RESEARCH PAPER | Mathematics | Volume : 5 | Issue : 4 | April 2015 | ISSN - 2249-555X

Unsteady MHD Stagnation Point Flow and Heat Transfer over a Stretching Sheet with Suction or Injection

KEYWORDS: Stagnation flow, stretching sheet, heat transfer, MHD, Unsteady similarity variables.

R. N. Jat	Navin Kumar
Department of Mathematics, University of Rajasthan, Jaipur-302004, India.	Department of Mathematics, University of Rajasthan, Jaipur-302004, India.

ABSTRACT *In the present investigation The unsteady two-dimensional magnetohydrodynamics (MHD) stagnation point flow and heat transfer over a stretching sheet with suction/injection is studied. By using similarity transformation the governing partial differential equations are converted into nonlinear ordinary differential equations using a similarity transformation and then solved numerically. Results for the skin friction coefficient, local Nusselt number, velocity, and temperature profiles are presented for different values of the governing parameters like as Magentic, Prandtl number and Eckert number have been discussed in detail with Graphical representation.*

INTRODUCTION

The study of flow and heat transfer over a stretching/shrinking sheet receives considerable attention from many researchers due to its variety of application in industries such as extrusion of plastic sheets, wire drawing, hot rolling and glass fiber production. First of all Sakiadis etc [1, 2] performed the pioneering work of boundary layer flow over a continuous moving surface and similarity solutions were obtained for the governing equations. Crane [3] studied the flow over a linearly stretching sheet in an ambient fluid and gave a closed-form solution for stead two-dimensional flow of an incompressible viscous fluid caused by the stretching of an elastic sheet, which moves in its own plane with a velocity which varies linearly with distance from a fixed point. P.S. Gupta and A.S. Gupta [4] extended the work of Crane [3] by investigating the effect of mass transfer on a stretching sheet with suction or blowing for linear surface velocity subject to uniform temperature. Chaim [5] studied stagnation poiny flow towards the stretching sheet. Mahapatra and Gupta [6,7] investigated the steady two-dimensional magnetohydrodynamics (MHD) stagnation point flow and heat transfer toward a stretching surface. of an incompressible viscous electrically conducting fluid toward a stretching surface. They obtained the exact similarity solution of the Navier-Stokes equations and observed that the flow displays a boundary-layer structure when the stretching velocity of the surface is less than the free stream velocity. Wang [8] investigated the steady two-dimensional flow and axisymmetric stagnation point flow with heat transfer over a shrinking/sretching sheet and found that solutions do not exist for the larger shrinking rates. Nik Long et al. [9,10] found that the solution is unique over the stretching sheet. Ishak, Nazar etc [11] studied MHD stagnation point flow towards a stretching sheet with prescribed surface heat flux. Bhattacharyya [12,13] Heat transfer analysed in unsteady boundary layer stagnation-point flow and towards a shrinking/stretching sheet. Recently Jat, Chand [14] studied the viscous dissipation and radiation effects on MHD flow and heat transfer over stretching sheet. Further, Jat et al. [15] studied the above problem with micro polar fluid.

Formulation

Consider the unsteady stagnation point flow over a stretching or shrinking sheet immersed in an incompressible viscous fluid of ambient temperature T_∞

It is assumed that the free stream velocity is in the form $U_\infty(x,t)=ax(1-\lambda t)^{-1}$, the sheet is stretched with velocity $U_w(x,t)=bx(1-\lambda t)^{-1}$ and the surface heat flux is $T_w(x,t)=T_\infty+cx(1-\lambda t)^{-1}$. The x-axis runs along the sheet and y-axis is measured normal to it. These assumptions along with the boundary-layer approximations and neglecting the viscous dissipation, the governing equations are given by

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U_\infty}{\partial t}+U_\infty\frac{\partial U_\infty}{\partial x}$$
$$+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\beta}{\rho}(u-U_\infty) \qquad (2)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial U_\infty}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (3)$$

With the boundary condition

$$u=U_w,\quad v=V_w,\quad T=T_w,\quad \text{at } y=0 \qquad (4)$$
$$u\to U_\infty,\quad T\to T_\infty,\quad \text{as } y\to\infty$$

where u and v are velocity components in x and y directions; respectively, υ is kinematic viscosity; α is thermal diffusivity; T is fluid temperature. Introducing the following similarity

transformations

$$\psi=\left(\frac{a\upsilon}{1-\lambda t}\right)^{1/2}xf(\eta),$$

$$\eta=\left(\frac{a}{\nu(1-\lambda t)}\right)^{1/2}y,$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \qquad (5)$$

where η is similarity variable and ψ is stream function defined as $u=\frac{\partial\psi}{\partial y}$ and

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$v=-\frac{\partial\psi}{\partial x}$,

thus we have

$$u=\frac{ax}{1-\lambda t}f'(\eta)$$

$$v=-\left(\frac{a\nu}{1-\lambda t}\right)^{1/2}f(\eta) \quad (6)$$

There for, the mass transfer velocity V_w can take the form

$$V_w(t)=-\left(\frac{a\nu}{1-\lambda t}\right)^{1/2}f_0 \quad (7)$$

where prime denotes differentiation with respect to η. With these values of u and v, (1) is identically satisfied whereas (2) and (3) reduce to the following nonlinear ordinary differential equations

$$f'''+ff''+1-f'^2+A\left(1-f'^2-\frac{1}{2}\eta f''\right)'+M(1-f')=0 \quad (8)$$

$$\frac{1}{\Pr}\theta''+f\theta'-f'\theta-A\left(\theta+\frac{1}{2}\eta\theta'\right)+Ecf''^2=0 \quad (9)$$

The boundary conditions (5) becomes

$$f(0)=f_0,$$
$$f'(0)=\frac{b}{a}=\varepsilon,$$
$$\theta(0)=1,$$
$$\theta(\infty)\to 0,$$
$$f'(\infty)\to 1 \quad (10)$$

Where $\varepsilon(=b/a)$ is the ratio of stretching shrinking sheet velocity parameter and free stream velocity parameter, $f_0>0$ and $f_0<0$ are the suction and injection parameters, respectively, $\Pr=\frac{\nu}{\alpha}$ is the Prandtl number, and $A=\frac{\lambda}{a}$ is unsteadiness parameter. The quantities of physical interest to be obtained are the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'$ü which are defined as

$$C_f=\frac{\tau_w}{\rho U_\infty^2/2},$$

$$Nu_x=\frac{xq_w}{K(T_w-T_\infty)} \quad (11)$$

And dimensionless parameters are

$$M=\frac{\sigma\beta_0^2}{\rho a}(1-\lambda t)$$

(Magnetic parameter)

$$Ec=\frac{U^2}{C_p(T_w-T_\infty)}$$

(Eckert number)

$$\Pr=\frac{\mu C_p}{\kappa}$$

(Prandtl number)

$$A=\frac{\lambda}{a}$$

(Unsteady parameter)

where the surface shear stress τ_w and the surface heat flux q_w are defined as

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0},$$

$$q_w=-\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (12)$$

with and κ being the dynamic viscosity and thermal conductivity, respectively. Using the

similarity variables equation (4) we obtain

$$\frac{1}{2}C_f\,\mathrm{Re}_x^{1/2}=f''(0),$$

$$\frac{Nu_x}{\mathrm{Re}_x^{1/2}}=-\theta'(0) \quad (13)$$

where $\mathrm{Re}_x=\frac{U_\infty x}{\upsilon}$ is the local Reynolds number.

Results and Discussions

The set of non linear ordinary differential equation (8) and (9) with boundary conditions (10) were solved numerically using Runga-Kutta forth order algoritm with a systematic guessing of $f''(0)$ and $\theta'(0)$ by the shooting technique until the boundary conditions at infinity are satisfied. The step size $\Delta\eta$=0.001 is used while obtaining the numerical solution and accuracy up to the seventh decimal place i.e. 1×10^{-7}, which is very sufficient for convergence. In this method, we choose suitable finite values of $\eta\to\infty$ say η_∞ which depend on the values of the parameter used. The computation are done by generating a program in Matlab. For the validation of numerical results, the case A=0 and Pr.=0.7 with no effect of suction or injection (f_0=0) are considered firstly and compared to those of Wang[10] and M.Suail [20]. These quantitative comparisons are shown in table 1 for the variation of k and found to be in favorable agreement. The computation through employed numerical scheme has been carried out for various values of the parameters such as Unsteadiness parameter A, Permeability parameter K, Magnetic parameter M, Prandtl number Pr and Eckert number Ec. It is observed from the figures that the boundary conditions are satisfied asymptotically in all the cases, which supporting the accuracy of the numerical results obtained. The velocity profile $f'(\eta)$ for different values of the unsteady parameter A is shown in fig.3. It is observed that the velocity increases with the increasing values of unsteady parameter A. It is interesting to note that the thickness of boundary decreases with increasing values of A. This is due to the fluid flow caused solely by the stretching sheet. velocity profile ( ) for different values of the magnetic parameter M is shown in fig.4. It is observed that velocity increases with the increasing values of magnetic parameter M. As M increases, the Lorentz force, which opposes the flow, also increases and leads to enhanced deceleration of the flow, and from fig.4, we observed that velocity decreases for increasing values of permeability parameter K. The temperature profiles for different values of A, Pr, Ec and M are presented in figure 5 to figure 8. From fig.5, we observed that temperature of the fluid is decreases with the increasing values of unsteady parameter A. Temperature at a point of surface

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decreases significantly with the increases of A i.e. rate of heat transfer increases with increasing values of A. Physically, it means that the temperature gradient at the surface increases as A increases, which imply the increases of heat transfer rate $-\theta'(\eta)$ at the surface. From fig.7 and fig.8, we observed that temperature of the fluid is increases as magnetic parameter Ec and Eckert number M increases. It is observed from the fig.6 temperature of the fluid is decreases with the increasing values of Prandtl number Pr, this is because of the increase in Prandtl number Pr, indicates the increase of the fluid heat capacity or the decrease of the thermal diffusivity hence cause a diminution of the influence of the thermal expansion to the flow. Which implies momentum boundary layer is thicker than thermal boundary layer.

Table.1 Variation of $f''(0)$ with k when f_0=0

A	K	Wang[8]	M. Suali[10]	Present
0	3		-4.276545	-4.276542
	2			1.887307
	0.5			0.713296
	0.2	1.05113	1.051130	1.051132
	0.1	1.14656	1.146561	1.146563

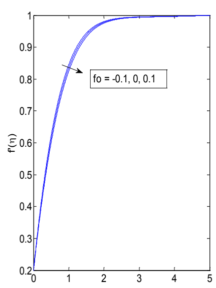

Figure.1 The velocity profiles f'(η) for different values of f_0 when A=0.01, k=0.1

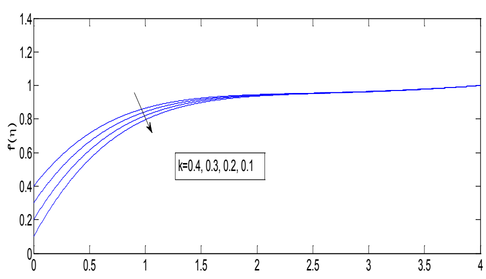

Figure.2 The velocity profiles f'(η) for different values of k

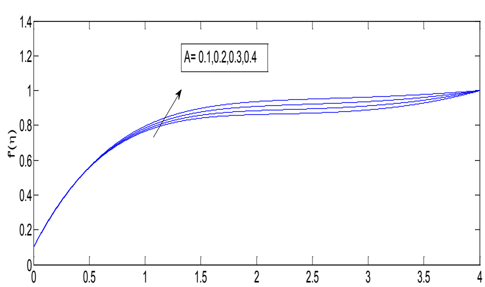

Figure.3 The velocity profiles f'(η) for different values of A

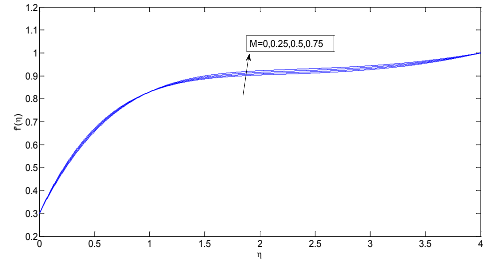

Figure.4 The velocity profiles f'(η) for different values of M

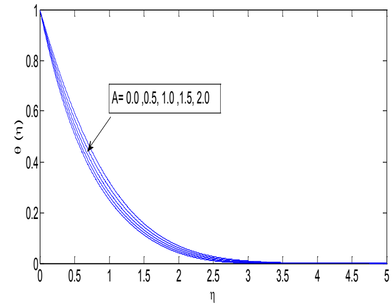

Figure.5 Temprature profile for various values of A when f_o=0.1, K=0.1

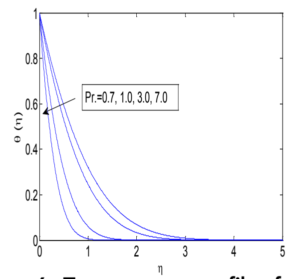

Figure.6 Temprature profile for various values of Pr when f_0=0.01, K=0.1

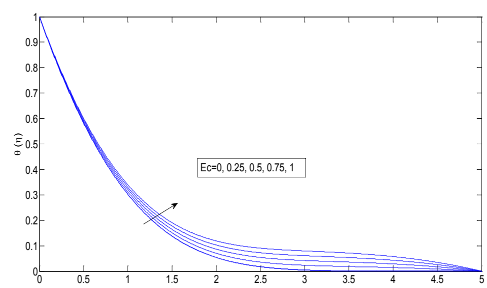

Figure.7 Temprature profile for various values of Ec when M=0,K=0.1,Pr=0.7

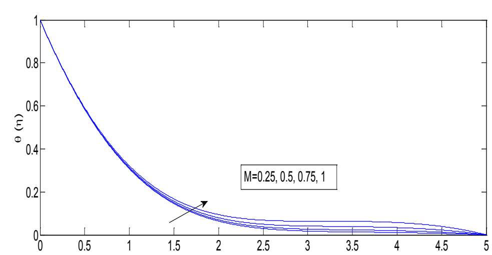

Figure.8 Temprature profile for various values of M when K=0.1, Pr=0.7,Ec=0.1

Volume : 5 | Issue : 4 | April 2015 | ISSN - 2249-555X

Acknowledgements
This work has been carried out with the financial support of UGC in the form of J.R.F (F-41-4/NET/RES/JRF-1296) awarded to one of the author (Navin Kumar).

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