RESEARCH PAPER | PHYSICS | Volume : 5 | Issue : 4 | April 2015 | ISSN - 2249-555X

Establishment of Soliton Concept

KEYWORDS: Solitons; inverse scattering method; soliton equation.

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ABSTRACT *Soliton is a nonlinear wave which propagates without change of its properties and remain stable against mutual collisions and retain their identities.*

The first is a solitary wave condition known in hydrodynamics since the 19th century. The second means that the wave has the property of a particle. In modern physics, a suffix-on is used to indicate the particle property, for example, phonon and photon. Zabusky and Kruskal [1] named a solitary wave with the particle property a `soliton'.

The history leading to the discovery of soliton is interesting and impressive. The first documented observation of the solitary wave was made in 1834 by the Scottish scientist and engineer, John Scott-Russell (1844):

I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped—not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished and after a chase of one or two miles I lost it in the windings of the channel. Such in the month of August 1834 was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation.

The word `solitary wave' was coined by Scott-Russell himself. This phenomenon attracted some attention of scientists including Stokes, Boussinesq and Rayleigh, but a theoretical confirmation had to wait until 1898 when two Dutch physicists, Korteweg and de Vries, presented their now famous equation [2] (KdV equation for short),

$$u_t + \alpha u u_x + \beta u_{xxx} = 0; \quad \alpha, \beta \text{ constants.} \qquad (1.1)$$

Here, u(x; t) represents the height from the average water surface and x is the coordinate moving with the velocity of a linearized wave.

In 1955, Fermi, Pasta and Ulam investigated how the equilibrium state is approached in a one-dimensional nonlinear lattice [3]. It was expected that the nonlinear interactions among the normal modes of the linear system would lead to the energy of the system being evenly distributed throughout all the modes. The results of numerical analysis contradicted this idea. The energy is not distributed equally into all the modes, but the system returns to the initial state after some period (the recurrence phenomena). In 1965, Zabusky and Kruskal solved the KdV equation numerically as a model for nonlinear lattice and observed the recurrence phenomena[5]. Further, they found an unexpected property of the KdV equation. From a smooth initial waveform, waves with sharp peaks emerge. Those pulse-waves move almost independently with constant speeds and pass through each other after collisions. A detailed analysis confirmed that each pulse is a solitary wave of sech2-type and the solitary waves behave like stable particles. Thus, the soliton was discovered.

2. Result and discussion of soliton concept

How do we confirm analytically the properties of solitons? Why are solitons stable like particles? Is soliton a specific phenomenon of the KdV equation? Solutions to these questions played an important role in establishing the soliton concept.

After suitable scaling of the independent and dependent variables the KdV equation has a form,

$$u_t \quad 6uu_x + u_{xxx} = 0: \qquad (2.1)$$

The second and third terms represent the nonlinear and dispersion effects, respectively. The nonlinear effect causes the steepening of waveform, while the dispersion effect makes the waveform spread. Due to the competition of these two effects, a stationary waveform (solitary wave) exists. The reason why each solitary wave is stable inspite of mutual inter-actions is that the KdV equation has an infinite number of conserved quantities. Dynamical properties of the system are severely restricted by the existence of an infinite number of conservation laws. The conserved quantities guarantee the time-independence of parameters which characterize the solitons, and therefore the solitons are stable. Corresponding

to an infinite number of conserved quantities (recall the field variable has infinite degrees of freedom), arbitrary number of solitons may coexist.

Fundamental properties of solitons are investigated by the inverse scattering method. In 1967, Gardner, Greene, Kruskal and Miura [4] introduced a linear problem (eigenvalues problem) where the potential u(x; t) is the solution of the KdV equation (2.1) ,

Volume : 5 | Issue : 4 | April 2015 | ISSN - 2249-555X

$$\psi_{xx} + u(x; t)\psi = \lambda\psi. \quad (2.2)$$

It can be shown that when u evolves obeying (2.1), the eigenvalue l does not depend on time. Equation (2.2) is nothing but the Schrödinger equation in quantum mechanics. A problem that for a given u one calculates the transmission coefficient 1 =a(k) , the reflection coefficient b(k)=a(k), discrete eigenvalues ln = hn2 etc. is called a scattering problem. Conversely, a problem that for a given scattering data a(k); b(k); l n etc. one determines the potential is called an inverse scattering problem. The latter problem for eq. (2.2) was solved by Gelfand-Levitan and Marchenko. It can be shown further that the time-development of the eigenfunction y(x; t) is

$$\psi_t = 4\psi_{xxx} + 3u_x\psi + 6u\psi_x. \quad (2.3)$$

The initial-value problem of the KdV equation could be solved. At that time, however, it seemed a fluke. Five years later, by extending the inverse scattering method, Zakharov and Shabat [6] solved the nonlinear Schr¨odinger (NLS) equation,

$$i\psi_t + \psi_{xx} + 2|\psi|^2\psi = 0; \quad (2.4)$$

and subsequently Wadati solved the modified KdV (mKdV) equation [7] ,

$$u_t + 6u^2u_x + u_{xxx} = 0; \quad (2.5)$$

Further, the Sine-Gordon equation has been solved [8] and we have now more than 100 soliton equations.

Through the inverse scattering method, we are in a position to define the soliton in a rigorous manner. A transformation from the field variables to the scattering data is a canonical transformation, and the action-angle variables are defined in the scattering data space. Thus, the soliton equation is a completely integrable system and the soliton is a fundamental mode of the system [9].

The discovery of the inverse scattering method is the most important development in the theory of solitons. When the scattering data space is regarded as the extension of the mo-mentum space, the inverse scattering method is considered as the extension of the Fourier transformation into nonlinear problems. The Fourier transformation was introduced in 1811 to solve the diffusion equation. About 150 years later, it was developed into a unified method to solve nonlinear evolution equations.

CONCLUSIONS:

Korteweg and de Vries and nonlinear Schrödinger (NLS) equation describes soliton motion and the compression of pulses occurs by SPM (Self phase modulation) and broadening of pulse occurs by GVD (Group velocity dispersion).

If these two mechanisms compensate each other the pulse do not change shape (called fundamental solitons).

Acknowledgments:
I shall always be grateful to the chairman Shri K.S Ravi Kumar and principal of JPNCE institution for all the possible support extended to my work.

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