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Bayesian one- Way Repeated Measurements Model Based on Markov Chain Monte Carlo

KEYWORDS

Ameera Jaber Mohaisen
Mathematics Department College of Education for Pure Science AL-Basrah University-Iraq

Khawla Abdul Razzaq Swadi
Mathematics Department College of Education for Pure Science AL-Basrah University-Iraq

ABSTRACT *A simple Bayesian approach to repeated measurement model which has only one Within units factor and one between units factor incorporating univariate random effects as well as the experimental error term. is described using Markov Chain Monte Carlo(MCMC). Bayesian approach is employed to making inferences on the one-way repeated measurements model.*

1. Introduction

In the Bayesian approach to inference, all unknown quantities contained in a probability model for the observed data are treated as random variables. Specifically, the fixed but unknown parameters are viewed as random variables under the Bayesian approach. Bayesian techniques based on Markov chain Monte Carlo provide what we believe to be the most satisfactory approach to fitting complex models as well as the direction that model is most likely to take in the future [3],[4],[5],[6],[8],[10],[11] .

Repeated measurements is a term used to describe data in which the response variable for each experimental units is observed on multiple occasions and possible under different experimental conditions . Repeated measures data is a common form of multivariate data, and linear models with correlated error which are widely used in modeling repeated measures data. Repeated measures is a common data structure with multiple measurements on a single unit repeated over time. Multivariate linear models with correlated errors have been accepted as one of the primary modeling methods for repeated measures data [1],[2] ,[7],[9] .

In this paper, we consider the linear one-way repeated measurements model which has only one within units factor and one between units factor incorporating univariate random effects as well as the experimental error term. Inferences about the parameters of the model such as it is estimation and estimation error as well as model checking are of interest.

2.Repeated Measurements Model and Prior Distribution

Consider the model

$$y_{ijk} = \mu + \tau_j + \delta_{i(j)} + \gamma_k + (\tau\gamma)_{jk} + e_{ijk} \qquad (1)$$

Where

$i=1,\ldots, n$ is an index for experimental unit within group j ,

$j=1,\ldots,q$ is an index for levels of the between-units factor (Group) ,

$k=1,\ldots,p$ is an index for levels of the within-units factor (Time) ,

y_{ijk} is the response measurement at time k for unit i within group j,

μ is the overall mean ,

τ_j is the added effect for treatment group j ,

$\delta_{i(j)}$ is the random effect for due to experimental unit i within treatment group j ,

γ_k is the added effect for time k ,

$(\tau\gamma)_{jk}$ is the added effect for the group j × time k interaction ,

e_{ijk} is the random error on time k for unit i within group j ,

For the parameterization to be of full rank, we imposed the following set of conditions

$\sum_{j=1}^{q} \tau_{j=0}$, $\sum_{k=1}^{p} \gamma_{k=0}$, $\sum_{j=1}^{q} (\tau\gamma)_{jk=0}$ for each k=1,…,p

$\sum_{k=1}^{p} (\tau\gamma)_{jk=0}$ for each j=1,…,q

And we assumed that the e_{ijk} and $\delta_{i(j)}$ are indepnd

Volume : 4 | Issue : 10 | October 2014 | ISSN - 2249-555X

$e_{ijk} \sim \text{i.i.d } N(0,\sigma_e^2)$,
$\delta_{i(j)} \sim \text{i.i.d } N(0,\sigma_\delta^2)$ (2)

Sum of squares due to groups, subjects(group), time, group*time and residuals are then defined respectively as follows:

$$SS_G = np\sum_{j=1}^{q}\left(\bar{y}_{.j.} - \bar{y}_{...}\right)^2,$$

$$SS_{U(G)} = p\sum_{i=1}^{n}\sum_{j=1}^{q}(\bar{y}_{ij.} - \bar{y}_{.j.})^2$$

$$SS_{time} = nq\sum_{k=1}^{p}(\bar{y}_{..k} - \bar{y}_{...})^2,$$

$$SS_{G\times time} = n\sum_{j=1}^{q}\sum_{k=1}^{p}(\bar{y}_{.jk} - \bar{y}_{.j.} - \bar{y}_{..k} + \bar{y}_{...})^2$$

$$SS_E = \sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}(y_{ijk} - \bar{y}_{.jk} - \bar{y}_{ij.} + \bar{y}_{.j.})^2$$

Where

$\bar{y}_{...} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p} y_{ijk}}{nqp}$ is the overall mean.

$\bar{y}_{.j.} = \frac{\sum_{i=1}^{n}\sum_{k=1}^{p} y_{ijk}}{np}$ is the mean for group j

$\bar{y}_{ij.} = \frac{\sum_{k=1}^{p} y_{ijk}}{p}$ is the mean for the i^{th} subject in group j.

$\bar{y}_{..k} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{q} y_{ijk}}{nq}$ is the mean for time k.

$\bar{y}_{.jk} = \frac{\sum_{i=1}^{n} y_{ijk}}{n}$ is the mean for group j at time k.

Table1:ANOVA table for one-way Repeated measures model

Source of variation	d.f	SS	MS	E(MS)
Group	$q-1$	SS_G	$\frac{SS_G}{q-1}$	$\frac{np}{(q-1)}\sum_{j=1}^{q}\tau_j^2 + p\sigma_\delta^2 + \sigma_e^2$
Unit (Group)	$q(n-1)$	$SS_{U(G)}$	$\frac{SS_{U(G)}}{q(n-1)}$	$p\sigma_\delta^2 + \sigma_e^2$
Time	$p-1$	SS_{time}	$\frac{SS_{time}}{p-1}$	$\frac{nq}{(p-1)}\sum_{k=1}^{p}\gamma_k^2 + \sigma_e^2$
Group*Time	$(q-1)(p-1)$	$SS_{G\times time}$	$\frac{SS_{G\times time}}{(q-1)(p-1)}$	$\frac{n}{(p-1)(q-1)}\sum_{j=1}^{q}\sum_{k=1}^{p}(\tau\gamma)_{jk}^2 + \sigma_e^2$
Residual	$q(p-1)(n-1)$	SS_E	$\frac{SS_E}{q(p-1)(n-1)}$	σ_e^2

We assume that the prior distribution on one-way repeated measurements model coefficients as following

$\mu \sim N(0,\sigma_\mu^2)$, $\tau_j \sim N(0,\sigma_\tau^2)$,
$\gamma_k \sim N(0,\sigma_\gamma^2)$, $(\tau\gamma)_{jk} \sim N(0,\sigma_{(\tau\gamma)}^2)$
$\sigma_\delta^2 \sim IG(\alpha_\delta,\beta_\delta)$, $\sigma_e^2 \sim IG(\alpha_e,\beta_e)$ (3)

3.Posterior Calculation

The likelihood function for the model (1) can derive as follows

$$L(y|\mu,\tau_j,\delta_{ij},\gamma_k,(\tau\gamma)_{jk},\sigma_\delta^2,\sigma_e^2) \propto \prod_{i=1}^{n}\prod_{j=1}^{q}\prod_{k=1}^{p}\frac{1}{\sqrt{2\pi\sigma_e^2}} \exp\left[\frac{-(y_{ijk}-\mu-\tau_j-\delta_{ij}-\gamma_k-(\tau\gamma)_{jk})^2}{2\sigma_e^2}\right]$$

$$\rightarrow L(y|\mu,\tau_j,\delta_{i(j)},\gamma_k,(\tau\gamma)_{jk},\sigma_\delta^2,\sigma_e^2) \propto (2\pi(\sigma_e^2))^{-\frac{nqp}{2}} \times \exp\left[\frac{-\sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}(y_{ijk}-\mu-\tau_j-\delta_{i(j)}-\gamma_k-(\tau\gamma)_{jk})^2}{2\sigma_e^2}\right]$$

since

$$\sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}(y_{ijk}-\mu-\tau_j-\delta_{i(j)}-\gamma_k-(\tau\gamma)_{jk})^2 = \sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}[y_{ijk}+y_{i.k}-y_{i.k}+y_{..k}-y_{..k}+y_{ij.}-y_{ij.}+y_{i..}-y_{i..}-\mu-\tau_j-\delta_{i(j)}-\gamma_k-(\tau\gamma)_{jk}]^2$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}[(y_{ijk}-\mu)^2+(y_{i.k}-\tau_j)^2+(y_{..k}-\delta_{i(j)})^2+(y_{ij.}-\gamma_k)^2+(y_{i..}-(\tau\gamma)_{jk})^2-(y_{i.k}+y_{..k}+y_{ij.}+y_{i..})^2]$$. Then

$$L(y|\mu,\tau_j,\delta_{i(j)},\gamma_k,(\tau\gamma)_{jk},\sigma_\delta^2,\sigma_e^2) \propto (2\pi(\sigma_e^2))^{-\frac{nqp}{2}}\exp\left[-\frac{\sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}(y_{ijk}-\mu)^2}{2(\sigma_e^2)} - \frac{\sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}(y_{i.k}-\tau_j)^2}{2(\sigma_e^2)} - \frac{\sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}(y_{..k}-\delta_{i(j)})^2}{2(\sigma_e^2)} - \frac{\sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}(y_{ij.}-\gamma_k)^2}{2(\sigma_e^2)} - \frac{\sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}(y_{i..}-(\tau\gamma)_{jk})^2}{2(\sigma_e^2)} + \frac{\sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}(y_{i.k}+y_{..k}+y_{ij.}+y_{i..})^2}{2(\sigma_e^2)}\right] \quad (4)$$

Then we have the posterior density of one-way repeated measurements model coefficients and the variances (σ_e^2) and (σ_δ^2) as follows

$$\pi_1(\mu|\tau_j,\delta_{i(j)},\gamma_k,(\tau\gamma)_{jk},\sigma_\delta^2,\sigma_e^2) \propto L(y|\mu,\tau_j,\delta_{i(j)},\gamma_k,(\tau\gamma)_{jk},\sigma_\delta^2,\sigma_e^2)\,\pi_0(\mu),$$

where π_0 and π_1 represents prior and posterior density respectively, then

The posterior of μ is $\mu|\tau_j,\delta_{i(j)},\gamma_k,(\tau\gamma)_{jk},\sigma_\delta^2,\sigma_e^2$

Volume : 4 | Issue : 10 | October 2014 | ISSN - 2249-555X

$$\pi_1\left(\mu|\tau_j, \delta_{i(j)}, \gamma_k, (\tau\gamma)_{jk}, \sigma_\delta^2, \sigma_e^2\right) \propto$$

$$\left(2\pi(\sigma_e^2)\right)^{-\frac{nqp}{2}} \exp\left[-\frac{\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p\left(y_{ijk}-\mu\right)^2}{2(\sigma_e^2)} - \frac{\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p\left(y_{i.k}-\tau_j\right)^2}{2(\sigma_e^2)} - \frac{\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p\left(y_{..k}-\delta_{i(j)}\right)^2}{2(\sigma_e^2)}\right.$$

$$\left. - \frac{\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p\left(y_{ij.}-\gamma_k\right)^2}{2(\sigma_e^2)} - \frac{\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p\left(y_{i..}-(\tau\gamma)_{jk}\right)^2}{2(\sigma_e^2)} + \frac{\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p\left(y_{i.k}+y_{..k}+y_{ij.}+y_{i..}\right)^2}{2(\sigma_e^2)}\right] \times \left(2\pi\sigma_\mu^2\right)^{-\frac{1}{2}} \exp\left[\frac{-\mu^2}{2\sigma_\mu^2}\right]$$

$$= \left(2\pi(\sigma_e^2)\right)^{-\frac{nqp}{2}}\left(2\pi\sigma_\mu^2\right)^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\mu^2\left(\frac{nqp}{\sigma_e^2}+\frac{1}{\sigma_\mu^2}\right)+\mu\left(\frac{\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p y_{ijk}}{\sigma_e^2}\right)\right]$$

$$= \exp\left[-\frac{1}{2}\left(\mu^2 - 2\mu\frac{\frac{\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p y_{ijk}}{\sigma_e^2}}{\frac{nqp}{\sigma_e^2}+\frac{1}{\sigma_\mu^2}}\right)\times\left(\frac{nqp}{\sigma_e^2}+\frac{1}{\sigma_\mu^2}\right)\right]$$

$$= \exp\left[\frac{-\frac{1}{2}\left(\mu-\frac{\frac{\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p y_{ijk}}{\sigma_e^2}}{\frac{nqp}{\sigma_e^2}+\frac{1}{\sigma_\mu^2}}\right)^2}{\frac{1}{\frac{nqp}{\sigma_e^2}+\frac{1}{\sigma_\mu^2}}}\right]$$

$$\therefore \mu|\tau_j, \delta_{i(j)}, \gamma_k, (\tau\gamma)_{jk}, \sigma_\delta^2, \sigma_e^2 \sim N\left[\frac{\frac{\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p y_{ijk}}{\sigma_e^2}}{\frac{nqp}{\sigma_e^2}+\frac{1}{\sigma_\mu^2}}, \frac{1}{\frac{nqp}{\sigma_e^2}+\frac{1}{\sigma_\mu^2}}\right] \quad (5)$$

By the same way we can find the posterior of the other parameters is

$$\tau_j|\mu, \delta_{i(j)}, \gamma_k, (\tau\gamma)_{jk}, \sigma_\delta^2, \sigma_e^2 \sim N\left[\frac{\frac{\sum_{i=1}^n\sum_{k=1}^p y_{i.k}}{\sigma_e^2}}{\frac{np}{\sigma_e^2}+\frac{1}{\sigma_\tau^2}}, \frac{1}{\frac{np}{\sigma_e^2}+\frac{1}{\sigma_\tau^2}}\right] \quad (6)$$

$$\gamma_k\,|\mu, \tau_j, \delta_{i(j)}, (\tau\gamma)_{jk}, \sigma_\delta^2, \sigma_e^2 \sim N\left[\frac{\frac{\sum_{i=1}^n\sum_{j=1}^q y_{ij.}}{\sigma_e^2}}{\frac{nq}{\sigma_e^2}+\frac{1}{\sigma_\gamma^2}}, \frac{1}{\frac{nq}{\sigma_e^2}+\frac{1}{\sigma_\gamma^2}}\right] \quad (7)$$

$$(\tau\gamma)_{jk}\,|\mu, \tau_j, \delta_{i(j)}, \gamma_k, \sigma_\delta^2, \sigma_e^2 \sim N\left[\frac{\frac{\sum_{i=1}^n y_{i..}}{\sigma_e^2}}{\frac{n}{\sigma_e^2}+\frac{1}{\sigma_{\tau\gamma}^2}}, \frac{1}{\frac{n}{\sigma_e^2}+\frac{1}{\sigma_{\tau\gamma}^2}}\right] \quad (8)$$

$$\delta_{i(j)}\,|\mu, \tau_j, \gamma_k, (\tau\gamma)_{jk}, \sigma_\delta^2, \sigma_e^2 \sim N\left[\frac{\frac{\sum_{k=1}^p y_{..k}}{\sigma_e^2}}{\frac{p}{\sigma_e^2}+\frac{1}{\sigma_\delta^2}}, \frac{1}{\frac{p}{\sigma_e^2}+\frac{1}{\sigma_\delta^2}}\right] \quad (9)$$

$$\sigma_\delta^2|\mu, \tau_j, \delta_{i(j)}, \gamma_k, (\tau\gamma)_{jk}, \sigma_e^2 \sim IG[\alpha_\delta, \beta_\delta]. \quad (10)$$

$$\sigma_e^2|\mu, \tau_j, \delta_{i(j)}, \gamma_k, (\tau\gamma)_{jk}, \sigma_\delta^2 \sim IG\left(\alpha_e + \frac{nqp}{2}, \beta_e + \frac{RSS}{2}\right) \quad (11)$$

Where

$$RSS=\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p\left[(y_{ijk}-\mu)^2 + (y_{i.k}-\tau_j)^2 + (y_{..k}-\delta_{i(j)})^2 + (y_{ij.}-\gamma_k)^2 + (y_{i..}-(\tau\gamma)_{jk})^2 - (y_{i.k}+y_{..k}+y_{ij.}+y_{i..})^2\right]$$

4. Model checking and Bayes factors

We would like to choose between a Bayesian mixed repeated measurements model and its fixed counterpart by the criterion of the Bayes factor for tow hypotheses :

$$\left.\begin{array}{l} H_0:\ y_{ijk} = \mu + \tau_j + \gamma_k + (\tau\gamma)_{jk} + e_{ijk} \\ \qquad\qquad \text{versus} \\ H_1:\ y_{ijk} = \mu + \tau_j + \delta_{i(j)} + \gamma_k + (\tau\gamma)_{jk} + e_{ijk} \end{array}\right\} \quad (12)$$

We compute the Bayes factor, B_{01}, of H_o relative to H_1 for testing problem (12) as following

$$B_{01}(y_{ijk}) = \frac{m(y_{ijk}|H_o)}{m(y_{ijk}|H_1)}, \quad (13)$$

where $m(y_{ijk}|H_i)$ is the predictive (marginal) density of y_{ijk} under model $H_i, i = 0, 1$.

We have

$$m(y_{ijk}|H_o) = \frac{1}{\left(2\pi\left(\sigma_\mu^2+\sigma_\tau^2+\sigma_\gamma^2+\sigma_{(\tau\gamma)}^2+\sigma_e^2\right)\right)^{\frac{1}{2}}} exp\left[\frac{-\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p y_{ijk}^2}{2(\sigma_\mu^2+\sigma_\tau^2+\sigma_\gamma^2+\sigma_{(\tau\gamma)}^2+\sigma_e^2)}\right],$$

and

$$m(y_{ijk}|H_1) = \frac{1}{\left(2\pi\left(\sigma_\mu^2+\sigma_\tau^2+\sigma_\delta^2+\sigma_\gamma^2+\sigma_{(\tau\gamma)}^2+\sigma_e^2\right)\right)^{\frac{1}{2}}} exp\left[\frac{-\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p y_{ijk}^2}{2(\sigma_\mu^2+\sigma_\tau^2+\sigma_\delta^2+\sigma_\gamma^2+\sigma_{(\tau\gamma)}^2+\sigma_e^2)}\right],$$

$$\therefore B_{01}(y_{ijk}) = \frac{\sqrt{\left(\sigma_\mu^2+\sigma_\tau^2+\sigma_\delta^2+\sigma_\gamma^2+\sigma_{(\tau\gamma)}^2+\sigma_e^2\right)}}{\sqrt{\left(\sigma_\mu^2+\sigma_\tau^2+\sigma_\gamma^2+\sigma_{(\tau\gamma)}^2+\sigma_e^2\right)}}$$

$$\frac{\exp\left[\frac{-\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p y_{ijk}^2}{2(\sigma_\mu^2+\sigma_\tau^2+\sigma_\gamma^2+\sigma_{(\tau\gamma)}^2+\sigma_e^2)}\right]}{\exp\left[\frac{-\sum_{i=1}^n\sum_{j=1}^q\sum_{k=1}^p y_{ijk}^2}{2(\sigma_\mu^2+\sigma_\tau^2+\sigma_\delta^2+\sigma_\gamma^2+\sigma_{(\tau\gamma)}^2+\sigma_e^2)}\right]} \quad (14)$$

5.Example (The storage experiment)

In this section, we illustrate the effectiveness of the our methodology. We have choosing the data set which an experiment was conducted during winter season 2008-2009 in one of unhted plastic house which belong to tomato development project in Basrah / Agriculture directorate of Basrah (Khor Al – zubiar) in

Volume : 4 | Issue : 10 | October 2014 | ISSN - 2249-555X

order to investigate the effect of calcium on growth and yield of cultivars of cucumber (Sayff) and the storage temperature on storage capability and quality. The field experiment included 84 variable treatments which were the interaction of three factors of three storage temperatures, which were room temperature, $5C^{\circ}$ and 12 C° and two concentration of calcium chloride 0, 1, 2, 3 % and three storage period 0 , 10 , 15 , 20 , 25 , 30 day. After harvest, the fruit was treated with $CaCl_2$. $2H_2O$ for five minute, then it was stored in three temperatures. Then the chemical changes characteristics of the fruit was reviewed during the storage at 5 days periods. the design of the experiment was done according to the model (1). Table (2) below show the results for the analysis of variance for model, from this table we can see that the calculated F-values is greater than the tabulated F-values at 0.05 level significant that is means there is significant effect for calcium chloride on storage capability for cucumber fruits under different temperatures. The values of parameters $(\mu, \tau_j, \delta_{i(j)}, \gamma_k, (\tau\gamma)_{jk}, \sigma_\delta^2, \sigma_e^2)$ for the model (1) based on ANOVA table shown in table (3).

Table2:ANOVA table for one-way Repeated measures model

Source of variation	d.f	SS	MS	E(MS)	F-Test
Group	2	5.9874	2.9937	4.637	$F_c = \frac{MS_G}{MS_{U(G)}} = 8.9498*$
Unit (Group)	9	3.0102	0.3345	1.643	$F_t(2,9,0.05)=4.26$
Time	6	13.825	2.3042	2.556	$F_c \frac{MS_T}{MS_E} = 9.1618*$ $F_t(6,34,0.05)=2.36$
Group* Time	12	9.2196	0.7683	1.019	$F_c = \frac{MS_{G\times T}}{MS_E} = 3.0549*$ $F_t(12,34,0.05)=2.03$
Residual	34	8.5518	0.2515	0.252	
Total	63	40.594			

Table(3) estimation values for parameters $(\mu, \tau_j, \delta_{i(j)}, \gamma_k, (\tau\gamma)_{jk}, \sigma_\delta^2, \sigma_e^2)$ by ANOVA table

$\hat{\mu}$	$\hat{\tau}_j$	$\widehat{\delta_{i(j)}}$	$\hat{\gamma}_k$	$\widehat{(\tau\gamma)}_{jk}$	$\widehat{\sigma_\delta^2}$	$\widehat{\sigma_e^2}$
4.162	8.117	36.836	25.726	62.423	0.199	0.252

We next applied our methodology (Bayesian method) to the storage experiment data. Figure(1) represent the posterior density of coefficients for the model (1). Figure (2) shows the number for iterations of the Gibbs sampler which used in this study, which was 5000 iterations for this data, while figure (3) shows density estimates based on 5000 iterations of σ_e^2 and σ_δ^2.Table(4) presents the values of the parameters$(\mu, \tau_j, \delta_{i(j)}, \gamma_k, (\tau\gamma)_{jk}, \sigma_\delta^2, \sigma_e^2)$ based on Bayesian method. From table(3) and table(4), we can see that the values of parameters obtained in both ANOVA and Gibbs sampling are nearly alike and encouraging.

Table(4)estimation values for parameters$(\mu, \tau_j, \delta_{i(j)}, \gamma_k, (\tau\gamma)_{jk}, \sigma_\delta^2, \sigma_e^2)$ by Bayesian method

$\hat{\mu}$	$\hat{\tau}_j$	$\widehat{\delta_{i(j)}}$	$\hat{\gamma}_k$	$\widehat{(\tau\gamma)}_{jk}$	$\widehat{\sigma_\delta^2}$	$\widehat{\sigma_e^2}$
4.158	9.494	37.23	26.37	62.72	0.1994	0.231

The model checking approach based on Bayes factor which its value was $B_{01}(y_{ijk}) = 1.5854 \times 10^{-5}$ that is mean the Bayes factor favors H_1 with strong evidence for the storage experiment data.

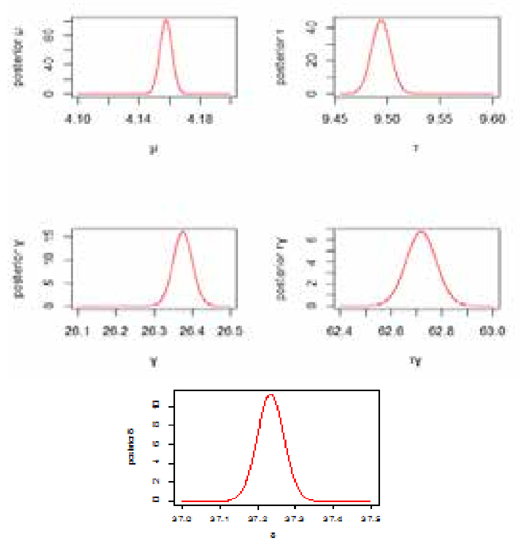
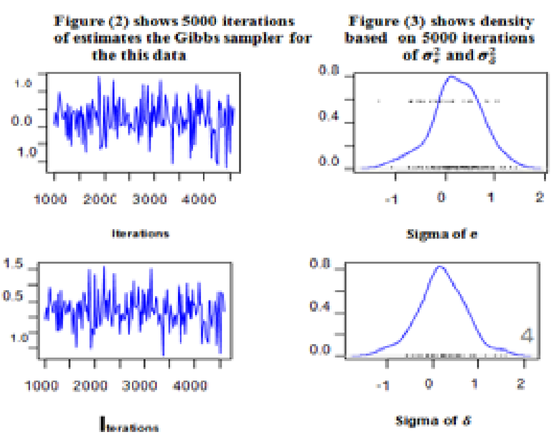


Figure (1) the posterior density of one-way repeated measurements model coefficients $(\mu, \tau_j, \delta_{i(j)}, \gamma_k, (\tau\gamma)_{jk})$



6.Conclusions

1-The posterior density of μ is

$$\mu|\tau_j,\delta_{i(j)},\gamma_k,(\tau\gamma)_{jk},\sigma_\delta^2,\sigma_e^2 \sim N\left[\frac{\frac{\sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}y_{ijk}}{\sigma_e^2}}{\frac{nqp}{\sigma_e^2}+\frac{1}{\sigma_\mu^2}},\frac{1}{\frac{nqp}{\sigma_e^2}+\frac{1}{\sigma_\mu^2}}\right]$$

2- The posterior density of τ_j is

$$\tau_j|\mu,\delta_{i(j)},\gamma_k,(\tau\gamma)_{jk},\sigma_\delta^2,\sigma_e^2 \sim N\left[\frac{\frac{\sum_{i=1}^{n}\sum_{k=1}^{p}y_{i.k}}{\sigma_e^2}}{\frac{np}{\sigma_e^2}+\frac{1}{\sigma_\tau^2}},\frac{1}{\frac{np}{\sigma_e^2}+\frac{1}{\sigma_\tau^2}}\right].$$

3- The posterior density of $\gamma_k$is

$$\gamma_k\ |\mu,\tau_j\ ,\delta_{i(j)},(\tau\gamma)_{jk},\sigma_\delta^2,\sigma_e^2 \sim N\left[\frac{\frac{\sum_{i=1}^{n}\sum_{j=1}^{q}y_{ij.}}{\sigma_e^2}}{\frac{nq}{\sigma_e^2}+\frac{1}{\sigma_\gamma^2}},\frac{1}{\frac{nq}{\sigma_e^2}+\frac{1}{\sigma_\gamma^2}}\right].$$

4- The posterior density of $(\tau\gamma)_{jk}$ is

$$(\tau\gamma)_{jk}\ |\mu,\tau_j\ ,\delta_{i(j)},\gamma_k,\sigma_\delta^2,\sigma_e^2 \sim N\left[\frac{\frac{\sum_{i=1}^{n}y_{i..}}{\sigma_e^2}}{\frac{n}{\sigma_e^2}+\frac{1}{\sigma_{\tau\gamma}^2}},\frac{1}{\frac{n}{\sigma_e^2}+\frac{1}{\sigma_{\tau\gamma}^2}}\right].$$

5- The posterior density of $\delta_{i(j)}$ is

$$\delta_{i(j)}\ |\mu,\tau_j\ ,\gamma_k,(\tau\gamma)_{jk},\sigma_\delta^2,\sigma_e^2 \sim N\left[\frac{\frac{\sum_{k=1}^{p}y_{..k}}{\sigma_e^2}}{\frac{p}{\sigma_e^2}+\frac{1}{\sigma_\delta^2}},\frac{1}{\frac{p}{\sigma_e^2}+\frac{1}{\sigma_\delta^2}}\right].$$

6- The posterior density of σ_δ^2 is

$\sigma_\delta^2|\mu,\tau_j,\delta_{i(j)},\gamma_k,(\tau\gamma)_{jk},\sigma_e^2 \sim IG[\alpha_\delta,\beta_\delta]$.

7- The posterior density of σ_e^2 is

$\sigma_e^2|\mu,\tau_j,\delta_{i(j)},\gamma_k,(\tau\gamma)_{jk},\sigma_\delta^2 \sim IG\left(\alpha_e+\frac{nqp}{2},\beta_e+\frac{RSS}{2}\right).$

8- The Bayes factor for checking the Bayesian repeated measurements model is

$$B_{01}(y_{ijk}) = \frac{\sqrt{\left(\sigma_\mu^2+\sigma_\tau^2+\sigma_\delta^2+\sigma_\gamma^2+\sigma_{(\tau\gamma)}^2+\sigma_e^2\right)}}{\sqrt{\left(\sigma_\mu^2+\sigma_\tau^2+\sigma_\gamma^2+\sigma_{(\tau\gamma)}^2+\sigma_e^2\right)}}\ \frac{\exp\left[\frac{-\sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}y_{ijk}^2}{2(\sigma_\mu^2+\sigma_\tau^2+\sigma_\gamma^2+\sigma_{(\tau\gamma)}^2+\sigma_e^2)}\right]}{\exp\left[\frac{-\sum_{i=1}^{n}\sum_{j=1}^{q}\sum_{k=1}^{p}y_{ijk}^2}{2(\sigma_\mu^2+\sigma_\tau^2+\sigma_\delta^2+\sigma_\gamma^2+\sigma_{(\tau\gamma)}^2+\sigma_e^2)}\right]}.$$

9- There is significant effect for calcium chloride on storage capability for cucumber fruits under different temperatures.

10- The values of parameters obtained in both ANOVA and Gibbs sampling are nearly alike and encouraging.

11- The Bayes factor favors H_1 with strong evidence for the storage experiment data that is mean the correct model is mixed one-way repeated measurements model .

REFERENCE 1] Al-Mouel, A.H.S.,(2004) . Multivariate Repeated Measures Models and Comparison of Estimators, Ph.D.Thesis, East China Normal University, China. [2] Al-Mouel , A.H.S., and Wang, J.L,(2004).One–Way Multivariate Repeated Measurements Analysis of Variance Model, Applied Mathematics a Journal of Chinese Universities, 19,4,pp435- 448. [3] Berger, J. O. (1985) . Statistical Decision Theory and Bayesian Analysis. Springer, New York. [4] Ghosh, J.K., Delampady, M. and Samanta, T. (2006). Introduction to Bayesian Analysis: Theory and Methods. Springer, New York. [5] Gilks, W.R. ,Richardson, S. and Spiegel halter, D.J. ,(1996). Markov Chain Monte Carlo in Practice, Boca Raton, FL: Chapman and Hall. [6] Jon, W.,(2013). Bayesian and frequentist regression methods, Springer New York Heidelberg Dordrecht London. [7] Neter, J. and Wasserman, W. , (1974). Applied linear statistical models, regression analysis of variance and experimental designs, Richard. D. IRWIN, INC. [8] Papoulis,A.,(1997). Markoff Sequences in Probability, Random Variables, and Stochastic Processes, 2nd ed. New York . McGraw-Hill, PP. 528-535, 1984. [9] Vonesh, E.F. and Chinchilli, V.M.,(1997). Linear and Nonlinear Models for the Analysis of Repeated Measurements, Marcel Dakker, Inc., New York. [10] Walsh, B. , (2002). Introduction to Bayesian Analysis, Lecture Notes for EEB 596z . [11] Walsh, B. ,(2004) . Markov Chain Monte Carlo and Gibbs Sampling, Lecture Notes for EEB 581.