RESEARCH PAPER | Mathematics | Volume : 4 | Issue : 9 | September 2014 | ISSN - 2249-555X

Modified Approach on Shortest Path in Intuitionistic Fuzzy Environment

KEYWORDS: TIFN, SPP, Order relation, Graded Mean Integration Representation.

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ABSTRACT *A new method is proposed for finding the Shortest Path Problem(SPP) with Triangular Intuitionistic Fuzzy Numbers(TIFN). Here, to find the smallest edge by the intuitionistic fuzzy distance using graded mean integration. We discuss the SPP from a specified vertex to all other vertices in a network. An illustrative example is given to demonstrate our proposed approach.*

1.Introduction

Many researchers have paid much attention to the fuzzy SPP since it is central to lots of applications. In the fuzzy SPP, the fuzzy shortest length and the corresponding shortest path are the useful information for the decision makers. In this paper, we proposed a new approach that can obtain the important information. The SPP in a network has attracted many previous researchers since it is important to many applications such as communications, routing and transportation [4]. In a network problem, the arcs are assumed to represent transportation time or cost. In the real world, the transportation time or cost may be known only approximately due to vagueness of information. To deal with this imprecise information, the probability concepts could be employed. However, to conduct probability distributions requires either a priori predictable regularity or a posteriori frequency determination. The fuzzy SPP was first discussed by Dubois and Prade [6]. Although the shortest path distance can be obtained, a corresponding shortest path cannot be identified [7]. Liu and Kao's algorithm for finding the shortest path can obtain a non-dominated shortest path [11].

This paper is organized as follows. In section 2, preliminary concepts and definitions are given. The procedure for finding SPP using TIFN developed in section 3.An illustrative example is provided in section 4 to find the shortest path. The last section draws some concluding remarks.

2.1 Positive fuzzy number A fuzzy number $\tilde{a}$ is called a positive fuzzy number if its membership function is such that $\mu_{\tilde{a}}(x)$ = 0 for all x < 0 .

2.2 Intuitionistic Fuzzy Number (IFN) Let $A=\{x,\mu_A(x),\gamma_A(x)/x\in X\}$ be an IFS ,then we call $(\mu_A(x),\gamma_A(x))$ an IFN .We denote it by $(\langle a,b,c\rangle,\langle e,f,g\rangle)$ where $\langle a,b,c\rangle$ and $\langle e,f,g\rangle \in F(I)$, $I=[0,1], 0\le c+g\le 1$.

2.3 Triangular Intuitionistic Fuzzy Number (TIFN) and its arithmetic A triangular

IFN 'A' is given by $A=(\langle x,y,z\rangle,\langle l,m,n\rangle)$ with $\langle l,m,n\rangle \le \langle x,y,z\rangle^c$ i.e., either l ≥ y , m ≥ z or m ≤ x , n ≤ y are membership and non- membership fuzzy numbers of A.

The addition of two TIFN are as follows.

For two triangular intuitionistic fuzzy numbers A = $(\langle a_1,b_1,c_1\rangle:\mu_A,\langle e_1,f_1,g_1\rangle:\gamma_A)$ and B = $(\langle a_2,b_2,c_2\rangle:\mu_B,\langle e_2,f_2,g_2\rangle:\gamma_B)$ with $\mu_A\ne\mu_B$ and $\gamma_A\ne\gamma_B$, define

A+B = $(\langle a_1+a_2,b_1+b_2,c_1+c_2\rangle:Min(\mu_A,\mu_B),\langle e_1+e_2,f_1+f_2,g_1+g_2\rangle:Max(\gamma_A,\gamma_B))$

2.4 Order Relation Consider an order relation among fuzzy numbers. A variety of methods for the ordering and ranking of fuzzy numbers has been proposed in the literature. These methods have been reviewed and tested by Bortolan and Degani [2].Let $\tilde{a}$ and $\tilde{b}$ be two TIFN such that $\tilde{a}$ = $(\langle a_1,a_2,a_3\rangle,\langle a^1{}_1,a^1{}_2,a^1{}_3\rangle)$ and $\tilde{b}$ = $(\langle b_1,b_2,b_3\rangle,\langle b^1{}_1,b^1{}_2,b^1{}_3\rangle)$ then $\tilde{a}$ ≤ $\tilde{b}$ iff the inequalities are a1 ≤ b1 , a2 ≤ b2 , a3 ≤ b3 , a11 ≤ b11 , a12 ≤ b12 , a13 ≤ b13.

2.5 Graded Mean Integration Representation Suppose A = $(\langle a_1,a_2,a_3\rangle,\langle a^1{}_1,a^1{}_2,a^1{}_3\rangle)$ is a TIFN . The graded mean integration representation of A becomes

Q(A) = $\frac{(a_1+a^1{}_1)+4(a_2+a^1{}_2)+(a_3+a^1{}_3)}{6}$.

2.6 Fuzzy Distance Let A = $(\langle a_1,a_2,a_3\rangle,\langle a^1{}_1,a^1{}_2,a^1{}_3\rangle)$ and B = $(\langle b_1,b_2,b_3\rangle,\langle b^1{}_1,b^1{}_2,b^1{}_3\rangle)$ be two TIFN and their graded mean integration representation are Q(A), Q(B) respectively. Assume

Pi = $\frac{\tilde{a}_i-Q(A)+\tilde{b}_i-Q(B)}{2}$,

i = 1,2,3 ; C_i = |Q(A) - Q(B)| + P_i , i = 1,2,3 ; then the fuzzy distance of A, B is C = (c_1, c_2, c_3).

3. Algorithm

Step 1: Assume S = $(\langle 0,0,0\rangle,\langle 0,0,0\rangle)$ then compute fuzzy distance.

Step 2: Calculate the graded mean value for each adjacent vertex from the current vertex.

Graded mean value is given by

Q(A) = $\frac{(a_1+a^1{}_1)+4(a_2+a^1{}_2)+(a_3+a^1{}_3)}{6}$

Step 3: Evaluate the fuzzy distance between two edges using

P_i = $\frac{\tilde{a}_i-Q(A)+\tilde{b}_i-Q(B)}{2}$,

Volume : 4 | Issue : 9 | September 2014 | ISSN - 2249-555X

$i = 1,2,3$ and $C_i = |Q(A) - Q(B)| + P_i$, $i = 1,2,3$ and then the fuzzy distance of A, B is $C = (c_1, c_2, c_3)$ for each adjacent node.

Step 4: Compare fuzzy distance among all adjacent nodes using order relation (2.4).

Step 5: Select the smallest fuzzy distance edge among them and continue until destination node is reached.

Step 6: Fuzzy shortest path is obtained for the given network.

4. Numerical Example The problem is to find the shortest path between source node (say node1) and destination node (say node 6) on the network consists of 6 vertices {1,2,3,4,5,6}and 11 edges $\{e_{12}, e_{13}, e_{14}, e_{23}, e_{24}, e_{34}, e_{35}, e_{36}, e_{45}, e_{46}, e_{56}\}$ the arc lengths of the network , shown in the following figure are all TIFN and given by

$e_{12} = (\langle 3,6,6\rangle, \langle 7,8,9\rangle)$; $e_{13} = (\langle 2,4,5\rangle, \langle 6,7,8\rangle)$; $e_{14} = (\langle 1,3,5\rangle, \langle 4,6,7\rangle)$; $e_{23} = (\langle 3,5,7\rangle, \langle 6,8,9\rangle)$

$e_{24} = (\langle 5,6,7\rangle, \langle 7,8,10\rangle)$; $e_{34} = (\langle 7,8,9\rangle, \langle 9,10,11\rangle)$; $e_{35} = (\langle 1,4,6\rangle, \langle 5,7,8\rangle)$; $e_{36} = (\langle 2,3,4\rangle, \langle 4,7,9\rangle)$

$e_{45} = (\langle 2,5,7\rangle, \langle 6,8,10\rangle)$; $e_{46} = (\langle 3,4,5\rangle, \langle 5,6,7\rangle)$; $e_{56} = (\langle 7,8,9\rangle, \langle 9,10,11\rangle)$

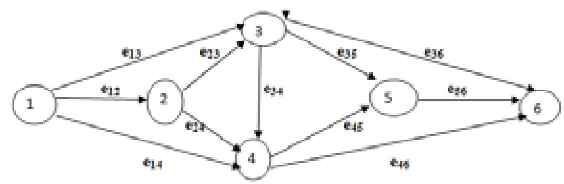

Figure 1. A network

Now start with node 1, the distance $S = (\langle 0,0,0\rangle, \langle 0,0,0\rangle)$. From node 1, the adjacent nodes are 2,3 and 4.

$$Q(1,1) = (\langle 0,0,0\rangle, \langle 0,0,0\rangle) = \frac{0}{6} = 0; \quad Q(1,2) = \frac{10 + 4(14) + 15}{6} = 13.5;$$

$$P_1 = [0 - 0 + 10 - 13.5]/2 = -1.75; \quad P_2 = [0 - 0 + 14 - 13.5]/2 = 0.25;$$

$$P_3 = [0 - 0 + 15 - 13.5] = 0.75; \quad C_1 = |0 - 13.5| - 1.75 = 11.75;$$

$$C_2 = |0 - 13.5| + 0.25 = 13.75; \quad C_3 = |0 - 13.5| + 0.75 = 14.25.$$

Hence $C = (11.75, 13.75, 14.25)$.

$$\text{Now, } Q(1,3) = \frac{8 + 4(11) + 13}{6} = 10.8$$

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$$\text{Now, } Q(1,3) = \frac{8 + 4(11) + 13}{6} = 10.8$$

We compare (1, 2), (1,3) and (1,4) fuzzy distance among edges. We get (1,4) is the smallest among them according to order relation. From node 4, the only one adjacent node is 5. Then choose (4,5) edge and then from node 5, the only one adjacent node is 6 which is the destination node. Now the process is terminated and fuzzy shortest path 1 – 4 – 5- 6 is obtained.

5. Conclusion The fuzzy shortest length and the corresponding shortest path are the useful information for the decision makers in a intuitionistic fuzzy SPP. This algorithm can be implemented using TIFN graded mean integration chosen by the decision maker, the algorithm can return a single path as solution. It provides the better output for different types of network.

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