RESEARCH PAPER | Statistics | Volume : 4 | Issue : 10 | October 2014 | ISSN - 2249-555X

Inventory Model For Weibull Deteriorating Items With Stock Dependent Demand, Time Varying Holding Cost And Variable Selling Price

KEYWORDS Inventory, Deterioration, Stock dependent demand, Time varying holding cost, Variable selling price

Raman Patel	Reena U. Parekh
Department of Statistics, Veer Narmad South Gujarat University, Surat-395007 (INDIA)	Department of Statistics, Veer Narmad South Gujarat University, Surat-395007 (INDIA)

ABSTRACT *An inventory model for Weibull deteriorating items with variable selling price and stock dependent demand is developed. Holding cost is linear function of time. Shortages are not allowed. Numerical example is considered and sensitivity analysis is also carried out for parameters.*

INTRODUCTION:

In recent years much work has been done regarding inventory models for deteriorating items. Research in the area of deteriorating items inventory model started with the work of Within [16]) who considered fashion goods deteriorating at the end of prescribed storage period. Ghare and Schrader [4] considered inventory problem under constant demand and constant deterioration. Shah and Jaiswal [12] considered an order level inventory model for items deteriorating at a constant rate. Aggarwal [1]) discussed an order level inventory model with constant rate of deterioration. Dave and Patel [3]) developed the deteriorating items inventory model with linear trend in demand. They considered demand as linear function of time. Burewell [2] developed economic lot size model for price dependent demand under quantity and freight discounts. Salameh and Jaber [11] developed a model to determine the total profit per unit of time and the economic order quantity for a product purchased from the supplier. Mukhopadhyay et al. [7] developed an inventory model for deteriorating items with a price-dependent demand rate. The rate of deterioration was taken to be time-proportional and a power law form of the price-dependence of demand was considered. Teng and Chang [13] considered the economic production quantity model for deteriorating items with stock level and selling price dependent demand. Other research work related to deteriorating items can be found in, for instance (Raafat [9], Goyal and Giri [5], Ruxian et al. [10]).

Patra et al. [8] developed a deterministic inventory model when deterioration rate was time proportional. Demand rate was taken as a nonlinear function of selling price, deterioration rate, inventory holding cost and ordering cost were all functions of time. Wang et al. [15] considered the problem of determining the optimal replenishment policy for deteriorating items with variable selling price under stock dependent demand. Tripathy and Mishra [14] dealt with development of an inventory model when the deterioration rate follows Weibull two parameter distribution, demand rate is a function of selling price and holding cost is time dependent. The model was developed by taking care of with and without shortage both cases. Mathew [6] developed an inventory model for deteriorating items with mixture of Weibull rate of decay and demand as function of both selling price and time.In this paper we have developed an inventory model under time varying holding cost and stock dependent demand with variable selling price. Shortages are not allowed. Numerical example is provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.

NOTATIONS AND ASSUMPTIONS:

The following notations are used here:

NOTATIONS:

D(t) : Demand is a function of inventory level I(t)
A : Ordering cost per order
c : Unit purchasing cost per item
HC : Holding cost per unit time is a linear function of time t (x+yt, x>0, 0<y<1)
DC : deterioration cost
MC : Manufacturing cost
SR : Sales Revenue
I(t) : Inventory level at any instant of time t, $0 \le t \le T$
Q : Order quantity
T : Cycle length
α : Scale parametrs ($0 < \alpha < 1$)
β : Shape parameter ($\beta > 0$)
π : Total relevant profit

e

$$I(t) = \begin{pmatrix} -at - \frac{a\alpha}{(\beta+1)}t^{\beta+1} + \frac{1}{2}bt^2 + aT + \frac{a\alpha}{(\beta+1)}T^{\beta+1} + \frac{1}{2}abT^2 \\ +a\alpha t^{\beta+1} + \frac{1}{2}ab\alpha t^{\beta+2} - a\alpha t^{\beta}T - \frac{1}{2}ab\alpha t^{\beta}T^2 + \frac{ab\alpha}{(\beta+1)}t^{\beta+2} \\ +\frac{1}{2}ab^2t^3 - abtT - \frac{ab\alpha}{(\beta+1)}tT^{\beta+1} - \frac{1}{2}ab^2tT^2 \end{pmatrix} \quad (2)$$

(by neglecting higher powers of α and β)

Putting t=0 in equation (2) we get the order quantity Q as:

$$Q = a\left(T + \frac{\alpha}{(\beta+1)}T^{\beta+1} + \frac{1}{2}bT^2\right) \quad (3)$$

Based on the assumptions and descriptions of the model, the total relevant profit, include the following elements:

(i) Ordering cost (OC) = A (4)

(ii) $HC = \int_0^T (x+yt)I(t)dt$

The total profit per unit time is given by

Volume : 4 | Issue : 10 | October 2014 | ISSN - 2249-555X

ASSUMPTIONS:

The following assumptions are used in the development of the model:

- The demand of the product is declining as a function of inventory level I(t).
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- The deteriorated units can neither be repaired nor replaced during the cycle time.
- The deterioration of the items follows a Weibull deterioration with parameter α and β
- The variable selling price S(t) is a function of inventory,

  i.e. $S(t) = S_0 - \rho(D(t)$

  where S_0, ρ, a and b are positive constants.

THE MATHEMATICAL MODEL AND ANALYSIS:

The inventory level of the product at time t over the period (0,T) can be represented by the differential equation

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI(t)}{dt}\ \acute{a}\hat{a}t\ \ I(t) = -(a + bI(t)), \qquad 0 \le t \le T$$

, (1)

with the boundary conditions I(0) = Q, I(T) = 0.

The solution of equation (1) using boundary condition is:

(2)

(by neglecting higher powers of α and β)

Putting t=0 in equation (2) we get the order quantity Q as:

$$Q = a\left(T + \frac{\acute{a}}{(\hat{a}+1)}T^{\hat{a}+1} + \frac{1}{2}bT^2\right)$$

(3)

$$= -\frac{2}{105}\frac{1}{\left(\begin{matrix}\left(\frac{1}{2}+\beta\right)\left(\frac{3}{2}+\beta\right)\left(\frac{5}{2}+\beta\right)(\beta+1)^2\\(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)\end{matrix}\right)}\left(aT\left(\begin{matrix}\frac{35}{2}ab^2(5+\beta)(6+\beta)T^2\alpha^2\rho\\ \left(\begin{matrix}(3+b^2T^2-3bT)\beta^6+\left(-30bT+\frac{129}{4}+\frac{57}{4}b^2T^2\right)\beta^5\\+\left(\frac{301}{4}b^2T^2-\frac{903}{8}bT+\frac{993}{8}\right)\beta^4\\+\left(\frac{1605}{8}-\frac{3327}{16}bT+\frac{381}{2}b^2T^2\right)\beta^3\\+\left(\frac{225}{2}-\frac{3159}{16}bT+\frac{491}{2}b^2T^2\right)\beta^2\\+\left(-\frac{315}{4}bT+\frac{621}{4}b^2T^2\right)\beta+\frac{153}{4}b^2T^2\end{matrix}\right)\end{matrix}\right)\right)$$

$$= \int_0^T (x+yt)\left(\begin{matrix}-at-\frac{a\alpha}{(\beta+1)}t^{\beta+1}+\frac{1}{2}bt^2+aT+\frac{a\alpha}{(\beta+1)}T^{\beta+1}+\frac{1}{2}abT^2\\+a\alpha t^{\beta+1}+\frac{1}{2}ab\alpha t^{\beta+2}-a\alpha t^{\beta}T-\frac{1}{2}ab\alpha t^{\beta}T^2+\frac{ab\alpha}{(\beta+1)}t^{\beta+2}\\+\frac{1}{2}ab^2t^3-abtT-\frac{ab\alpha}{(\beta+1)}tT^{\beta+1}-\frac{1}{2}ab^2tT^2\end{matrix}\right)dt$$

$$= -\frac{1}{15}\frac{1}{(\beta+1)(\beta+2)(\beta+3)(\beta+4)}\left(aT^2\left(\begin{matrix}-15\left(\begin{matrix}(-2yT-2x)\beta^2\\+\left(-14x+ybT^2-11yT\right)\beta-24x+3ybT^2-12yT\end{matrix}\right)\alpha T^{\beta}\\+\left(\begin{matrix}5\left(ybT^2+\left(-\frac{3}{2}y+\frac{3}{2}xb\right)T-3x\right)\alpha T^{\beta}\\+\left(\begin{matrix}yb^2T^3+\frac{15}{8}\left(-\frac{1}{3}y+xb\right)bT^2\\+\left(-\frac{5}{2}xb-\frac{5}{2}y\right)T-\frac{15}{2}x\end{matrix}\right)(\beta+1)\end{matrix}\right)(\beta+2)(\beta+3)(\beta+4)\end{matrix}\right)\right)$$

(5)

(iii) Deterioration cost:

$$DC = c\left[Q - \int_0^T D(t)dt\right] = c\left[Q - \int_0^T (a + bI(t))dt\right]$$

$$= \frac{1}{8}\frac{ac}{(\beta+1)(\beta+2)}\left(\begin{matrix}4\alpha\left(b^2T^2+2+2bT\right)(\beta+2)T^{\beta+1}\\+bT^2\left(\begin{matrix}-4\alpha\left((2+bT)\beta+2bT\right)T^{\beta}\\+(\beta+2)\left(\begin{matrix}\left(-8\alpha+4b\alpha T\right)T^{\beta}\\+Tb(\beta+1)\left(-\frac{4}{3}+bT\right)\end{matrix}\right)\end{matrix}\right)\end{matrix}\right)$$

(6)

(iv) Manufacturing cost is given by

$$MC = cQ = ca\left(T + \frac{\alpha}{(\beta+1)}T^{\beta+1} + \frac{1}{2}bT^2\right) \qquad (7)$$

(v) $SR = \int_0^T S(t)D(t)dt$

$$= \int_0^T \left(S_0 - \rho(a+bI(t))\right)(a+bI(t))dt$$

$$-\frac{2}{105}\frac{1}{\left(\begin{matrix}\left(\frac{1}{2}+\beta\right)\left(\frac{3}{2}+\beta\right)\left(\frac{5}{2}+\beta\right)(\beta+1)^2\\(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)\end{matrix}\right)}\left(aT\left(\begin{matrix}\left(\frac{1}{2}+\beta\right)\left(\frac{3}{2}+\beta\right)\left(\frac{5}{2}+\beta\right)(\beta+1)^2\\ \\(\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)\\ \left(\begin{matrix}b^6T^6\rho a-\frac{7}{4}b^5T^5\rho a-\frac{7}{2}b^4T^4\rho a\\-\frac{7}{2}b^4T^4\rho a+\frac{105}{16}S_0b^3T^3\\+35b^2\left(-\frac{1}{4}S_0+\rho a\right)T^2\\+\frac{105}{2}\left(-\frac{1}{2}S_0+\rho a\right)bT+\frac{105}{2}\rho a-\frac{105}{2}S_0\end{matrix}\right)\end{matrix}\right)\right) \qquad (8)$$

Volume : 4 | Issue : 10 | October 2014 | ISSN - 2249-555X

$$\eth(T) = \frac{SR - OC - HC - DC - MC}{T} \quad (9)$$

Putting values from equations (4) to (8) in equation (9), we get the average profit.

The optimal value of T= T* (say), which maximizes profit $\eth(T)$ can be obtained by differentiating equation (9) with respect to T and equate it to zero

$$\text{i.e. } \frac{d\eth(T)}{dT} = 0, \quad (10)$$

provided it satisfies the condition

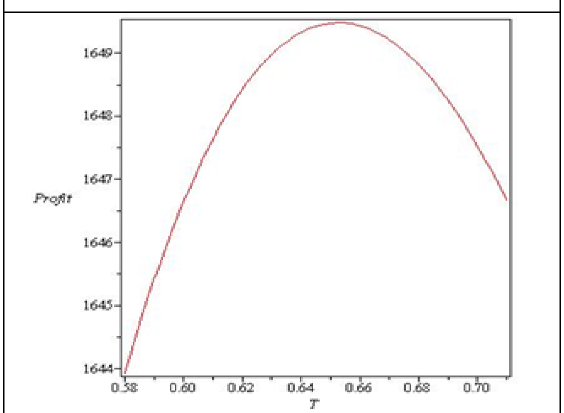
$$\frac{d^2\eth(T)}{dT^2} < 0. \quad (11)$$

NUMERICAL EXAMPLES:

Case I: Considering A= Rs.250, a = 600, b=0.05, c=Rs. 5, S_0= Rs. 15, α=0.01, β =2, x = Rs. 1.7, y=0.05, p=0.01, in appropriate units. The optimal value of T*=0.6528, Q* = 398.6286 and Profit π* = Rs. 1649.4843.

The second order condition given in equation (11) is also satisfied. The graphical representation of the concavity of the profit function is also given.

T and Profit



Graph 1

SENSITIVITY ANALYSIS:
On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Sensitivity Analysis

$$-\frac{2}{105}\frac{1}{\left(\left(\frac{1}{2}+\beta\right)\left(\frac{3}{2}+\beta\right)\left(\frac{5}{2}+\beta\right)(\beta+1)^2 (\beta+2)(\beta+3)(\beta+4)(\beta+5)(\beta+6)\right)}$$

$$aT\left(7b\alpha\left(\frac{1}{2}+\beta\right)\left(\frac{3}{2}+\beta\right)\left(\frac{5}{2}+\beta\right)T^{\beta+1}\left(\left(15\rho a+\frac{15}{4}S_0bT + a\rho b^4T^4 - \frac{5}{2}b^3T^3\rho a - \frac{15}{2}S_0\right)\beta^5 + \left(270\rho a - 50b^3T^3\rho a + 20b^4T^4\rho a - 135S_0 + 75S_0bT\right)\beta^4 + \left(170b^4T^4\rho a - \frac{685}{2}b^3T^3\rho a - 15b^2T^2\rho a - 45b\left(-\frac{155}{12}S_0+\rho a\right)T + 1785\rho a - \frac{1785}{2}S_0\right)\beta^3 + \left(700b^4T^4\rho a - 985b^3T^3\rho a - 270b^2T^2\rho a - 675b\left(-\frac{29}{9}S_0+\rho a\right)T + 5139\rho a - 2565S_0\right)\beta^2 + \left(1269b^4T^4\rho a - 1320b^3T^3\rho a - 1605b^2T^2\rho a - 3330b\left(-\frac{87}{74}S_0+\rho a\right)T + 5400\rho a - 2700S_0\right)\beta + 720\left(b^3T^3\rho a - \frac{3}{2}b^2T^2\rho a - \frac{35}{8}bT\rho a + \frac{15}{4}S_0 - \frac{15}{2}\rho a\right)bT\right)\right)$$

Volume : 4 | Issue : 10 | October 2014 | ISSN - 2249-555X

Para-meter	%t	T	Cost	Q
a	+20%	0.5802	1169.3471	424.2721
	+10%	0.6141	1445.0820	412.0379
	-10%	0.6975	1782.6981	383.8286
	-20%	0.7498	1844.9154	367.3248
x	+20%	0.6031	1584.7942	367.7547
	+10%	0.6265	1616.4914	382.2793
	-10%	0.6827	1683.9414	417.2475
	-20%	0.7169	1720.0700	438.5861
p	+20%	0.6335	905.8065	386.6283
	+10%	0.6430	1277.5546	392.5334
	-10%	0.6631	2021.6042	405.0386
	-20%	0.6740	2393.9236	411.8265
α	+20%	0.6499	1647.7146	396.9343
	+10%	0.6513	1648.5983	397.7507
	-10%	0.6543	1650.3744	399.5058
	-20%	0.6558	1651.2687	400.3823
β	+20%	0.6544	1651.7351	399.4810
	+10%	0.6536	1650.7054	399.0487
	-10%	0.6521	1648.0261	398.2857
	-20%	0.6512	1646.2703	397.8374
A	+20%	0.7125	1576.2473	435.8382
	+10%	0.6834	1612.0665	417.6838
	-10%	0.6206	1688.7473	378.6152
	-20%	0.5863	1730.1726	357.3393

From the table we observe that as parameter a, A and p increases/ decreases average total profit decreases/ increases.

From the table we observe that with increase/ decrease in parameters x, α and β, there is corresponding small decrease/ increase in total profit.

CONCLUSION:

In this paper, we have developed a inventory model for deteriorating items with linear demand under variable selling price. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding decrease/ increase in the value of profit.

REFERENCE 1. Aggarwal, S.P. (1978): A note on an order level inventory model for a system with constant rate of deterioration; Opsearch, Vol. 15, pp. 184-187. | 2. Burewell, T.H. (1997): Economic lot size model for price dependent demand under quantity and freight discounts; International J. of Prod. Eco., Vol. 48, pp. 141-155.. | 3. Dave, U. and Patel, L.K. (1981): (T, si)-policy inventory model for deteriorating items with time proportional demand; J. Oper. Res. Soc., Vol. 32, pp. 137-142. | 4. Ghare, P.N. and Schrader, G.F. (1963): A model for exponentially decaying inventories; J. Indus. Engg., Vol. 15, pp. 238-243. | 5. Goyal, S.K. and Giri, B. (2001): Recent trends in modeling of deteriorating inventory; Euro. J. Oper. Res., Vol. 134, pp. 1-16. | 6. Mathew, R.J. (2013): Perisable inventory model having mixture of Weibull lifetime and demand as function of both selling price and time; International J. of Scientific and Research Publication, Vol. 3(7), pp. 1-8. | 7. Mukhopadhyay, R.N., Mukherjee, R.N. and Chaudhary, K.S. (2004): Joint pricing and ordering policy for deteriorating inventory; Computers and Industrial Engineering, Vol. 47, pp. 339-349 | 8. Patra, S.K., Lenka, T.K. and Ratha, P.C. (2010b): An order level EOQ model for deteriorating items in a single warehouse system with price depended demand in nonlinear (quadratic) form; International J. of Computational and Applied Mathematics, Vol. 5, pp. 277-288. | 9. Raafat, F. (1991): Survey of literature on continuous deteriorating inventory model, J. of O.R. Soc., Vol. 42, pp. 27-37. | 10. Ruxian, L., Hongjie, L. and Mawhinney, J.R. (2010): A review on deteriorating inventory study; J. Service Sci. and management; Vol. 3, pp. 117-129. | 11. Salameh, M.K. and Jaber, M.Y. (2000): Economic production quantity model for items with imperfect quality; J. Production Eco., Vol. 64, pp. 59-64. | 12. Shah, Y.K. and Jaiswal, M.C. (1977): An order level inventory model for a system with constant rate of deterioration; Opsearch; Vol. 14, pp. 174-184. | 13. Teng, J.T. and Chang, H.T. (2005): Economic production quantity model for deteriorating items with price and stock dependent demand; Computers and Oper. Res., Vol. 32, pp. 279-308. | 14. Tripathy, C.K. and Mishra, U. (2010): An inventory model for Weibull deteriorating items with price dependent demand and time varying holding cost; Applied Mathematical Sciences, Vol. 4, pp. 2171-2179. | 15. wang, Y.W., Yang, C.T., Tsai, A. and Hsu, C.P. (2010): Inventory models for deteriorating items with variable selling price under stock dependent demand; Computational Collective Intelligence, Technologies and Applications, Lecture Notes in Computer Science, Vol. 6423, pp. 342-353. | 16. Within, T.M. (1957): Theory of inventory management, Princeton University Press, Princeton, NJ. |