RESEARCH PAPER | STATISTICS | Volume : 5 | Issue : 4 | April 2015 | ISSN - 2249-555X

Estimating Scale Model: Type II Censored Sample

KEYWORDS: Equivariant estimation, Type-II censored sampling, Optimal estimation, Scale model and Uniform model.

T. Leo Alexander

Associate Professor, Department of Statistics, Loyola College, Chennai

ABSTRACT *Lehmann and Casella (1998) have discussed in detail onequivariant estimation of the parameters of location, scale, location-scale models. Edwin Prabakaranand Chandrasekar(1994) developed simultaneous equivariant estimation approach. The problem of Equivariant estimation of scale parameter of uniform model in three situations based on type-II right censored sampling is consider in this paper.*

1. Introduction

Equivariance is a desirable property used for restricting the class of estimators whenever the model possesses symmetry. Zacks (1971) and Lehmann and Casella (1998) have discussed in detail on study of the problem of equivariant estimation for certain models. In the case of location-scale model, Lehmann and Casella(1998) develops marginal Equivariant procedure for estimating the parameters. Edwin Prabakaranand Chandrasekar (1994) have proposed a simultaneous Equivariant estimation for estimating the parameters of a location-scale model. For a detailed discussion on simultaneous equivariant estimation and related results the reader is referred to Edwin Prabakaran (1995). Contributions to simultaneous Equivariant estimation based on censored samples studied in Leo Alexander(2000).

In this paper, we obtain optimal estimators for the parameter(s) of uniform model under Type II censoring by invoking the above procedures. The problem of Equivariant estimation for the uniform scale model considering three different loss functions namely Squared error loss function, Absolute error loss function and Linex loss function is discussed Section 2.

1.1 Preliminaries

Suppose N randomly selected units were placed on a test simultaneously, the failure times of the first n units to fail were observed. Thus the number of completely determined life spans is n and the number of censored ones is (N-n). let$X_{i:N}$, i=1,2,…,n denote the failure times of the completely observed items. Then the joint probability density function (pdf) of $(X_{1:N}, X_{2:N},..., X_{n:N})$ (BAIN, 1978) is

$$g_\theta(x_1, x_2,..., x_n) = \frac{N!}{(N-n)!}\prod_{i=1}^{n} f_\theta(x_i)\left\{1 - F_\theta(x_i)\right\}^{N-n}$$

Here f_θ and F_θ denote the common pdf and the distribution function of the failure times of the units selected randomly, which are put to test. Further n is assumed to be known in advance.

2 uniform scale model

Suppose $\mathbf{X} = (X_1, X_2,..., X_N)$ have the joint pdf

$$f(x;\tau)=1/\tau^{N}\ ,\ 0\le x_i\le\tau,\quad i=1,2,\ldots,N;\tau>0.$$

Thus (1.1) reduces to

$$g_\tau(x_{1:N},\ldots,x_{n:N})=\frac{N!}{(N-n)!}\frac{1}{\tau^N}(1-\frac{x_{n:N}}{\tau})^{N-n},\quad 0\le x_{1:N}\le x_{n:N}\le\tau;\tau>0\quad\ldots(2.1)$$

Note that the above pdf belongs to a scale model. We are interested in deriving MRE estimator of τ^m by considering three loss functions. Following Lehmann Casella (1998, p.169) the MRE estimator of τ^m is given by

$$\delta^*(\mathbf{X})=\frac{\delta_0(\mathbf{X})}{w^*(\mathbf{Z})},$$

where δ_0 is a scale equivariant estimator and $w(\mathbf{z})=w^*(\mathbf{z})$ minimizes

$$E_1[\gamma\{\frac{\delta_0(\mathbf{x})}{w(\mathbf{z})}\}\,|\,\mathbf{z}].$$

Case (i): If the loss function is of the form

$$\gamma(\frac{\delta}{\tau^m})=(\frac{\delta}{\tau^m}-1)^2$$

then $w^*=\dfrac{E_1(\delta_0{}^2\,|\,\mathbf{z})}{E_1(\delta_0\,|\,\mathbf{z})}$.

Clearly $\delta_0(\mathbf{X})=X_{n:N}^m$ is a scale equivariant estimator which is a sufficient statistic Further δ_0 is not complete sufficient. Since we are interested in the evaluation of conditional distribution under $\tau=1$, we take $\tau=1$ in (2.1) . In order to find w^*,

consider the transformation

$$Z_N=X_{n:N}\quad and\quad Z_i=X_{i:N}/X_{n:N},\ i=1,2,\ldots,n-1.$$

Now $X_{n:N}=Z_n$ and

$X_{i:N}=Z_nZ_i,\ i=1,2,\ldots,n-1$

the Jacobian of the transformations is Z_n^{n-1}.

Thus the joint pdf of $\mathbf{Z}=(Z_1,Z_2,\ldots,Z_n)$ is given by

$$h(z_1,\ldots,z_n)=\frac{N!}{(N-n)!}z_n{}^{n-1}(1-z_n)^{N-n},\quad 0<z_1<\ldots<z_n<1,\ \ 0<z_n<1.$$

Also the joint pdf of $\mathbf{Z}=(Z_1,Z_2,\ldots,Z_{n-1})$ is given by

$$h_1(z_1,\ldots,z_{n-1})=\frac{N!}{(N-n)!}\int_0^1 z_n{}^{n-1}(1-z_n)^{N-n}\,dz_n=(n-1)!\,,\quad 0<z_1<\ldots<z_n<1.$$

Thus the condition pdf of Z_n given $(Z_1,\ldots,Z_{n-1})$ is given by

$$h^*(z_n\,|\,z_1,\ldots,z_{n-1})=\frac{N!}{(N-n)!}z_n{}^{n-1}(1-z_n)^{N-n},\quad 0<z_n<1\quad\ldots(2.2)$$

Note that Z_n is independent of $(Z_1,\ldots,Z_{n-1})'$.

Volume : 5 | Issue : 4 | April 2015 | ISSN - 2249-555X

$$Now \quad w^* = \frac{E_1(\delta_0{}^2 \mid \mathbf{z})}{E_1(\delta_0 \mid \mathbf{z})}.$$

Consider

$$E_1(\delta_0{}^2 \mid \mathbf{z})$$
$$= \frac{N!}{(n-1)!(N-n)!} \int_0^1 z_n{}^{2m+n-1} (1-z_n)^{N-n}\, dz_n$$
$$= \frac{N!}{(n-1)!} \frac{(2m+n-1)!}{(2m+N)!} \quad . \quad .$$

Similarly

$$E_1(\delta_0 \mid \mathbf{z}) = \frac{N!}{(n-1)!} \frac{(m+n-1)!}{(m+N)!} \quad ...(2.3)$$

Thus

$$w^* = \frac{(2m+n-1)! \; (m+N)!}{(m+n-1)! \; (2m+N)!} \quad .$$

Therefore the MRE estimator of τ^m is given by

$$\delta^*(\mathbf{X}) = \frac{(m+n-1)! \; (2m+N)!}{(2m+n-1)! \; (m+N)!} \; X^m_{n:N} \quad ...(2.4)$$

Moreover, when the loss is squared error, the MRE estimator $\delta^*(\mathbf{X})$ can be evaluated more explicitly by the Pitman form (Lehmann and Casella p.179).

Therefore the Pitman estimator is given by

$$\delta^*(\mathbf{X}) = \frac{\int_0^{1/x_{n:N}} v^{n+m-1}(1-vx_{n:N})^{N-n}\, dv}{\int_0^{1/x_{n:N}} v^{n+2m-1}(1-vx_{n:N})^{N-n}\, dv} \quad .$$

Taking $vx_{n:N} = u$, so that

$$\delta^*(\mathbf{X}) = \frac{\int_0^1 u^{n+m-1}(1-u)^{N-n}\, du}{\int_0^1 u^{n+2m-1}(1-u)^{N-n}\, du} X^m_{n:N}.$$

Thus

$$\delta^*(\mathbf{X}) = \frac{(m+n-1)!(2m+N)!}{(2m+n-1)!(m+N)!} X^m_{n:N}.$$

This estimator coincides with the one given in (2.4).

Remark 2.1 If n=N and m=1, the above estimator reduces to

$$\delta^*(\mathbf{X}) = \frac{N+2}{N+1} x_{N:N} \,,$$

which is same as the complete sample case (Lehmann 1983, p.177).

Case (ii): If the loss function is of the form

$$\gamma\left(\frac{\delta}{\tau}\right) = \frac{|\delta - \tau|}{\tau},$$

then w^* = scale-median of $\delta_0(\mathbf{X})$ under the conditional distribution of X given Z and with $\tau = 1$, so that w^* can be found by solving the following equation for $c = w^*$

Here $\delta^*(\mathbf{X}) = X_{n:N}$ which has the pdf

$$f(x_{n:N}) = N\, x_{n:N}^{N-1}, \qquad 0 < x_{n:N} < 1.$$

So the equation reduces to

Volume : 5 | Issue : 4 | April 2015 | ISSN - 2249-555X

$$N\int_0^c x_{n:N}\, x_{n:N}^{N-1}\, dx_{n:N} = N\int_c^1 x_{n:N}\, x_{n:N}^{N-1}\, dx_{n:N}$$

This implies $w^* = (1/2)^{1/(N+1)}$.

Therefore the MRE estimator of τ is given by

$$\delta^*(\mathbf{X}) = \frac{\delta_0(\mathbf{X})}{w^*}$$
$$= 2^{1/(N+1)} X_{n:N}$$

Case (iii): Following Varian(1975), MRE estimator of τ under Linex loss function is provided. Consider the scale invariant Linex loss function,

$$R(\delta \mid \mathbf{z}) = e^{-a} E_1(e^{a/w\delta_0} \mid z) - a/w\, E_1(\delta_0 \mid z) + a -$$
$$= e^{-a}\frac{N!}{(n-1)!(N-n)!}\int_0^1 e^{a/wz_n} z_n^{\,n-1}(1-z_n)$$
$$-(a/w)(n/(N+1)) + a - 1$$

In view of (2.2) and (2.3) when m=1.

Thus w^* is to obtained as the value of w minimizing $R(\delta \mid \mathbf{z})$.

Therefore the MRE estimator of τ is given by

$$\delta^*(\mathbf{X}) = \frac{\delta_0(\mathbf{X})}{w^*} = \frac{X_{n:N}}{w^*} .$$

$$L(\tau;\delta) = e^{a(\delta/\tau - 1)} - a(\delta/\tau - 1) - 1, \quad a > 0 .$$

Here $\delta_0(\mathbf{X}) = X_{n:N}$. In order to find w^* , consider

Acknowledgement

The author thanks Dr. B. Chandrasekar, Department of Statistics, Loyola College, Chennai for his valuable guidance and suggestions.

REFERENCE 1. Bain, L. J., 1978: Statistical Analysis of reliability and life-testing models, Marcel Dekker. Inc., New York. | 2. Edwin Prabakaran, T. andChandrasekar, B. (1994): simultaneous Equivariant estimation for location-scale models. J.Statist. Plann. Inference 40, 51-59. | 3. Edwin Prabakaran, T., 1995: Contributions to theory of simultaneous Equivariant estimation. Ph.D. thesis, University of Madras. | 4. Lehmann, E.L. and Casella, G. (1998). Theory of Point Estimation, Second edition, Springer – Verlag, New York. | 5. Leo Alexander, T. (2000). Contributions to simultaneous equivariant estimation based on censored samples, Ph.D Thesis, University of Madras, India. | 6. Varian, H.R. (1975). A Bayesian Approch to Real Estate Assessment In : Studies in Bayesian Econometric and Statistics in Honor of Leonard J. Savage, eds. S.E. Frienberg and A.Zellner. North Holland, Amsterdam, 195-208. | 7. Zacks, S., 1971: The Theory of Statistical Inference. John Wiley and Sons, New York. |