RESEARCH PAPER | Mathematics | Volume : 5 | Issue : 4 | April 2015 | ISSN - 2249-555X

# On the Convergence of a Fourth-Order Method for Simultaneous Finding Polynomial Zeros

KEYWORDS: simultaneous methods, polynomial zeros, local convergence, error estimate.

**Slav I. Cholakov**
Faculty of Mathematics and Informatics, University of Plovdiv. Plovdiv-4000, Bulgaria

**Milena D. Petkova**
Faculty of Mathematics and Informatics, University of Plovdiv. Plovdiv-4000, Bulgaria

**ABSTRACT** *In 2011, Petković, Rančić and Milošević [5] presented a new fourth-order iterative method for finding all zeros of a polynomial simultaneously. In this paper we establish a new local convergence theorem with error estimates for this method. In particular, an estimate of the radius of the convergence ball of the method is obtained.*

## 1. INTRODUCTION

Throughout this paper $(K,|\cdot|)$ denotes an arbitrary normed field and $K[z]$ denotes the ring of polynomials over $K$. Let $f \in K[z]$ be a polynomial of degree $n \geq 2$. A vector $\xi \in K^n$ is called a *root vector* of $f$ if $f(z) = a_0 \prod_{i=1}^{n} (z - \xi_i)$ for all $z \in K$, where $a_0 \in K$.

In the literature there are a lot of iterative methods for finding all zeros of polynomial simultaneously (see, e.g., Sendov, Andreev and Kjurkchiev [18], Kyurkchiev [2], McNamee [3], Petković [4] and references therein). In 2011, Petković, Rančić and Milošević [5] presented a new fourth-order iterative method for simultaneous finding polynomial zeros. Their method is defined by the following iteration

$$(1) \qquad x^{k+1} = T x^k, \qquad k = 1, 2, \ldots,$$

where the operator $T : D \subset K^n \to K^n$ is defined by $Tx = (T_1(x), \ldots, T_n(x))$ with

$$(2) \qquad T_i(x) = x_i - u_i - \frac{u_i^2 \left( \dfrac{f''(x_i)}{f'(x_i)} - u_i \left( S_i^2 - G_i \right) \right)}{2\,(1 - u_i S_i)^2}$$

for $i = 1, \ldots, n$, where

$$u_i = \frac{f(x_i)}{f'(x_i)}, \quad S_i = \sum_{j \neq i} \frac{1}{x_i - x_j}, \quad G_i = \sum_{j \neq i} \frac{1}{(x_i - x_j)^2}.$$

The domain $D$ of $T$ is the set

$$D = \left\{ x \in K^n : x_i \neq x_j \ for\ i \neq j, f'(x_i) \neq 0, 1 - u_i S_i \neq 0 \right\}.$$

Petković, Rančić and Milošević [5] proved the following asymptotic convergence theorem for the method (1).

**Theorem 1.1 ([5]).** Let $f \in C[z]$ be a polynomial of degree $n \geq 3$ which has $n$ simple zeros in $C$. If an initial guess $x^0 \in C^n$ is sufficiently close to a root vector $\xi$ of $f$, then the iteration (1) converges to $\xi$ with order of convergence four.

In this paper, we establish a new local convergence theorem with error estimates for the method (1) which improves Theorem 1.1. In particular, we obtain an estimate of the radius of convergence ball of this method.

Volume : 5 | Issue : 4 | April 2015 | ISSN - 2249-555X

## 2. PRELIMINARIES

Throughout this paper we follow the terminology from [9, 10, 17]. Let the vector space $K^n$ be endowed with the $\| x \|_p = \left( \sum_{i=1}^{n} |x_i|^p \right)^{1/p}$ for some $1 \le p \le \infty$. Let $(R^n, \| \cdot \|_p)$ be equipped with coordinate-wise ordering $\preceq$ defined by $x \preceq y$ if and only if $x_i \le y_i$ for $i = 1, \ldots, n$. Then $(R^n, \| \cdot \|_p, \preceq)$ is a solid vector space. Furthermore, let $K^n$ be endowed with the cone norm $\| \cdot \| : K^n \to R^n$ defined by

$$\| x \| = (|x_1|, \ldots, |x_n|).$$

Then $(K^n, \| \cdot \|)$ is a cone norm space over $R^n$.

In the sequel, for two vectors $x \in K^n$ and $y \in R^n$ we denote by $\frac{x}{y}$ a vector in $R^n$ defined by

$$\frac{x}{y} = \left( \frac{|x_1|}{y_1}, \ldots, \frac{|x_n|}{y_n} \right)$$

provided that $y$ has only nonzero components. Finally, the function $d : K^n \to R^n$ is defined by $d(x) = \left( d_1(x), \ldots, d_n(x) \right)$, where

$$d_i(x) = \min_{j \ne i} |x_i - x_j| \qquad (i = 1, \ldots, n).$$

**Lemma 2.1 ([12]).** Let $u, v \in K^n$ and $1 \le p \le \infty$. If $v$ is a vector with distinct components, then for all $i, j = 1, \ldots, n$,

$$|u_i - v_j| \ge \left( 1 - \left\| \frac{u - v}{d(v)} \right\|_p \right) |v_i - v_j|,$$

$$|u_i - u_j| \ge \left( 1 - 2^{1/q} \left\| \frac{u - v}{d(v)} \right\|_p \right) |v_i - v_j|,$$

where $1 \le q \le \infty$ is defined by $1/p + 1/q = 1$.

Recently, Proinov [8, 9, 11] has developed a general convergence theory for iterative methods of the type (1), where $T : D \subset X \to X$ is an iteration function of a cone metric space $X$. A central role in this theory is played by the concept *a function of initial conditions* of $T$. In this theory, the convergence of an iterative process is always studied with respect to a function of initial conditions. Recall that a function $E : D \to R_+$ is called a function of initial conditions of $T$ if there exist an interval $J \subset R_+$ containing $0$ and a gauge function $\varphi : J \to J$ such that $E(Tx) \le \varphi(E(x))$ for all $x \in D$ such that $Tx \in D$ and $E(x) \in J$. Examples of functions of initial conditions can be found in [1, 6-9, 11-16].

The following theorem of Proinov plays an important role in the proof of our main result. This theorem in a metric space was proved in [8]. In cone metric space setting the proof can be found in [11].

**Theorem 2.1 ([11]).** Suppose $(X, \| \cdot \|)$ is a cone normed space over a solid vector space $(Y, \preceq)$. Let $T : D \subset X \to X$ be an operator of $X$ and $E : D \to R_+$ be a function of initial conditions of $T$ with a strict gauge function $\varphi$ of order $r > 1$ on an interval $J$. Suppose $T$ is an iterated contraction at $\xi \in D$ with respect to $E$ with control function $\phi$, i.e. $E(\xi) \in J$ and

(3) $$\| Tx - \xi \| \preceq \phi(E(x)) \| x - \xi \|$$

for all $x \in D$ with $E(x) \in J$, where $\phi : J \to R_+$ is a nondecreasing function such that

(4) $$\varphi(t) = t\,\phi(t) \text{ for all } t \in J.$$

Then $\xi$ is a unique fixed point of $T$ in the set $U = \{ x \in D : E(x) \in J \}$. Moreover, for each initial point $x_0$ of $T$ the iteration (1) remains in the set $U$ and converges to $\xi$ with error estimates

(5) $$\| x_{k+1} - \xi \| \preceq \lambda^{r^k} \| x_k - \xi \|, \quad \| x_k - \xi \| \preceq \lambda^{S_k(r)} \| x_0 - \xi \| \quad \text{for all } k \ge 0,$$

where $\lambda = \phi(E(x_0))$ and $S_k(r) = (r^k - 1)/(r - 1)$.

Volume : 5 | Issue : 4 | April 2015 | ISSN - 2249-555X

**3. AUXILIARY RESULTS**

Let $f \in K[z]$ be a polynomial of degree $n \geq 2$ which has $n$ simple zeros in $K$ and let $\xi \in K^n$ be a root vector of $f$. In this section we study the convergence of the iteration (1) with respect to the function of initial conditions $E : K^n \to R_+$ defined by

$$E(x) = \left\| \frac{x - \xi}{d(\xi)} \right\|_p \quad (1 \leq p \leq \infty). \tag{6}$$

For the sake of brevity, we use the following notation

$$a = (n-1)^{1/q}, \quad b = 2^{1/q}, \tag{7}$$

where $1 \leq q \leq \infty$ is defined by $1/p + 1/q = 1$.

**Lemma 3.1.** Let $f \in K[z]$ be a polynomial of degree $n \geq 2$ which has $n$ simple zeros in $K$ and let $\xi \in K^n$ be a root vector of $f$. Suppose $x \in D$ is a vector such that $f(x_i) \neq 0$ for some $1 \leq i \leq n$. Then

$$T_i(x) - \xi_i = \frac{A_i^2 + 2\sigma_i A_i^2 - 2\sigma_i A_i - B_i}{2(1+\sigma_i)(1-A_i)^2}(x_i - \xi_i), \tag{8}$$

where $T_i(x)$ is defined by (2), $\sigma_i$, $A_i$ and $B_i$ are defined with

$$\sigma_i = (x_i - \xi_i)\sum_{j \neq i} \frac{1}{x_i - \xi_j}, \quad A_i = \sum_{j \neq i} \frac{(x_i - \xi_i)(x_j - \xi_i)}{(x_i - \xi_j)(x_i - x_j)} \tag{9}$$

and

$$B_i = (x_i - \xi_i)^2 \sum_{j \neq i} \frac{(x_j - \xi_i)(2x_i - x_j - \xi_j)}{(x_i - \xi_j)^2 (x_i - x_j)^2}. \tag{10}$$

Define the real function $\phi$ by

$$\phi(t) = \frac{a(1-t)((a + (3-2n)b + 1)t + 2n) + 2a^2(n-1)t^2}{2(1-nt)(1-(b+1)t-(a-b)t^2)^2} t^3, \tag{11}$$

where $a$, $b$ are defined by (7).

**Lemma 3.2.** Let $f \in K[z]$ be a polynomial of degree $n \geq 2$ which has $n$ simple zeros in $K$, $\xi \in K^n$ be a root vector of $f$ and $1 \leq p \leq \infty$. Suppose a vector $x \in K^n$ satisfies

$$E(x) < \alpha = \min\left\{ \frac{2}{b + 1 + \sqrt{(b-1)^2 + 4a}}, \frac{1}{n} \right\}, \tag{12}$$

where $E : K^n \to R_+$ is defined by (6) and $a, b$ are defined by (7). Then the following statements hold true:

(i) $x \in D$;

(ii) $\| Tx - \xi \| \preceq \phi(E(x)) \| x - \xi \|$, where the function $\phi$ is defined by (11);

(iii) $E(Tx) \leq \varphi(E(x))$, where $\varphi$ is defined by $\varphi(t) = t\phi(t)$.

**Proof.** (i) Let $i \neq j$. By Lemma 2.1 with $u = x$ and $v = \xi$ and from (12), we get

$$\begin{aligned} |x_i - x_j| &\geq \left(1 - b\left\| \frac{x - \xi}{d(\xi)} \right\|_p\right) |\xi_i - \xi_j| \\ &\geq (1 - bE(x)) d_j(\xi) > 0. \end{aligned} \tag{13}$$

Hence, $x_i \neq x_j$. Now we have to prove that $f'(x_i) \neq 0$ for all $i = 1, \ldots, n$. If $x_i = \xi_i$, then $f(x_i) = 0$ and $f'(x_i) \neq 0$, since $f$ has only simple zeros. Let $x_i \neq \xi_i$. From Lemma 2.1 with $u = x$ and $v = \xi$, and (12), we get for $i \neq j$

$$\begin{aligned} |x_i - \xi_j| &\geq \left(1 - \left\| \frac{x - \xi}{d(\xi)} \right\|_p\right) |\xi_i - \xi_j| \\ &\geq (1 - E(x)) d_i(\xi) > 0, \end{aligned} \tag{14}$$

which means that $x_i \neq \xi_j$. In the case $x_i$ is not zero of $f$. Then,

$$\frac{f'(x_i)}{f(x_i)} = \frac{1}{x_i - \xi_i} + \sum_{j \neq i} \frac{1}{x_i - \xi_j} = \frac{1 + \sigma_i}{x_i - \xi_i}. \tag{15}$$

It follows from (15) that $f'(x_i) \neq 0$ if and only if $\sigma_i \neq -1$. From (9), the triangle inequality and (14), we obtain

Volume : 5 | Issue : 4 | April 2015 | ISSN - 2249-555X

(16)
$$|\sigma_i| \le |x_i - \xi_i| \sum_{j\ne i} \frac{1}{|x_i - \xi_j|} \le \frac{|x_i-\xi_i|}{(1-E(x))d_i(\xi)} \sum_{j\ne i} 1 \le \frac{(n-1)E(x)}{1-E(x)}.$$

From (16) and (12), we get the following estimate:

(17) $$|1+\sigma_i| \ge 1-|\sigma_i| \ge \frac{1-nE(x)}{1-E(x)} > 0.$$

Therefore, $\sigma_i \ne -1$. Taking into account the definition of $D$, it remains to prove that

(18) $$1 - \frac{f(x_i)}{f'(x_i)} \sum_{j\ne i} \frac{1}{x_i - x_j} \ne 0.$$

It follows from (15) and (9) that

(19) $$1 - \frac{f(x_i)}{f'(x_i)} \sum_{j\ne i} \frac{1}{x_i - x_j} = \frac{1-A_i}{1+\sigma_i},$$

which means that (18) holds true if and only if $A_i \ne 1$. Using the definition of $A_i$ in (9), the triangle inequality, (14), (13) and Hölder's inequality, we obtain

(20)
$$|A_i| \le |x_i - \xi_i| \sum_{j\ne i} \frac{|x_j-\xi_j|}{|x_i-\xi_j||x_i-x_j|} \le \frac{|x_i-\xi_i|}{(1-E(x))(1-bE(x))d_i(\xi)} \sum_{j\ne i} \frac{|x_j-\xi_j|}{d_j(\xi)} \le \frac{aE(x)^2}{(1-E(x))(1-bE(x))}.$$

From this and (12), we get the following estimate:

(21)
$$|1-A_i| \ge 1-|A_i| \ge \frac{1-(b+1)E(x)-(a-b)E(x)^2}{(1-E(x))(1-bE(x))} > 0,$$

which implies $A_i \ne 1$. Hence, $x \in D$.

(ii) We have to prove that

(22) $$|T_i(x)-\xi_i| \le \phi(E(x))|x_i - \xi_i|,$$

for all $i = 1,\ldots,n$. Let $1 \le i \le n$. The case $x_i = \xi_i$ is trivial since $T_i(x) = \xi_i$. Let $x_i \ne \xi_i$. From Lemma 3.1 and the triangle inequality, we get

(23)
$$|T_i(x)-\xi_i| \le \frac{|A_i|^2 + 2|\sigma_i||A_i|^2 + 2|\sigma_i||A_i| + |B_i|}{2(1-|\sigma_i|)(1-|A_i|)^2} |x_i - \xi_i|.$$

It follows from (10), the triangle inequality and (14) that

(24)
$$|B_i| \le |x_i-\xi_i|^2 \sum_{j\ne i} \frac{|x_j-\xi_j||2x_i-x_j-\xi_j|}{|x_i-\xi_j|^2|x_i-x_j|^2} \le \frac{|x_i-\xi_i|^2}{(1-E(x))^2 d_i(\xi)^2} \sum_{j\ne i} \frac{|x_j-\xi_j|(|x_i-x_j|+|x_i-\xi_j|)}{|x_i-x_j|^2}.$$

Using the triangle inequality and Hölder's inequality, we obtain

(25)
$$|x_i-x_j| \le |\xi_i-\xi_j| + |x_i-\xi_i| + |x_j-\xi_j| \le (1+bE(x))|\xi_i-\xi_j|.$$

Similarly of (25), we get that

(26) $$|x_i-\xi_j| \le (1+E(x))|\xi_i-\xi_j|.$$

From (24), (25), (26), (13) and Hölder's inequality, we obtain the following estimate:

(27) $$|B_i| \le \frac{a(2+(b+1)E(x))E(x)^3}{(1-E(x))^2(1-bE(x))^2}.$$

Combining (23), (20), (16) and (27), we get (22).

(iii) Dividing both sides of the inequality by $d_i(\xi)$ and taking the $p$-norm, we obtain (iii).

## 4. MAIN RESULTS

**Theorem 4.1.** Let $f \in K[z]$ be a polynomial of degree $n \ge 2$ which has $n$ simple zeros in $K$, $\xi \in K^n$ be a root vector of $f$ and $1 \le p \le \infty$. Suppose $x^0 \in K^n$ is an initial guess satisfying the following conditions,

(28) $$E(x^0) < \alpha \quad \text{and} \quad \phi(E(x^0)) < 1,$$

where the function $E : K^n \to R_+$ is defined by (6), $\alpha$ is defined in (12) and the real function $\phi$ is defined by (11). Then the iteration (1) is well defined and converges with fourth-order to $\xi$ with error estimates

Volume : 5 | Issue : 4 | April 2015 | ISSN - 2249-555X

$$(29)\quad \begin{matrix} \| x^{k+1} - \xi \| \preceq \lambda^{4^k} \| x^k - \xi \| \\ \| x^k - \xi \| \preceq \lambda^{(4^k-1)/3} \| x^0 - \xi \| \end{matrix} \quad \text{for all } k \geq 0,$$

where $\lambda = \phi(E(x^0))$.

**Proof.** We shall prove the theorem by applying Theorem 2.1 to the operator $T : D \subset K^n \to K^n$. Let $J = [0, R)$, where $R$ be the unique solution of the equation $\phi(t) = 1$ in interval $[0, \alpha)$. Then (28) is equivalent to $E(x^0) \in J$. It is easy to show that the function $\varphi$ defined by $\varphi(t) = t\phi(t)$ is quasi-homogeneous of degree four on $[0, \alpha)$ and $R$ is a fixed point of $\varphi$ in $(0, \alpha)$. Hence, $\varphi$ is a strict gauge function of order $r = 4$ on $J$. Then by Lemma 3.2 we conclude that the function $E : D \to R_+$ defined by (6) is a function of initial condition of $T$ with strict gauge function $\varphi$. The same lemma shows that $T$ satisfies the contractive condition (3).

Now we shall show that $x^0$ is an initial point of $T$. According to the assumptions of the theorem, we have $x^0 \in K^n$ and $E(x^0) \in J$. Note that $R < \alpha$. Then by Lemma 3.2 we conclude that $x^0 \in D$. According to Proposition 4.1 of [9] to show that $x^0$ is an initial point of $T$ we have to prove that

$$(30)\quad x \in D \text{ with } E(x) \in J \quad \text{implies } Tx \in D.$$

Since $x \in D$, then $Tx \in K^n$. On the other hand, $E(x) \in J$ implies $E(Tx) \in J$ since $\varphi : J \to J$ and $E(Tx) \leq \varphi(E(x))$. Thus, we have $Tx \in K^n$ and $E(Tx) \in J$. Applying Lemma 3.2 to $Tx$, we deduce $Tx \in D$ which proves (30).

It follows from Theorem 2.1 that iteration (1) is well defined and converges to $\xi$ with error estimates (29) which completes the proof. □

Setting $p = \infty$ in theorem Theorem 4.1 we get the following result.

**Theorem 4.2.** Let $f \in K[z]$ be a polynomial of degree $n \geq 2$ which has $n$ simple zeros in $K$ and let $\xi \in K^n$ be a root vector of $f$. Suppose $x^0 \in K^n$ is an initial guess satisfying the following conditions

$$(31)\quad E(x^0) = \left\| \frac{x^0 - \xi}{d(\xi)} \right\|_\infty < \min\left\{ \frac{2}{3 + \sqrt{4n - 3}}, \frac{1}{n} \right\}, \qquad \phi(E(x^0)) < 1,$$

where the real function $\phi$ is defined by

$$(32)\quad \phi(t) = \frac{(n-1)(2n - (5n-6)t + (2n^2 - n - 4)t^2)}{2(1 - nt)(1 - 3t - (n-3)t^2)^2} t^3$$

Then the iteration (1) is well defined and converges with fourth-order to $\xi$ with error estimates

$$(33)\quad \begin{matrix} \| x^{k+1} - \xi \| \preceq \lambda^{4^k} \| x^k - \xi \| \\ \| x^k - \xi \| \preceq \lambda^{(4^k-1)/3} \| x^0 - \xi \| \end{matrix} \quad \text{for all } k \geq 0,$$

where $\lambda = \phi(E(x^0))$.

**Corollary 4.1.** Let $f \in K[z]$ be a polynomial of degree $n \geq 2$ which has $n$ simple zeros in $K$ and let $\xi \in K^n$ be a root vector of $f$. If $x^0 \in K^n$ is an initial guess satisfying

$$(34)\quad E(x^0) = \left\| \frac{x^0 - \xi}{d(\xi)} \right\|_\infty \leq \frac{20}{37n},$$

then the iteration (1) is well defined and converges with fourth-order to $\xi$ with error estimates (33).

**Proof.** We shall prove that $x^0$ satisfies the condition (31) of Theorem 4.2. It is easy to show that $x^0$ satisfies the first inequality in (31). We have to show that

$$(35)\quad \phi_n = \phi(20/(37n)) < 1.$$

The sequence $(\phi_n)$ is decreasing for $n \geq 2$. Then, the inequality (35) follows from $\phi_2 < 1$. □

We end the paper with a corollary which improves Theorem 1.1. We denote by $\operatorname{sep}(f)$ the *separation number of* $f$ which is defined to be the minimum distance between two distinct zeros of $f$.

Volume : 5 | Issue : 4 | April 2015 | ISSN - 2249-555X

We denote by $\bar{U}(x,\rho)$ the closed ball in $(R^n, \|\cdot\|_\infty)$ with center $x$ and radius $\rho$.

**Corollary 4.2.** Let $f \in K[z]$ be a polynomial of degree $n \geq 2$ which has $n$ simple zeros in $K$ and let $\xi \in K^n$ be a root vector of $f$. Then for each initial guess $x^0$ in the closed ball $\bar{U}(\xi,\rho)$ with radius

$$\rho = \frac{20}{37n}\operatorname{sep}(f) \ ,$$

the iteration (1) is well defined and converges to $\xi$ with fourth-order and with error estimates (33).

**REFERENCE** [1] S. Cholakov, Local convergence of Chenyshev-like method for simultaneous finding polynomial zeros, C. R. Bulg. Acad. Sci., 66 (2013), 1081-1090. | [2] N. Kyurkchiev, Initial Approximations and Root Finding Methods, Mathematical Research, Vol. 104, Wiley, Berlin, 1998. | [3] J. McNamee, Numerical methods for roots of polynomials, Part I, Studies in Computational Mathematics, Vol. 14, Elsevier, Amsterdam, 2007. | [4] M. Petković, Point Estimation of Root Finding Methods, Lecture Notes in Mathematics. Lecture Notes in Mathematics, Vol. 1933, Springer, Berlin, 2008. | [5] M. Petković, L. Rančić, M. Milošević, On the new fourth-order methods for the simultaneous approximation of polynomial zeros, J. Comput. Appl. Math., 235 (2011), 4059-4075. | [6] P. Proinov, A new semilocal convergence theorem for the Weierstrass method from data at one point, C. R. Acad. Bulg. Sci., 59 (2006), 131-136. | [7] P. Proinov, Semilocal convergence of two iterative methods for simultaneous computation of polynomial zeros, C. R. Acad. Bulg. Sci., 59 (2006), 705-712. | [8] P. Proinov, General local convergence theory for a class of iterative processes and its applications to Newton's method, J. Complexity, 25 (2009), 38-62. | [9] P. Proinov, New general convergence theory for iterative processes and its applications to Newton-Kantorovich type theorems, J. Complexity, 26 (2010), 3-42. | [10] P. Proinov, A unified theory for cone metric spaces and its applications to the fixed point theory, Fixed Point Theory Appl., 2013 (2013), Article ID 103. | [11] P. Proinov, General convergence theorems for iterative processes and applications to the Weierstrass root-finding method, arXiv:1503.05243, 2015. | [12] P. Proinov, S. Cholakov, Semilocal convergence of Chebyshev-like root-finding method for simultaneous approximation of polynomial zeros, Appl. Math. Comput., 236 (2014), 669-682. | [13] P. Proinov, S. Cholakov, Convergence of Chebyshev-like method for simultaneous computation of multiple polynomial zeros, C. R. Acad. Bulg. Sci., 67 (2014), 907-918. | [14] P. Proinov, S. Ivanov, On the convergence of Halley's method for simultaneous computation of polynomial zeros, J. Numer. Math., (2014), accepted. | [15] P. Proinov, M. Petkova, A new semilocal convergence theorem for the Weierstrass method for finding zeros of a polynomial simultaneously, J. Complexity, 30 (2013), 366-380. | [16] P. Proinov, M. Petkova, Convergence of the two-point Weierstrass root-finding method, Japan J. Industr. Appl. Math., 31 (2014), 279-292. | [17] P. Proinov, I. Nikolova, Iterative approximation of fixed points of quasi-contraction mappings in cone metric spaces, J. Inequal. Appl., 2014 (2014), Article ID 226. | [18] Bl. Sendov, A. Andreev, N. Kjurkchiev, Numerical Solution of Polynomial Equations. In: Handbook of Numerical Analysis (P. Ciarlet, J. Lions, eds.), Vol. III, 625-778, Elsevier, Amsterdam, 1994. |