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Static Sphere of Dust of Uniform Densityusing Anisotropic Line Element

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ABSTRACT *The Schwarchild problem of static sphere of perfect fluid is solved without using the relation* $T^k_{;k}=0$*. Hence solution for a static sphere of dust of uniform density has been obtained by putting pressure* $P=0$ *in the solution for the fluid. A new field equation is discussed.*

Introduction:

We shall first solve the Schwarzchild problem i.e the problem of static sphere of perfect fluid without using equation $T^k_{;k}=0$ or

$$\frac{dP}{dr}=(\rho+P)\frac{v'}{2} \text{ ---------- (A)}$$

Condition (A) is used as a field equation in the books by Naralikar[1] and Tolman[2]. Eddington[3] has given the solutions and shown that the solutions satisfy Einstein's field equations and boundary conditions. There is however a misunderstanding that condition (A) which arises because of the relation $T^k_{;k}=0$.is necessary condition to solve Einstein's field equations. As a matter of fact Einstein has converted the equation $T^k_{;k}=0$ into an identity. $T^k_{;k}=0$ Hence we first show that Einstein's field equations can be solved without using $T^k_{;k}=0$ and then show that solutions for a static sphere of dust of uniform density can be obtained from corresponding solutions of a static sphere of perfect fluid by merely putting pressure $P=0$

Einstein's field equations for a static sphere of perfect fluid:

The line element is given by

$$ds^2=-e^{\lambda}dr^2-r^2d\theta^2-r^2\mathrm{Sin}^2\theta\cdot d\phi^2+e^{v}dt^2$$

The field equations are given by

$$8\pi P=e^{-\lambda}\left(\frac{v'}{r}+\frac{1}{r^2}\right)-\frac{1}{r^2} \quad (1)$$

$$8\pi P=e^{-\lambda}\left(\frac{v''}{2}-\frac{\lambda'v'}{4}+\frac{v'}{4}+\frac{v'-\lambda'}{2r}\right) (2)$$

$$8\pi\rho=e^{-\lambda}\left(\frac{v'}{r}-\frac{1}{r^2}\right)+\frac{1}{r^2} (3)$$

In these equations $\rho=\rho_0+3P$ and is regarded as constant inside the fluid.

One can integrate equation (3) by using integrating factor $-r^2$ to give

$$re^{-\lambda}=r-\frac{8\pi\rho}{3}r^3+k$$

$$e^{-\lambda}=1-\frac{8\pi\rho}{3}r^2+\frac{k}{r}$$

$k=0$ for regularity at $r=0$

Hence $e^{-\lambda}=(1-\alpha r^2)$ with

$$\alpha=\frac{8\pi\rho}{3} \quad (4)$$

From equations (1) and (2) one obtains on subtraction

$$\frac{v''}{2}-\frac{\lambda'v'}{4}+\frac{\lambda'}{4}+\frac{v'-\lambda'}{2r}-\frac{v'}{r}+\frac{e^{\lambda}-1}{r^2}=0$$
(5)

From equation (4) one can show that $\frac{e^{-\lambda}-1}{r^2}=\frac{\lambda'}{2r}$ (6)

From (5) and (6) one gets

$$\frac{v''}{2}-\frac{v'\lambda'}{4}+\frac{v'^2}{4}-\frac{v'}{2r}=0 (7)$$

Now substitute $e^{v}=w^2$ or

$v=2\mathrm{Log}w$

Equation (7) reduces to

$$w''-\frac{w'\alpha r}{1-\alpha r^2}-\frac{w'}{r}=0$$

Further substitution of $q=w'$ gives

$$q'-q\left\{\frac{\alpha r}{1-\alpha r^2}+\frac{1}{r}\right\}=0 (8)$$

Equation (8) can be easily integrated to give where k_1 is the constant of integration.

give $q=\frac{dw}{dr}=\frac{k_1 r}{\sqrt{1-\alpha r^2}}$ where k_1 is the constant of integration.

Hence $w=k_2\sqrt{1-\alpha r^2}+k_3$ where k_2 and k_3 are constants of integration.

$\therefore e^{v}=A-B\sqrt{1-\alpha r^2}$ ------- (9) with

$$\alpha=\frac{8\pi\rho}{3}$$

This solution agrees with the solution given by Tolman[4]. The constants of integration A and B can be found by using boundary conditions.

a) $P=0$ at the boundary $r=a$ and P can be obtained from equation (1)

b) e^{v} is continuous at $r=a$ .We use the same exterior solution as Schwarchild exterior solution i.

exterior solution i.e. $e^{\nu} = \left(1 - \frac{2m}{r}\right)$

where $m \equiv$ Newtonian mass $\equiv \left(\frac{4\pi\rho a^3}{3}\right)$

and $\rho = \rho_0 + 3P$

Using these boundary conditions one gets the same solution as

$$e^{\nu} = \left[\frac{3}{2}\sqrt{1-\alpha a^2} - \frac{1}{2}\sqrt{1-\alpha r^2}\right]^2 \quad (10)$$

Result and Discussion:

Here we have not used the relation $T^{k}_{;k} = 0$.

For dust $P = 0$ Hence LHS of equations (1) and (2) are zero but equation (5) remains unaltered. Similarly ρ in equation (3) is equal to ρ_0 for dust and is regarded as constant for static sphere of dust. The corresponding solutions for static sphere of dust are altered accordingly. One has to replace $\rho = \rho_0 + 3P$ for fluid by $\rho = \rho_0$ for dust. Thus in equation (4) and (10) α becomes $\alpha = \frac{8\pi\rho_0}{3}$ for dust. The constant m for exterior solution becomes $\frac{4\pi\rho_0 a^3}{3}$ for dust.

Kelkaret al[5] have shown that a star (perfect fluid) of uniform density can rotate with any shape according to Einstein but rotating star has unique ellipticity of shape according to Newton. Further a star of spherical shape can rotate according to Einstein. In this paper we have shown that static sphere of dust can exist according to Einstein's theory. For radial motion of a star see Kelkaret al6. The star can move with uniform velocity according to Einstein but is accelerated inward according to Newton.

Conclusion:

These calculations done by us give strong motivation forsuggesting a change in Einstein's field equation.

We now give new field equations as

1) $T^{k}_{;k} = 0$ This is an independent equation which gives hydrodynamics of the fluid.

2) $R_{ik} \equiv -4\pi\rho_0 g_{ik} + \eta_{ik}$ where η_{ik} is a small correction term.

η_{ik} is given[7] by

$$\eta^{ik} = 4\pi P\left[\frac{dx^i}{ds}\frac{dx^j}{ds} - g^{ik}\right]$$

so that $g^{ik}\eta_{ik} = 0 = \eta_{ik}g^{ik}$ and

$\eta_{ik} << 4\pi\rho_0 g_{ik}$

This last condition ensures the Newtonian approximation of the field namely

$R_4 = -\frac{1}{2}\nabla^2 g_4 = -4\pi\rho$ as shown by Eddington[8]. Similarly $T^{k}_{;k} = 0$ gives Newtonian approximation of the dynamics of fluid as shown by Eddington[9]. One will have to take the equation of state of the fluid as an additional equation. This equation will prevent the collapse of star to a point. The factor 4π in the expression for η_{ik} has been obtained by Kelkar and Shrivastav[10] using the principles of least action and the energy momentum aspect.

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