.Fuzzy Transportation Problem

The fuzzy transportation problems, in which a decision maker is uncertainabout the precise values of transportation cost,availability and demand, can be formulated as follows

$$Minimize\tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c_{ij}} \widetilde{x_{ij}} \,,$$

$$Subjectto, \quad \sum_{j=1}^{n} \widetilde{x_{ij}} = \tilde{a}_{i\;,i=1,2......m}$$

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$$\sum_{i=1}^{m}\widetilde{x_{ij}} = \tilde{b}_j, \quad j=1,2,\ldots\ldots n$$

$\sum_{i=1}^{m}\tilde{a}_i = \sum_{j=1}^{n}\tilde{b}_j, i = 1,2,\ldots\ldots m and j = 1,2,\ldots\ldots n$

$and \widetilde{x_{ij}} \geq \tilde{0}$

Where m= total number of sources

n=total number of destinations

$\tilde{a}_i$=the fuzzy availability of the product at i[th] source

$\tilde{b}_j$= the fuzzy demand of the product at j[th]destination

$\widetilde{c_{ij}}$= the fuzzy transportation cost for unit quantity of the product from i[th] source to j[th]destination

$\widetilde{x_{ij}}$=the fuzzy quantity of the product that should be transported from i[th] source to j[th]destination to minimize the total fuzzy transportation cost

$\sum_{i=1}^{m}\tilde{a}_i$= total fuzzy availabilityof the product

$\sum_{j=1}^{n}\tilde{b}_j$= total fuzzy demand of the product

$\sum_{i=1}^{m}\sum_{j=1}^{n}\widetilde{c_{ij}}\widetilde{x_{ij}}$= total fuzzy transportation cost

If $\sum_{i=1}^{m}\tilde{a}_i = \sum_{j=1}^{n}\tilde{b}_j$ then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem.

This problem can be represented as follows:

Table 1

Destinations

		1	....	n	Supply
Sources	1	$\widetilde{c_{11}}$	....	$\widetilde{c_{1n}}$	$\widetilde{a_1}$
	: :	: :	....	: :	: :
	m	$\widetilde{c_{m1}}$	....	$\widetilde{c_{mn}}$	$\widetilde{a_m}$
	Demand	$\widetilde{b_1}$	....	$\widetilde{b_n}$	

5.METHODOLOGY

Step 1: Construct the transportation problem from fuzzy transportation problem. Suppose the values of transportation problem are not integers, round off into integers.

Step II: Select the minimum odd cost from all cost in the matrix. Suppose all the costs are even, multiply by 1/2 each column.

StepIII: Subtract selected least odd cost only from odd cost in the matrix.Now there will be at least one zero and remaining all cost become even.

Step IV: Allocate this minimum of supply / demand at the place of zero.

StepV: After the allotment, multiply by 1/2 each column.

Step VI: Again select minimum odd cost in the remaining column except zeros in that column.

Step VII: Go to step III and repeat step IV and V till optimal solution are obtained.

StepVIII: Finally total minimum cost is calculated as sum of the product of the cost and corresponding allocated value of supply/demand

Total cost= $\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}$

6.NUMERICAL EXAMPLE:

A factory has three originsO_1, O_2, O_3 four destinations D_1 ,D_2, D_3 ,D_4. The fuzzy transportation cost for unit quantity of the product from i[th] source to j[th] destination is C_{ij} where

Table 2

$(C_{ij})_{3x4}$

(1,5,9)	(4,9,14)	(9,13,17)	(1,2,3)
(9,11,13)	(9,18,27)	(18,20,22)	(1,3,5)
(8,14,20)	(10,15,20)	(10,16,22)	(2,7,12)

Fuzzy availability of the product at sources are (20,50,80), (25,50,75), (30,50,70) and the fuzzy demand of the product at destinations are (10,30,50), (20,40,60), (35,55,75), (10,25,40) respectively.

The fuzzy transportation problem is given by

Table 3

	D_1	D_2	D_3	D_4	Fuzzy supply
O_1	(1,5,9)	(4,9,14)	(9,13,17)	(1,2,3)	(20,50,80)
O_2	(9,11,13)	(9,18,27)	(18,20,22)	(1,3,5)	(25,50,75)
O_3	(8,14,20)	(10,15,20)	(10,16,22)	(2,7,12)	(30,50,70)
Fuzzy demand	(10,30,50)	(20,40,60)	(35,55,75)	(10,25,40)	(75,150,225)

In conformation to model thefuzzy transportation problem can be formulated in the following mathematical form

Min Z= $R(1,5,9)x_{11}+R(4,9,14)x_{12}+R(9,13,17)x_{13}+R(1,2,3)x_{14}+R(9,11,13)x_{21}+R(9,18,27)x_{22}+$
$R(18,20,22)x_{23}+R(1,3,5)x_{24}+R(8,14,20)x_{31}+R(10,15,20)x_{32}+R(10,16,22)x_{33}+R(2,7,12)x_{34}$

Step1:

Applying Robust ranking method ,we get

$$R(\tilde{a}) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U)d\alpha$$

Where $\left(a_{\alpha,}^L a_{\alpha,}^U\right) = \{(b-a)\alpha + a, c-(c-b)\alpha\}$

$$R(1,5,9) = \int_0^1 (0.5)(4\alpha + 1, 9 - 4\alpha)\,d\alpha$$

$$R(1,5,9) = \int_0^1 (0.5)(10)\,d\alpha = 5$$

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Similarly we get,

R(4,9,14)=9, R(9,13,17)=13, R(1,2,3) = 2, R(9,11,13)=11, R(9,18,27)=18, R(18,20,22)=20, R(1,3,5)=3, R(8,14,20)=14, R(10,15,20)=15, R(10,16,22)=16, R(2,7,12)=7

Rank of all supplies: R (20,50,80) =50, R(25,50,75)=50, R (30,50,70)=50

Rank of all demands: R(10,30,50)=30, R (20,40,60)=40 , R(35,55,75)=55, R (10,25,40)=25

Substitute these values in fuzzy transportation problem we get the crisp transportation problem which is shown infollowing table

Table 4

	D_1	D_2	D_3	D_4	Supply
O_1	5	9	13	2	50
O_2	11	18	20	3	50
O_3	14	15	16	7	50
Demand	30	40	55	25	150

Step 2: Herethe minimum odd cost is 3.

Step 3: Subtract 3 from all odd cost, which is shown in below table. which is depicted in below.

Table 5

	D_1	D_2	D_3	D_4	Supply
O_1	2	6	10	2	50
O_2	8	18	20	0	50
O_3	14	12	16	4	50
Demand	30	40	55	25	150

Allocate the cell(O_2, D_4), Min(25,50)=25, we get x_{24}=25 and delete column D_4 as for demand exhausted supply is (50-25)=25 which is shown in following table

Table 6

	D_1	D_2	D_3	Supply
O_1	2	6	10	50
O_2	8	18	20	25
O_3	14	12	16	50
Demand	30	40	55	

Step IV: Multiply by 1/2 each column which is shown in following table

Table 7

	D_1	D_2	D_3	Supply
O_1	1	3	5	50
O_2	4	9	10	25
O_3	7	6	8	50
Demand	30	40	55	

Now the minimum odd cost is 1. Go to step III and repeat step IV and V Until and unless all the demands are satisfied and all the supplies are exhausted.

Table 8

	D_1	D_2	D_3	D_4	Supply
O_1	5 **[30]**	9 **[20]**	13	2	50
O_2	11	18	20 **[25]**	3 **[25]**	50
O_3	14	15 **[20]**	16 **[30]**	7	50
Demand	30	40	55	25	150

The minimum transportation cost associated with this solution is

$$Z = (5x30)+(9x20)+(20x25)+(3x25)+(15x20)+(16x30)$$

$$= 1685$$

7.Conclusion

In thispaper, Fuzzy set theory has been applied to obtain the optimal solution for transportation problem. Moreover the fuzzy problem has been converted into crisp problem by Robust ranking indices. A simple algorithm has been developed to find the optimal solution in transportation problem. The proposed method helps to get directly optimal solution with less iteration.so it will be very helpful for decision makers who are dealing with logistic and supply chain problems.

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