Brick Masonry Shallow Dome as an alternative to Conventional Floor Slab

KEYWORDS: shallow dome, meridional stress, hoop stress, brick masonry, superimposed load, axis of rotation, anisotropic.

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ABSTRACT *Spherical domes have been built to cover large circular areas in many old buildings. Use of these domes is often limited to the topmost / last floor due to large value of rise at the centre. Domes can be used as floors also, provided the rise at the centre is comparatively small, so that the curved top surface could be leveled with little filling. Analysis of spherical dome for uniformly distributed load due to self weight, varying load of the fill and uniformly distributed load on the projected plan area due to floor finish / superimposed load, is presented in this paper. Graphs of meridional and hoop stress with respect to increase in rise at the centre are presented for shallow spherical domes built with 115 mm thick brick masonry. Study reveals that brick masonry shallow dome is a potential alternative to conventional floor slab for circular areas.*

Nomenclature

P-B	:	arc of circle with radius *r*
ϕ	:	angle measured from axis of rotation up to any point on arc *P-B* on surface of dome.
ϕ_1	:	angle measured from axis of rotation up to point B.
$d\phi$	:	small increment of the angle
r	:	radius of sphere
R	:	radius of circle
s	:	unit radial pressure on the circle
S	:	Ring tension
T	:	Meridional thrust per unit length of circle of latitude through point B.
W_u	:	Total self weight of dome up to latitude ϕ_1
W_v	:	Total weight due to fill required to level the top surface of dome up to circle of latitude ϕ_1 passing through point B
W_{F+L}	:	Total weight due to floor finish and superimposed load up to latitude ϕ_1

1.0 Introduction

Brick masonry is man's oldest man made building material. Egyption civilization is thought to be the first to use brick for building material (1971). This was in the beginning of 3100 B. C. The process of un-burnt brick-making has been depicted on a wall painting in the Tomb of Rekhniara (1500 B.C.) (1961). Most of the old temples, mosques, churches and the pyramids of Middle Kingdome were made of bricks (1996). For centuries brick was the main building material and historical, monumental

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architecture is dominated by extensive use of brick masonry. In Western countries the cubic meters of clay units per residential unit ranges from 10 m^3 to 63 m^3 (1998). Masonry structure has become the predominant form of residential buildings in North America, Europe and Australia (1998). Brick masonry is also a simple versatile material capable of being used with greater sophistication and flair (1998). Francis et. al. (1970), Hilsdorf (1969), Khoo and Hendry (1973), Mojsilovic, N. (2005), Tikalsky et. al. (1995), Frunzio et. al. (1997), Gumaste K. S., et. al., (2007), Hemant B. Kaushik, et. al. (2007), Sarangapani G, et. al., (2005) and many other researchers have worked on the establishment of compressive strength of masonry, an anisotropic material, considering the contribution of factors influencing its compressive strength. da Porto F., et. al., (2005), Lopez-Almansa, et. al., (2010), Roca P., et. al., (2007) have done considerable work on the testing, design and construction of brick masonry vaults. Because of extensive research and the availability of brick masonry codes, compressive strength of brick masonry can be easily predicted. Therefore, brick masonry shallow domes can be successfully constructed.

Domes are subjected to two types of stresses, viz., meridional stress and hoop stress. Under the effect of uniformly distributed load on the surface of the dome, meridional stress, which is in radial direction along the surface of dome, is zero at the top and increases gradually to a maximum value at the base. Hoop stress is maximum compressive at the crown which reduces to zero at an angle of 51° 48′ and is tensile downwards for uniformly distributed load on the surface of dome (1992). Since the stresses (meridional and hoop), above the angle of 51° 48′ with the vertical axis passing through the crown, are compressive only, shallow domes can be more effective and economical in comparison to flat reinforced concrete slabs. Moreover, reinforced concrete can be replaced with material such as stone or brick masonry that can resist compression effectively.

1.1 Loads on Shallow Spherical Dome

To calculate stresses at any point, on the surface of a shallow dome, on a circle of latitude ϕ with the vertical axis of rotation, it is necessary to calculate the surface area and total vertical load over this area. Calculations of the area and loads are as follows:

1.1.1 Surface Area of the Dome

Let us consider a part of the dome between the crown *P* and plane of latitude through point *B* as shown in Fig. 1.1. Therefore,

$rd\phi$: length of the element arc

$rSin\phi$: horizontal distance from the element to the axis of rotation

$rd\phi * 2\pi rSin\phi$: area of the element of length $rd\phi$ rotated about axis

Total surface area of spherical dome described by rotating arc P-B about axis of rotation,

$$A = \int_0^{\phi_1} rd\phi * 2\pi rSin\phi$$

or $$A = 2\pi r^2 \int_0^{\phi_1} Sin\phi d\phi$$

or $$A = 2\pi r^2 \left[-Cos\phi\right]_0^{\phi_1}$$

or $$A = 2\pi r^2 \left[Cos\phi\right]_{\phi_1}^{0}$$

or $$A = 2\pi r^2 \left(Cos0 - Cos\phi_1\right)$$

or $$A = 2\pi r^2 \left(1 - Cos\phi_1\right) \qquad \text{............ 1.1}$$

1.1.2 Uniformly Distributed Load on Surface of the Dome

If w is the intensity of uniformly distributed load over the surface of the dome and is same for all elements, the total load W_u on the dome between points P and B is,

$$W_u = w * A = 2\pi r^2 * w(1 - Cos\phi_1) \qquad \text{............ 1.2}$$

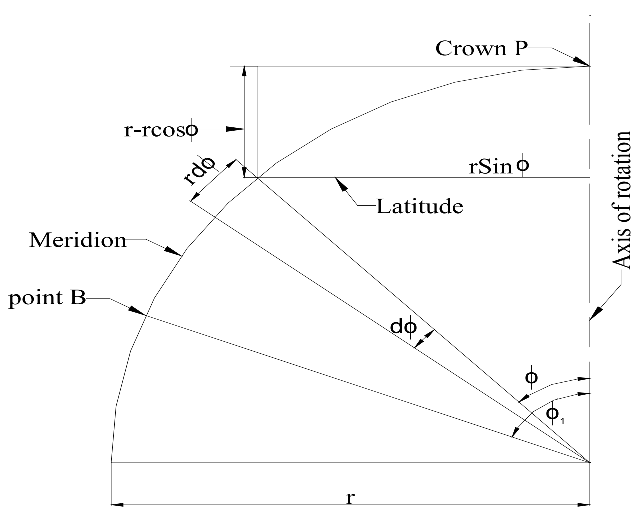

Fig. 1.1 Section of Shallow Spherical Dome Under Consideration

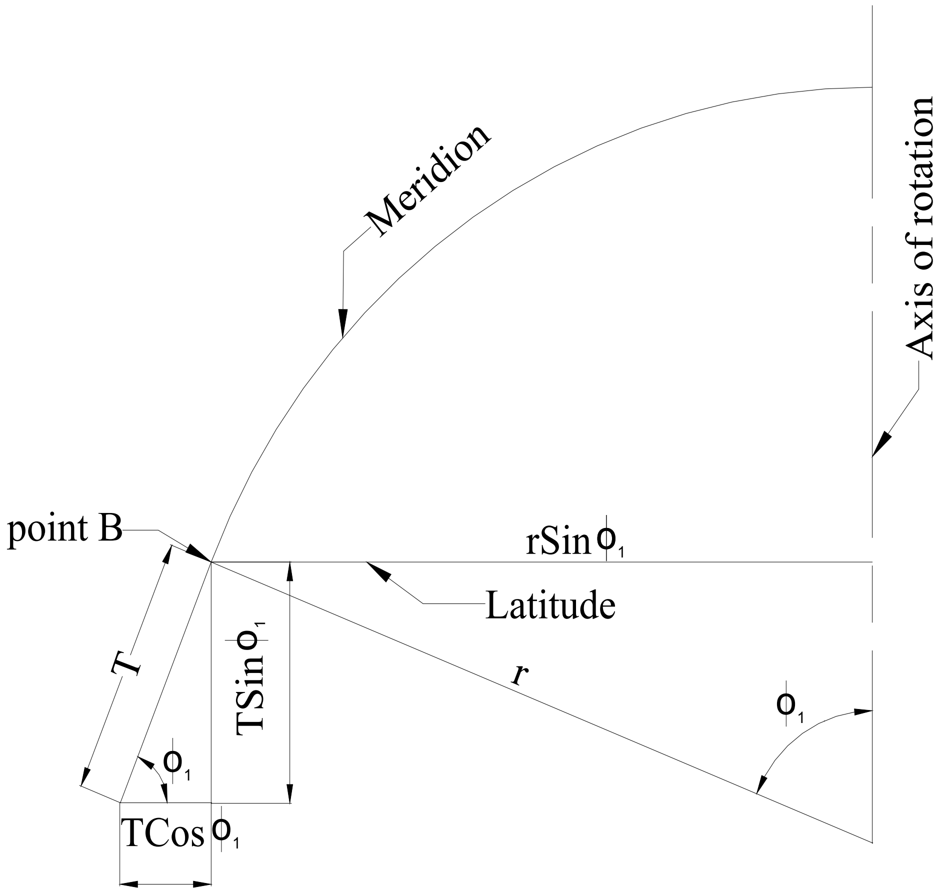

Fig. 1.2 Section of Shallow Spherical Dome Showing Forces at B

1.1.3 Load of Fill required to Level the Surface of Dome

If the top surface of the dome is required to be leveled by filling to use it as a floor, and density of fill material is w' then unit load on the element $rd\phi$ equals

$$w' r(1 - Cos\phi)$$

Load on the element of dome described by rotating $rd\phi$ about the axis equals

$$rd\phi * 2\pi r Sin\phi * w' r(1 - Cos\phi)$$

Total load of the fill on the spherical dome described by rotating arc *P-B* about the axis equals

$$W_v = \int_0^{\phi_1} rd\phi * 2\pi r Sin\phi * w' r(1 - Cos\phi)$$

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or $$W_v = 2\pi r^3 * w' \int_0^{\phi_1} (Sin\phi - Sin\phi Cos\phi) d\phi$$

or $$W_v = 2\pi r^3 * w' \int_0^{\phi_1} \left(Sin\phi - \frac{Sin2\phi}{2} \right) d\phi$$

or $$W_v = 2\pi r^3 * w' \left[(-Cos\phi) - \left(-\frac{Cos2\phi}{4} \right) \right]_0^{\phi_1}$$

or $$W_v = 2\pi r^3 * w' \left[\frac{Cos2\phi}{4} - Cos\phi \right]_0^{\phi_1}$$

or $$W_v = 2\pi r^3 * w' \left[\frac{Cos2\phi_1}{4} - Cos\phi_1 - \left(\frac{Cos0}{4} - Cos0 \right) \right]$$

or $$W_v = 2\pi r^3 * w' \left[\frac{Cos2\phi_1 - 1}{4} + 1 - Cos\phi_1 \right]$$

or $$W_v = 2\pi r^3 * w' \left[\frac{Cos2\phi_1 + 3}{4} - Cos\phi_1 \right] \qquad \text{............ 1.3}$$

1.1.4 Load of Floor Finish and Superimposed Load on the projected area on Plan of Dome

If the dome is further subjected to a uniformly distributed load w_F from the floor finish and uniformly distributed load w_L due to live load on plan area of the dome, then unit load on the element $rd\phi$ is ,

$$(w_F + w_L) Cos\phi$$

Load on the element of dome described by rotating $rd\phi$ about the axis equals,

$$rd\phi * 2\pi r Sin\phi * (w_F + w_L) Cos\phi$$

Total load of the floor finish and the live load on the spherical dome described by rotating arc *P-B* about the axis equals,

$$W_{F+L} = \int_0^{\phi_1} rd\phi * 2\pi r Sin\phi * (w_F + w_L) Cos\phi$$

or $$W_{F+L} = 2\pi r^2 (w_F + w_L) \int_0^{\phi_1} Sin\phi Cos\phi d\phi$$

or $$W_{F+L} = 2\pi r^2 (w_F + w_L) \int_0^{\phi_1} \frac{Sin2\phi}{2} d\phi$$

or $$W_{F+L} = 2\pi r^2 (w_F + w_L) \left[-\frac{Cos2\phi}{4} \right]_0^{\phi_1}$$

or $$W_{F+L} = 2\pi r^2 (w_F + w_L) \left[\frac{Cos0 - Cos2\phi_1}{4} \right]$$

or $$W_{F+L} = \pi r^2 (w_F + w_L) \left[\frac{1 - Cos2\phi_1}{2} \right] \qquad \text{............ 1.4}$$

1.2 Meridional Thrust and Hoop Forces

Consider,

W : Load above plane of latitude through point B (refer Fig. 1.2)
T : Meridional thrust per unit length of circle of latitude through point B
$2\pi r Sin\phi_1$: Length of circle of latitude through point B

$TSin\phi_1$: Vertical component of T

Equating W with the vertical component of T, we get

$$W = 2\pi r Sin\phi_1 * TSin\phi_1$$

or $$W = 2\pi r Sin^2\phi_1 * T$$

or $$T = \frac{W}{2\pi r * Sin^2\phi_1} \qquad \text{............ 1.5}$$

If the dome is discontinued along circle of latitude through point B, a circular ring through that point is subject to a unit radial force of $TCos\phi_1$.

A unit radial pressure s on a circular ring with radius R causes a ring tension of $S = s * R$

Substituting for $s = TCos\phi_1$ and $R = rSin\phi_1$, we get

$$S = TCos\phi_1 * rSin\phi_1$$

or $$S = \frac{WCos\phi_1}{2\pi r * Sin^2\phi_1} * rSin\phi_1$$

or $$S = \frac{WCos\phi_1}{2\pi Sin\phi_1} \quad \text{............ 1.6}$$

At point *P*, on the axis of rotation, consider an elemental square subjected to a thrust *T* on each of its four sides. Considering the effect of uniformly distributed load w on surface of dome and substituting for W_u from Eq. (1.2) in Eq. (1.5) and $Sin^2\phi_1 = (1 - Cos\phi_1)(1 + Cos\phi_1)$ we get,

$$T = \frac{2\pi r^2 w(1 - Cos\phi_1)}{2\pi r(1 - Cos\phi_1)(1 + Cos\phi_1)}$$

Or $$T = \frac{wr}{1 + Cos\phi_1}$$

Or $$T = \frac{1}{2}wr \qquad (\because \phi_1 = 0 \,\&\, \therefore Cos\phi_1 = 1)$$

Considering the effect of all the loads together then total load W on plan area of dome around a circle of latitude is given by,

$$W = W_u + W_v + W_{F+L}$$

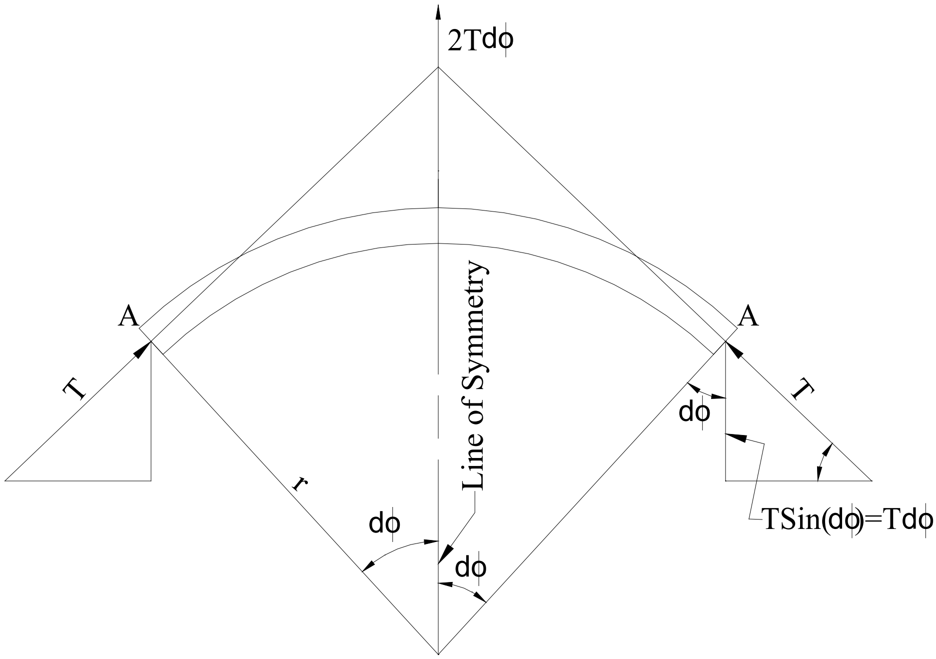

Fig. 1.3 Arc A-A, Part of Circular Ring of Radius '*R*'

Where W_u is load due to uniformly distributed load w on the surface of dome, and is given by Eq. (1.2) as follows:

$$W_u = 2\pi r^2 w(1 - Cos\phi_1)$$

W_v is load due to fill with a material of density w' , and is given by Eq. (1.3) as follows:

$$W_v = 2\pi r^3 * w'\left[\frac{Cos2\phi_1 + 3}{4} - Cos\phi_1\right]$$

And W_{F+L} is load due to floor finish and live load and is given by Eq. (1.4) as follows:

$$W_{F+L} = \pi r^2 (w_F + w_L)\left[\frac{1 - Cos2\phi_1}{2}\right]$$

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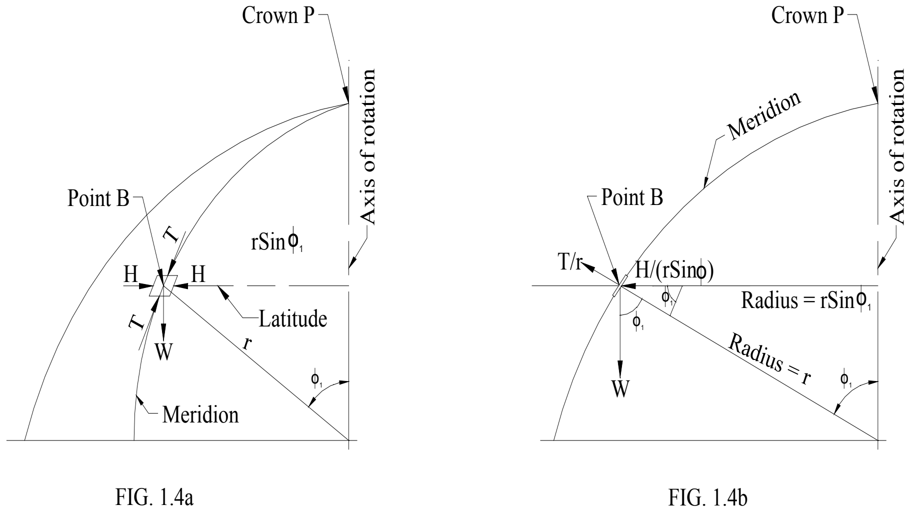

Fig. 1.4a & 1.4b Equilibrium of the Element of Dome at B and in Elevation

Meridian thrust *T* and ring tension *S* at plane of latitude through point *B* is given by,

Meridional Thrust $T = \dfrac{W}{2\pi r Sin^2(\phi_1)}$ 1.7

Ring Tension $S = \dfrac{W Cos\phi_1}{2\pi Sin\phi_1}$ 1.8

Where, W = total load above that circle

An elemental square at point B is subject to two reactive forces under the effect of vertical load *W* on the element. Two reactive forces are

T = Meridional thrust tangential to meridian and

H = Hoop force tangential to circle of latitude.

Load on the element, *W* = $w + \{w' r(1 - Cos\phi) + (w_F + w_L)\} Cos\phi$

Fig. 1.3 shows an arc $A - A$ which is part of a circular ring with radius r. The compression in the ring is T, and the angle subtending the arc is $2d\phi$ where $d\phi$ is considered such a small angle that $Sin(d\phi) = d\phi$. It is seen from the triangles in the figure that the component of T in the direction parallel to the symmetry line is $TSind\phi$ which equals $Td\phi$, since $d\phi$ is very small angle. The total radial component is $2Td\phi$ on

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an arc $A - A$ of length $2rd\phi$. Therefore, the radial component for an arc of unit length is,

$$\frac{2Td\phi}{2rd\phi} = \frac{T}{r}$$

Regarding the equilibrium of the element of the dome, the tangential force may be replaced by its radial component $\frac{T}{r}$. The H - force is tangential to the circle of latitude with radius $rSin\phi$ and may be replaced by its radial component lying in the plane of the circle and equaling $\frac{H}{rSin\phi_1}$. The two components and the load on the element lie in the same vertical plane. As the element is in equilibrium, sum of projections of the three forces must be equal to zero.

Projecting on the line through centre of the dome we get,

$$\frac{T}{r} + \frac{H}{rSin\phi_1} * Sin\phi_1 - \left[w + \left\{w'r\left(1 - Cos\phi_1\right) + \left(w_F + w_L\right)\right\} * Cos\phi_1\right]Cos\phi_1 = 0$$

or,

$$H = -T + \left[w + \left\{w'r\left(1 - Cos\phi_1\right) + \left(w_F + w_L\right)\right\}Cos\phi_1\right]rCos\phi_1 \quad .... \quad 1.9$$

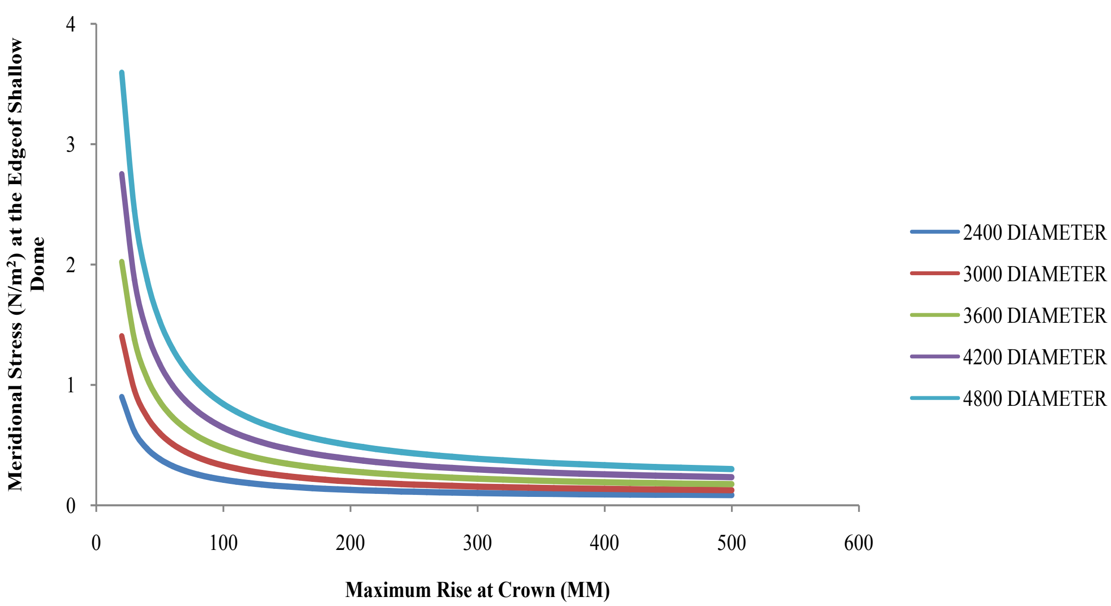

Fig. 1.5 Maximum Meridional Stress at Lower Edge of Shallow Dome versus Maximum Rise at Centre

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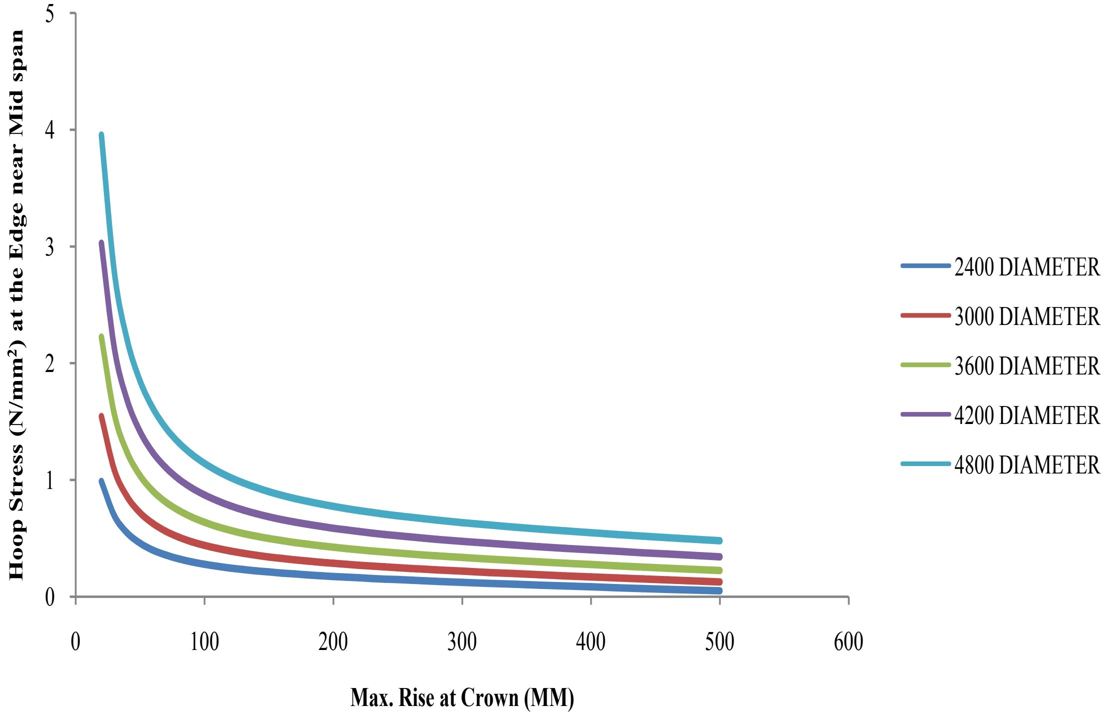

Fig. 1.6 Hoop Stress at Face of Beam at Mid Span versus Maximum Rise at Centre

115 mm thick brick masonry shallow spherical dome leveled with plain cement concrete is considered for calculation of stresses. Floor finish is of plain cement concrete 50 mm thick. Dome is subjected to superimposed load of 2.0 kN/m^2 on its surface in addition to self load, fill load and load of floor finish. Meridional and hoop stresses are worked out with respect to different values of rise at the crown and for different spans. Curves showing variation of stresses with respect to rise at centre, for different spans, are plotted in Fig. 1.5 and 1.6. It is observed that the maximum stress is limited to 0.9 N/mm^2 for covering circular area of 4.8 m diameter with a small rise of 100 mm at the crown. Meridional stress reduces with reduction in diameter and increase in rise at the centre. Hoop stress is also limited to 1.1 N/mm^2 for covering an area of 4.8 m diameter, with a maximum rise at the centre as 100 mm only. It reduces with reduction in diameter and increase in rise. It may also be noted that stresses increase rapidly with decrease in rise below 100 mm and the decrease is insignificant due to increase in rise till 200 mm at the centre. Thereafter, reduction in stress is very small with increase in rise.

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1.3 Conclusions

Following conclusions are drawn from above study :

i) Shallow domes can be very effectively used as floor slabs to cover circular areas of diameters up to 5.0 m, by filling to level, on the top surface of the dome, with suitable filling material.

ii) Appropriate value of rise at the centre may be between 100 mm to 200 mm. However, it may be worked out for a particular diameter and loading.

iii) Reduction in meridional and hoop stresses is very steep initially with increase of rise at the centre from 20 mm to 100 mm but further reduction in stresses is gradual for increase in rise from 200 mm to 500 mm and above.

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