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Salient Aspects of Soliton Physics

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ABSTRACT *The soliton concept has developed with mathematical methods. There are many new approaches such as the inverse scattering method, KdV and modified KdV equations but Soliton solutions of the sine-Gordon equation are far richer than those of the KdV and modified KdV equations.*

1.1.1: INTRODUCTION:
Study of salient aspects of soliton physics taking the Sine-Gordon (SG) equation:

We saw that the studies of solitary waves propagating in shallow waters such as a canal are related to the problem in nonlinear lattices and lead to the discovery of solitons. It was the KdV equation that encompassed the two different fields, hydrodynamics and lattice dynamics. This is merely one example of the universality and ubiquity of soliton equations.

We consider some salient aspects of soliton physics taking the Sine-Gordon (SG) equation as an example,

$$\phi_{tt} \quad \phi_{xx} + \sin\phi = 0: \qquad (1.1)$$

The SG equation has various interesting solutions: as x varies from ¥ to ¥, a solution which changes from 0 to 2p is a soliton (kink). A solution which changes from 2p to 0 is an anti-soliton (anti-kink) and the bound state of kink and anti-kink is a breather. In 1939, Frenkel and Kontrova [1] introduced the SG equation as a model for the dislocations in crystals. The displacement f (x; t) of atoms connected by linear springs may propagate as a kink in the periodic crystal field. Around 1960, Perring and Skyrme [3] considered the SG equation as a model for elementary particles (more rigorously, baryons). They examined collisions of kink-kink and kink-antikink and confirmed the particle-like stability of kinks (Historically, Seeger, Donth and Kochend¨orfer [2] found the kink-kink solution and the kink-antikink solution in the study of the SG equation as a dislocation model). Further, Perring and Skyrme defined a topological charge,

$$Q = \quad 2\pi \mathrm{f} \phi(\infty; t) \quad \phi(\ \infty; t) \mathrm{g}: \qquad (1.2)$$

1.1.2: Results and Discussion:
The topological charge is a conserved quantity under the boundary condition imposed on the field variable. In general, a discrete invariant quantity due to the topology of the field variables is called topological quantum number. In 1967, McCall and Hahn [3] discovered an interesting phenomenon in the field of nonlinear optics. Coherent light propagating in the system of 2-level atoms obeys the SG equation when the spectral widths are neglected (perfect resonance). The observed soliton-like behavior is called self-induced transparency (SIT). The 2p-pulse is the soliton and 0p-pulse is the breather. In the other limit, that is, the interaction between the medium and the light wave is not resonant, the envelop of the electric field is described by the NLS equation [4]. As an application of the SG equation related to new technology, a propagation of magnetic fluxes in the Josephson junction is important. The Josephson junction consists of two superconductors (1 and 2) and the insulator. We denote by f1 and f2 the phases of Cooper-pair wave functions in the superconducting plates. Due to the Josephson current J = J 0 sin f caused by the phase difference f = f1 f2 , the motion of f is described by the SG equations. There are many applications of the SG equation including one-dimensional organic conductors, one-dimensional ferromagnet, He3 and liquid crystals.

A wide applicability of the soliton equation implies soliton phenomena which are common in various fields of physics. This is the essence of soliton physics. Solitons appear in almost all branches of physics, such as hydrodynamics, plasma physics, nonlinear optics, condensed matter physics, low temperature physics, particle physics, nuclear physics, biophysics and astrophysics.

2. Further developments and open problems
The study of solitons is the first systematic research on nonlinear phenomena with a consistent leading principle. While the soliton concept makes a new viewpoint on nature, there are many problems to be studied.

1) Soliton mathematics
The soliton concept has developed with mathematical methods. There are many new approaches such as the inverse scattering method, Hirota method, the Darboux transformation and Painlev´ analysis. The most fundamental problem continues to be a criterion and a classification of the completely integrable systems[6].

2) Multi-dimensional solitons
Most of the soliton systems are (1 + 1) dimensional. In the cases of the Ernst equation and the cylindrical KdV equation, the number of independent variables is reduced because of the symmetry. Well known examples of (2 + 1) dimensional soliton systems are

$$(u_t - 6uu_x + u_{xxx})_x + \sigma u_{yy} = 0, \ \sigma = \pm 1 \qquad (2.1)$$

and

$$\sigma^2 v_{xx} + v_{yy} - (|u|^2)_{xx}, \ \sigma^2 = \pm 1 \qquad (2.2)$$

We call (2.1) the 2-dimensional KdV equation or Kadomtsev-Petviashvili equation, and (2.2) the Davey-Stewartson equation. A characterization of multi-dimensional solitons requires further investigations.

Conclusion

To control a nonlinear dynamical system in practice, one needs to investigate the effects due to external forces, noises, impurities and dissipations. For soliton systems with such effects, there are many reported numerical works and perturbative analyses. Among many, an interesting scenario is the competition of spatial order (solitons) and temporal disorder (chaos). On the other hand, the effect of the gaussian noise on the KdV solitons has been analytically studied.

Related to recent applications in nonlinear optics and condensed matter physics, non-linear wave propagations under the periodic potential are interesting. This would be a standard problem which is important both theoretically and experimentally.

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