

# Mathematical Modelling for Control Configuration of $NO_x$ Formation in Coal Combustion Processes

KEYWORDS	coal combustion, mathematical modelling, formation of $NO_x$				
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**ABSTRACT** The paper deals with the problems concentrated on formation of the most important types NOx. The main part of paper describes the Zel'dovič model of creation NO in coal and some results from mathematical calculations.

#### **1. INTRODUCTION**

Oxides of nitrogen, designated as  $NO_x$ , generally include NO (nitrogen monoxide, that is also known as nitric oxide) and  $NO_2$  (nitrogen dioxide) are investigated numerically using a finite volume method for the solution of the conservation and reaction equations governing the problem predicting  $NO_x$  emissions from the combustor good agreement with the experimental measurements.

#### 2. EMISSIONS OF NITROGEN OXIDES AS ONE OF MAJOR AIR POLLUTANTS

Pollutants are usually classified into several groups according their toxicity, spread, potential danger and emission sources. Nitrogen oxides together with sulphur dioxide, carbon monoxide, hydrocarbons, solid particles and photochemical oxides belong to major (critical) air pollutants. Nitrogen oxides, as primary air pollutants, are formed predominantly by the high/temperature fixation of nitrogen and oxygen in power plants and combustion equipment.

Fig.1 shows contribution of individual gases (Sulphur dioxide, Nitrogen oxides, Carbon monoxide) on emissions of major air pollutants in the years from 2011 to 2012. The measured values of Nitrogen oxides range from 105.6 thousand Tons/Year to 82.8 thousand Tons/Year.

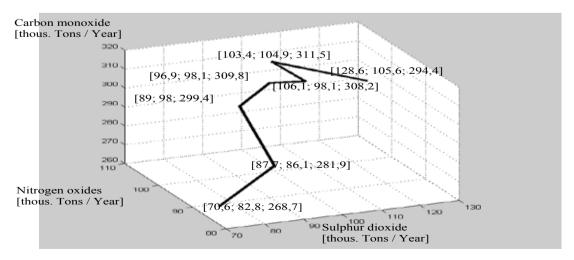


Fig.1 Contribution of individual gases (Sulphur dioxide, Nitrogen oxides, Carbon monoxide) on emissions of major air pollutants in the years from 2011 to 2012.

It is essential, that permanent economic growth has a negative influence on the quality of environment. Tightening environmental regulations increase the possibility for sustainable development of natural sources. It is also a motivation for development of both science and innovations in entrepreneur activities. Environmental protection is also reflected in a number of investments of both enterprises and municipalities.

## **3. CONTROL OF NO<sub>X</sub> FORMATION**

NO<sub>x</sub> formation depends especially on:

- temperature,
- oxygen partial pressure,
- coal properties.

One of the coal properties that might be important for  $NO_x$  formation is content of nitrogen in it. Also every boiler is specific from the view point of low  $NO_x$  combustion techniques implementation. Let us mention some of the different approaches to reduce the  $NO_x$  emissions from the boiler [2]:

- the burner quality (low NO<sub>x</sub> burner),
- fuel reburning,
- air staging in furnace,
- recirculation of gas flue.

## 3.1 NO<sub>x</sub> formation using coal reburning technology in industrial boilers

Coal reburning technology is one of those that try to set control of both  $CO_x$  and  $NO_x$  formation during the burning process in industrial boilers. Fig. 2 shows the technology where we can recognize three zones with the typical reactions. Both burning and reburning are sophisticated processes that are influenced also by the fuel quality. Some of the approximate reactions of burning and reburning processes are mentioned in the following [3].

Fuel combustion and NO<sub>x</sub> formation are typical reactions for primary combustion zone:

$$N (fuel) + O_2 \rightarrow NO_x \tag{1}$$

$$N_2(air) + O_2 \rightarrow NO_x \tag{2}$$

In general:

Fuel + 
$$O_2 \rightarrow CO_2 + CO + H_2O + O$$
ther Species (3)

Using the Zel'dovič mechanism [4], NO production can be expressed as (5):

$$\frac{d[NO]}{d\tau} = \frac{1}{1 + \frac{k_{-3}.[NO]}{k_4.[O_2] + k_5.[OH]}} \cdot \left[ 2.k_3.[N_2][O] - \frac{2.k_{-3}.k_{-4}.[NO]^2.[O]}{k_4.[O_2] + k_5.[OH]} - \frac{2.k_{-3}.k_{-5}.[NO]^2.[H]}{k_4.[O_2] + k_5.[OH]} \right]$$
(4)

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Using some approppriate mathematical modification [6],[7]:

$$\frac{d[NO]}{d\tau} = \frac{2[O](k_3.k_4.[O_2][N_2] - k_{-3}.k_{-4}.[NO]^2)}{k_4.[O_2] + k_{-3}.[NO]},$$
(5)

where  $k_3$ ,  $k_4$ ,  $k_5$ ,  $k_{-3}$ ,  $k_{-4}$ ,  $k_{-5}$  are rate constants with values given in cm<sup>3</sup>.mol<sup>-1</sup>.s<sup>-1</sup>:

$$k_{3} = 1,36.10^{14}.e^{-\frac{75400}{R.T}}$$

$$k_{4} = 6,43.10^{9}.T.e^{-\frac{6250}{R.T}}$$

$$k_{-3} = 3,1.10^{-3}.e^{-\frac{334}{R.T}}$$

$$k_{-4} = 1,55.10^{9}.T.e^{-\frac{38640}{R.T}}$$

Reburning technology, that is applied in industrial boilers, is based on steam injection into the reburning zone. Using the reburning techniques allows optimization of both combustion (flue) gas and air to control both CO and  $NO_x$  formation. Reburning technology is a process that uses hydrocarbon radicals and catalyst to covert nitrogen oxide (NO) to nitrogen (N<sub>2</sub>) and carbon dioxide (CO<sub>2</sub>):

$$2NO + 2CO + catalyst = N_2 + 2CO_2$$
(6)

In general:

Reburn Fuel 
$$\rightarrow$$
 CH<sub>x</sub> + Other Species (7)

$$CH_x + NO_x \rightarrow \bullet CN + \bullet NH_2 + H_2O$$
 (8)

 $NO_x + \bullet CN \rightarrow N_2 + CO$  (9)

Fig.2 Levels of burning processes in reburn technology with the stress on  $NO_x$  formation representation.

Typical processes in Burnout zone can be expressed by the reactions [8], [9]:

$$Air + CH_x + CO \rightarrow CO_2 + H_2O$$

$$\bullet CN + \bullet NH_2 + O_2 \rightarrow NO_x + CO_2 + H_2O$$
(10)
(11)

Taking into account reactions (10) and (11), it is clear, that if there is not enough air in the reaction (10), CO remains. If there is too much oxygen in the reaction (11), greater amount of  $NO_x$  is formed in the burner. Optimization of the amount of air and oxygen, during the appropriate temperature, is one of the ways to control both CO and  $NO_x$  formations [10].

## 4. MATHEMATICAL MODELLING FOR OPTIMIZATION OF THE RELATIONSHIPS BETWEEN AIR, OXYGEN, TEMPERATURE AND NO<sub>X</sub> FORMATION

Chosen values for amount of air excess, temperature, oxygen and  $NO_x$  obtained during the experimental measurements on pulse burner are used to find common relationships and specify constraints for  $NO_x$  formation [11].

Using the method of multiple regression, we will try to express common relationship among amounts of air excess, temperature, oxygen and  $NO_x$  formation.

Let be use the following signification:

 $x_1$  ... air excess,  $x_2$  ... temperature in °C,  $x_3$  ... oxygen in mol.cm<sup>-3</sup>, y ... NO<sub>x</sub> in mol.cm<sup>-3</sup>.

Assuming the linear relationship, we can express:

$$\hat{y} = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 \tag{12}$$

$$|y_1 - \hat{y}_1|$$
 (13)

The squares of residuals would be expressed as [12],[13]:

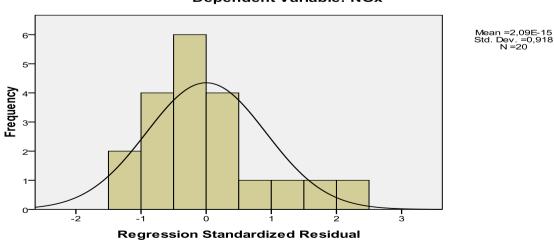
$$S(a_0, a_1, a_2, a_3) = \sum_{i=1}^n (y_i - \hat{y}_i)^2 = \sum_{i=1}^n (y_i - a_0 - a_1 x_{1i} - a_2 x_{2i} - a_3 x_{3i})^2$$
(14)

Table 1. Casewise Diagnostics with the residuals computed using the program system
SPSS. Casewise Diagnostics <sup>a</sup>

Case	Std.			
Number	Residual	NOx	Predicted Value	Residual
1	-1,275	,0000000375299	7,226727907167759E-9	-3,473737907167760E-9
2	2,128	,0000000986418	4,064593011073113E-9	5,799586988926887E-9
3	-,325	,00000000157045	2,456799226605485E-9	-8,863492266054848E-10
4	,031	,00000000191858	1,832806803389715E-9	8,577319661028473E-11
5	,118	,00000000206717	1,745400717720162E-9	3,217692822798382E-10
6	,041	,00000000206839	1,957495267974767E-9	1,108947320252328E-10
7	-,093	,00000000194660	2,199900874642326E-9	-2,533008746423262E-10
8	-,316	,00000000179162	2,653121987299688E-9	-8,615019872996883E-10
9	-,554	,00000000161116	3,120816444619380E-9	-1,509656444619380E-9
10	-,757	,00000000150157	3,563807055124524E-9	-2,062237055124524E-9
11	-,983	,0000000127801	3,956554801486063E-9	-2,678544801486063E-9
12	-1,154	,00000000113328	4,278888394612795E-9	-3,145608394612795E-9
13	1,929	,00000000978004	4,521867703494209E-9	5,258172296505791E-9
14	1,280	,0000000816945	4,681686598011462E-9	3,487763401988538E-9
15	,664	,0000000655869	4,749116475334722E-9	1,809573524665279E-9
16	,413	,0000000584794	4,722254270404578E-9	1,125685729595423E-9
17	-,101	,0000000433061	4,605829523296332E-9	-2,752195232963324E-10
18	-,182	,0000000380282	4,297573177043346E-9	-4,947531770433461E-10
19	-,279	,0000000325853	4,018884206156219E-9	-7,603542061562190E-10
20	-,586	,00000000205153	3,649485554543238E-9	-1,597955554543238E-9

a. Dependent Variable: NOx

Histogram



Dependent Variable: NOx

Fig.3 Histogram for dependent variable NO<sub>x</sub> with the value of mean and standard deviation.



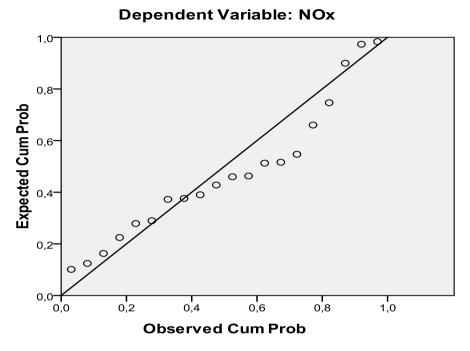


Fig.4 Normal P-P plot of regression standardized residual for dependent variable NO<sub>x</sub>.

To fined the proper values for the coefficients:  $a_0$ ,  $a_1$ ,  $a_2$  and  $a_3$  the sum (14) should be minimized:

$$\frac{\partial S(a_0, a_1, a_2, a_3)}{a_0} = 2\sum_{i=1}^n (y_i - a_0 - a_1 x_{1i} - a_2 x_{2i} - a_3 x_{3i})(-1)$$

$$\frac{\partial S(a_0, a_1, a_2, a_3)}{a_1} = 2\sum_{i=1}^n (y_i - a_0 - a_1 x_{1i} - a_2 x_{2i} - a_3 x_{3i})(-x_{1i})$$

$$\frac{\partial S(a_0, a_1, a_2, a_3)}{a_2} = 2\sum_{i=1}^n (y_i - a_0 - a_1 x_{1i} - a_2 x_{2i} - a_3 x_{3i})(-x_{2i})$$

$$\frac{\partial S(a_0, a_1, a_2, a_3)}{a_3} = 2\sum_{i=1}^n (y_i - a_0 - a_1 x_{1i} - a_2 x_{2i} - a_3 x_{3i})(-x_{2i})$$
(15)

Let each derivative equal zero, then (Abrikosov, 1975):

$$na_{0} + \sum_{i=1}^{n} x_{1i}.a_{1} + \sum_{i=1}^{n} x_{2i}.a_{2} + \sum_{i=1}^{n} x_{3i}.a_{3} = \sum_{i=1}^{n} y_{i}$$
$$\sum_{i=1}^{n} x_{1i}.a_{0} + \sum_{i=1}^{n} (x_{1i})^{2}.a_{1} + \sum_{i=1}^{n} x_{1i}x_{2i}.a_{2} + \sum_{i=1}^{n} x_{1i}x_{3i}.a_{3} = \sum_{i=1}^{n} x_{1i}y_{i}$$

$$\sum_{i=1}^{n} x_{2i}a_{0} + \sum_{i=1}^{n} x_{1i}x_{2i}a_{1} + \sum_{i=1}^{n} (x_{2i})^{2}a_{2} + \sum_{i=1}^{n} x_{2i}x_{3i}a_{3} = \sum_{i=1}^{n} x_{2i}y_{i}$$
(16)  
$$\sum_{i=1}^{n} x_{3i}a_{0} + \sum_{i=1}^{n} x_{1i}x_{3i}a_{1} + \sum_{i=1}^{n} x_{2i}x_{3i}a_{2} + \sum_{i=1}^{n} (x_{3i})^{2}a_{3} = \sum_{i=1}^{n} x_{3i}y_{i}$$

Solving the system of linear equations (16), we can get the values for the coefficients:  $a_0$ ,  $a_1$ ,  $a_2$  and  $a_3$ . There are many methods to do so. The program system SPSS was used (Table 3) together with the setting the Pearson correlations (Table 2).

Table 2. Pearson Correlations computed using the program system SPSS.(Correlations)

		NOx	Air	Temperature	Oxygen
Pearson Correlation	NOx	1,000	,132	-,105	-,334
	Air	,132	1,000	-,991	-,777
	Temperature	-,105	-,991	1,000	,694
	Oxygen	-,334	-,777	,694	1,000

Table 3. Values:  $a_0$  as a constant and coefficients  $a_1$ ,  $a_2$ ,  $a_3$  computed using the program system SPSS.

Mode	9	Unstandardized Coefficients	Standardized Coefficients	t	Sig.
1	(Constant)	1,581E-7		1,314	,207
	Air	-4,456E-8	-4,647	-1,263	,225
	Temperature	-4,522E-11	-3,805	-1,181	,255
	Oxygen	-,031	-1,300	-1,856	,082

Partial dependences are expressed on graphs: Fig. 3, Fig. 4 and Fig. 5.

#### **Partial Regression Plot**

**Dependent Variable: NOx** 

#### 7,500000000000E-9 0 0 5,000000000000E-9 0 Ň 2,500000000000E-9 0 0 0,0000000000000000E0 0 0 0 0 0 0 0 0 0 00 00 0 -2,5000000000000E-9 0 0,000 -0,020 0,020 0,040 Air

Fig.5 Partial regression plot, that expresses dependences between amounts of air excess and  $NO_x$  formation using the program system SPSS.



### Dependent Variable: NOx

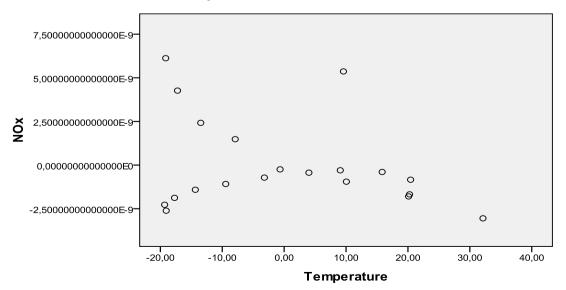


Fig.6 Partial regression plot, that expresses dependences between temperature and  $NO_x$  formation using the program system SPSS.

#### **Partial Regression Plot**

**Dependent Variable: NOx** 

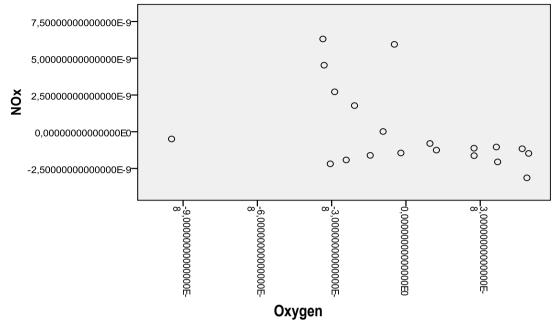


Fig.7 Partial regression plot, that expresses dependences between amounts of oxygen and NO<sub>x</sub> formation using the program system SPSS.

## Conclusion

Mathematical modelling was used to describe some processes of chemical reactions which are typical for industrial boiler aiming especially to control the formation of some pollutants, such as NO<sub>x</sub> and CO [14]. Control of both NO<sub>x</sub> and CO formations during burning and reburning processes can be easier knowing the relationships among the amounts of air excess, oxygen, temperature and NO<sub>x</sub> formation [15]. The common relationship is expressed in (12) and Table 3 comprises the concrete values of  $a_0$  and coefficients  $a_1$ ,  $a_2$ ,  $a_3$ , that were obtained using the program system SPSS.

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