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Construction of PBIB Designs Using Triplets and Quadruplets Generated From Polygons and Polyhedrons

KEYWORDS: PBIB designs, Triplets, Quadruplets, points, lines

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ABSTRACT *In this paper, we have constructed partially balanced incomplete block (PBIB) designs with two and three associate classes by establishing a link between Polygons, Polyhedrons through Triplets/Quadruplets and PBIB designs. Constructions of some new PBIB designs along with their association schemes has also been discussed. Efficiencies of the newly constructed PBIB designs are computed for the purpose of comparisons. In the study it was found that some newly constructed designs are more efficient as compared to the existing designs. In this paper, two and three class association scheme of the newly constructed designs with parameters of first kind and of second kind are also given in detail.*

1. Introduction

Partially balanced incomplete block (PBIB) designs with two and three associate classes using Triplets and Quadruplets generated from polygons and polyhedron has been constructed in this paper. These triplets, quadruplet combinations of symbols developed from an ordered pair of treatment symbols in constructing a PBIB design. Each of the combinations represents a treatment symbol of the new designs. Also it was observed that procedure of obtaining treatment symbols leads to an association scheme. Singh and Agarwal [2013] gave some methods of construction of rectangular designs and consequently, some series of rectangular designs are obtained by picking up one triplets of distinct treatments from each of 'm' sets to form a block of the design. Vinjay Kumar and Nandappa [2010] constructed some quadruplets designs associated with four associate class association scheme with maximum independent set of cubic graphs. Kumar [2005] constructed triplet designs with three associate classes through unreduced balanced incomplete (BIB) designs. Recently, Garg and Gurinder (2014) constructed three associate classes PBIB with three replicates using pairing in triplets system. Very, recently Garg and Syed (2014) constructed some new PBIB designs through chosen lines and Triangles of Graphs associated with two and three class association scheme.

2. Definitions and Preliminary Results

2.1 Polyhedron: Polyhedron is a three- dimensional solid and is entirely composed of polygons, each of these polygons is known as face. The segment where two polygons intersect is known as edge, and the edge is shared slide of the two polygons. The points where three or more edges intersect is known as a vertex. The vertexes of the polyhedron are the vertices of the polygons.

2.2 Polygons: Polygons are two-dimensional geometric figures and they are made of straight line segments. Each segment touches exactly two other segments, one at each of its endpoints. They are closed—they divide the plane into two distinct regions, one "inside" and the other "outside" the polygon.

2.3 Octahedron: An octahedron is a polyhedron with eight faces. A regular octahedron is a Platonic solid composed of eight equilateral triangles, four of which meet at each vertex. A regular octahedron is the dual polyhedron of a cube.

2.4 Hexahedron: A hexahedron is any polyhedron with six faces. A cube, for example, is a regular hexahedron with all its faces square, and three squares around each vertex.

3. P-Matrices

The P-matrices of the association scheme can be defined as follows:

$$P_1 = \begin{pmatrix} P^i_{11} & P^i_{12} & P^i_{13} & \dots & P^i_{1m} \\ P^i_{21} & P^i_{22} & P^i_{23} & \dots & P^i_{2m} \\ P^i_{31} & P^i_{32} & P^i_{33} & \dots & P^i_{3m} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ P^i_{m1} & P^i_{m2} & P^i_{m3} & \dots & P^i_{mm} \end{pmatrix} \quad ; i=1, 2, 3\dots m$$

These matrices are the essential part of an association scheme of PBIB designs which is necessary for the existence of any PBIB designs. The elements of P-matrices must satisfy the necessary conditions for the existence of an association scheme otherwise the association scheme may not be correct. P-matrices are very useful in the analysis of every m ($m \geq 2$) associate class PBIB designs. There will be as many P-matrices as the number of associate classes. This is also a check on the correctedness of any association scheme. The symmetrical structure of every P matrix indicates that the construction methodology of blocks of PBIB design is correct with respect to an association scheme.

4. Construction of designs

Theorem 4.1 The set of triplets in a regular Octahedron forms blocks of PBIB designs with two associate classes.

Proof: Consider a regular Octahedron which is a combination of two square pyramids whose faces are equilateral triangles as shown in fig (I) which Consists of vertices S ={1,2,3,4,5,6}. These vertices are termed as treatments. A set

T = {134, 136, 145, 156, 245, 256, 234, 236} are its faces, and 13, 14, 15, 16, 34, 45, 36, 56, 23, 24, 25, and 26 are its edges. These faces in a regular Octahedron are termed as blocks, such that every pair of elements of 's' appear together in a unique triple of T.

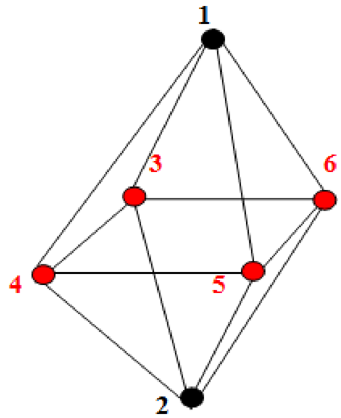

Fig. (1) Regular Octahedron

We observe that each of the symbols in the above triplets occurs in exactly r sets. Also, we verify the conditions vr=bk, as v=6, b=8, r=4, k=3. By considering all possible triplets as blocks, a PBIB designs with two associate class association schemes can be constructed:

Association Scheme: A pair of treatments says Θ and Φ together occur either λ_1 times or λ_2 times. Those occurring together λ_1 =0 times are first associates and all the remaining treatments λ_2 =2 times are second associates of each other.

The following table explains the association scheme with two associate classes:

Symbols	I[st] associates	2[nd] associates
1.	2	3,4,5,6
2.	1	3,4,5,6
3.	5	1,2,4,6
4.	6	1,2,3,5
5.	3	1,2,4,6
6.	4	1,2,3,5

The parameters of first kind (v, b, r, k, λ_1, λ_2) are given by v=6, b=8, r=4, k=3, λ_1 =0 λ_2 =2 and parameters of second kind (n_1, n_2) are given by n_1=1, n_2=4.

Also the P-matrices of the association scheme are

$$P_1 = \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$$

E_1=66.67%, E_2=80.00%, E=76.92%

Theorem 4.2 The set of quadruplets in a Hexahedron forms blocks of PBIB designs with three associate classes.

Proof: Consider Hexahedron as shown in fig (II) which consist of vertices S={ 1,2,3,4,5,6,7,8}. These vertices are termed as treatments. A set T ={1258, 3467, 1234, 2356, 5678, 1478} are its faces. These faces in a Hexahedron are considered as blocks, such that every pair of elements of 's' appear together in a unique quadruple of T.

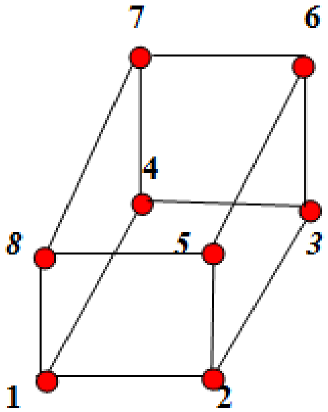

Fig (II) Hexahedron

In this hexahedron, we observe that each of the symbols in the above quadruplets occurs in exactly r sets. Also, we verify the conditions vr=bk, as v=8, b=6, r=3, k=4. By considering the quadruplets as blocks, a PBIB designs with three associate class association schemes can be constructed:

Association Scheme: A pair of treatment say (Φ, Ψ, Θ) occur together either λ_1 or λ_2 or λ_3 times. Those occurring together λ_1 =0 times are first associates, those occurring together λ_2 =1 times are second associates and all the remaining treatments λ_3 =2 are third associates of each other".

The following table explains the association scheme with two associate classes:

Symbols	*I[st] associates*	*2[nd] associates*	*3[rd] associates*
1.	6	3,5,7	2,4,8
2.	7	4,6,8	1,3,5
3.	8	1,5,7	2,4,6
4.	5	2,6,8	1,3,7

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5.	4	1,3,7	2,6,8
6.	1	2,4,8	3,5,7
7.	2	1,3,5	4,6 8
8.	3	2,4,6	1,5,7

The parameters of first kind (v, b, r, k, λ_1, λ_2, λ_3) are given by v=8, b=6, r=3, k=4, λ_1 =0, λ_2 =1, λ_3 =2 and parameters of second kind (n_1, n_2, n_3) are given by n_1=1, n_2=3,n_3=3.

Also the P-matrices of the association scheme are

$$P_1=\begin{bmatrix}0&0&0\\0&0&3\\0&3&0\end{bmatrix} \qquad P_2=\begin{bmatrix}0&0&1\\0&2&0\\1&0&2\end{bmatrix} \qquad P_3=\begin{bmatrix}0&1&0\\1&0&2\\0&2&0\end{bmatrix}$$

E_1=74.89%, E_2=81.69%, E_3=89.86%, E=83.87%

Theorem 4.3 The set of all triplets in the pattern (i, i+1, i+3); i=1,2,3,. . ., 8 in octagon graph forms blocks of PBIB designs with two associate classes.

Proof: Consider Octagon graph which consists of's' elements (say) as shown in fig (III) and a set 'T' consists of triples of's' (called blocks), we get the triples in the following pattern (i, i+1, i+3) if i>8, then reduce it by 8, such that every pair of elements of 's' appear together in a unique triple of T. The elements and triplets of octagon graph are given by

S= {1, 2, 3, 4, 5, 6, 7, 8}

T= {124, 235, 346, 457, 568, 671, 782, 813}

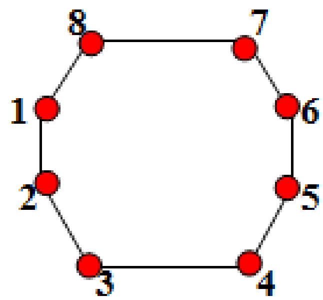

Fig (III) Octagon

We observe that each of the symbols in the above triplets occurs in exactly r sets. Also, we verify the conditions vr=bk, as v=8, b=8, r=3. k=3. By considering all possible triplets as blocks, a PBIB designs with two associate class association schemes can be constructed:

Association Scheme: A pair of treatments say Φ and Θ together occur together either λ_1 times or λ_2 times. Those occurring together λ_1=0 times are first associates and all the remaining treatments λ_2 =1are second associates of each other.

The following table explains the association scheme with two associate classes:

Symbols	*I^st^ associates*	*2^nd^ associates*
1.	5	2, 4, 6,8,3,7
2.	6	1, 3, 5,7,4,8
3.	7	2, 4, 6,8,1,5
4.	8	1, 3, 5,7,2,6
5.	1	2, 4, 6,8,3,7

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6.	2	1, 3, 5,7,4,8
7.	3	4, 5, 1,6,2,8
8.	4	1, 3, 5,7,2,6

The parameters of first kind (v, b, r, k, λ_1, λ_2) are given by v=8, b=8, r=3, k=3, λ_1 =0 λ_2 =1 and parameters of second kind (n_1, n_2) are given by n_1=1, n_2=6.

Also the P-matrices of the association scheme are

$$P_1 = \begin{bmatrix} 0 & 0 \\ 0 & 6 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 0 & 1 \\ 1 & 4 \end{bmatrix}$$

E_1=66.67%, E_2=76.19%, E=74.67%

Theorem 4.4 The set of all triplets in the pattern (i,i+1,i+3);i=1,2,3,. . ., 9 in Nonagon graph forms blocks of PBIB designs with two associate classes.

Proof: Consider a Nonagon graph which consists of's' elements (say) as shown in fig (IV) and a set 'T' consists of triples of's' (called blocks), we get the triples in this pattern (i, i+1, i+3) if i>9, then reduce it by 9, such that every pair of elements of's' appear together in a unique triple of T. The elements and triplets of Nonagon graph are given by

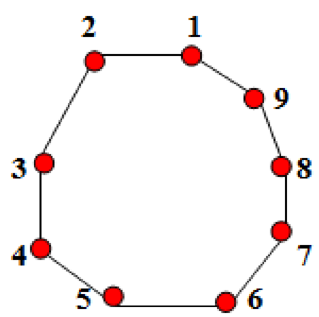

Fig (IV) Nonagon

S= {1, 2, 3, 4, 5, 6, 7, 8, 9}
T= {124, 235, 346, 457, 568, 679, 781, 892, 913}

We observe that each of the symbols in the above triplets occurs in exactly r sets. Also, we verify the conditions vr=bk, as v=9, b=9, r=3, k=3. By considering all possible triplets as blocks, a PBIB designs with two associate class association schemes can be constructed:

Association Scheme: A pair of treatments says Φ and Θ together occur either λ_1 times or λ_2 times. Those occurring together λ_1 =0 times are first associates and all the remaining treatments λ_2 =1 are second associates of each other.

The following table explains the association scheme with two associate classes:

Symbols	*1st associates*	*2nd associates*
1.	5,6	2,3,4,7,8,9
2.	6,7	1,3,4,5,8,9
3.	7,8	1,2,4,6,5,9
4.	8,9	1,2,3,5,6,7
5.	1,9	2,3,4,6,7,8
6.	1,2	3,4,5,8,7,9
7.	2,3	4,5,6,9,1,8
8.	3,4	6,7,2,8,1,3
9.	4,5	6,7,2,8,1,3

The parameters of first kind (v, b, r, k, λ_1, λ_2) are given by v=9, b=9, r=3, k=3, λ_1 =0 λ_2 =1 and parameters of second kind (n_1, n_2) are given by n_1=2, n_2=6.

Also the P-matrices of the association scheme are

$$P_1 = \begin{bmatrix} 0 & 1 \\ 1 & 5 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix}$$

E1=64.90%, E2= 83.99%, E3=82.77% and E=78.59%

Theorem 4.5 The set of all triplets in the pattern (i,i+1,i+6);i=1,2,3,. . ., 10 in Decagon graph forms blocks of PBIB designs with two associate classes.

Proof: Consider a Decagon graph which consists of 's' elements (say) as shown in fig.(V) and a set 'T' consists of triples of 's' (called blocks), we get the triples in the following pattern (i, i+2, i+6) if i>10, then reduce it by 10, such that every pair of elements of 's' appear together in a unique triple of T. The triplets of Decagon graph are given by

S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

T = {1,3,7} {3,5,9} {1,5,7} {3,7,9} {1,5,9}

{2,4,8} {4,6,10} {2,6,8} {4,8,10} {2,6,10}

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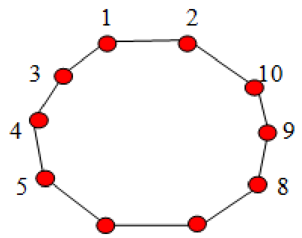

Fig (V) Decagon

We observe that each of the symbols in the above triplets occurs in exactly r sets. Also, we verify the conditions vr=bk, as v=10, b=10, r=3, k=3. By considering all possible triplets as blocks, a PBIB designs with two associate class association schemes can be constructed:

Association Scheme: A pair of treatment say (Φ, Ψ, Θ) occurs together either λ_1 or λ_2 or λ_3 times. Those occurring together λ_1 =1 times are first associates, those occurring together λ_2 =0 times are second associates and all the remaining treatments λ_3 =2 are third associates of each other".

The following table explains the association scheme with three associate classes:

Symbols	*I^st associates*	*2^nd associates*	*3^rd associates*
1.	3,9	2,4,6,8,10	5,7
2.	4,10	1,3,5,7,9	8,6
3.	1,5	2,4,6,8,10	7,9
4.	2,6	1,3,5,7,9	8,10
5.	3,7	2,4,6,8,10	1,9
6.	4,8	1,3,5,7,9	2,10
7.	5,9	2,4,6,8,10	1,3
8.	6,10	1,3,5,7,9	2,4
9.	1,7	2,4,6,8,10	3,5
10.	2,8	1,3,5,7,9	4,6

The parameters of first kind (v, b, r, k, λ_1, λ_2, λ_3) are given by v=10, b=10, r=3, k=3, λ_1 =0, λ_2 =1, λ_3 =2 and parameters of second kind (n_1, n_2, n_3) are given by n1=2 and n2=5.

Also the P-matrices of the association scheme are

$$P_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad P_2 = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \quad P_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

E1=64.90%, E2= 83.99%, E3=82.77% and E=78.59%

Conclusion

In this paper, we consider five configurations through quadruplets and triplets generated from polygons and polyhedrons and as a result we get new PBIB designs with two and three associate class. Efficiencies of the new designs are also computed for the purpose of comparison. Some newly constructed designs are more efficient, as compared to the existing designs.

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