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Application of variational iteration method to seepage flow derivatives in porous media using fractional calculus

KEYWORDS

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ABSTRACT *In this paper the variational iteration method is implemented to give exact solutions for seepage flow derivatives in porous media. A correction functional for the fractional partial equation is well constructed by a general Lagrange multipliers which can be identified optimally via variational theory. Some examples are given and comparisons are made with the Adomian Decomposition Method (ADM).The comparisons show that the method is very effective ,convenient and overcome the difficulty arising in calculating Adomian polynomials.*

I. INTRODUCTION

Infact its very difficult to solve or to approximate nonlinear problems. Common analytic procedures linearize the problem or assume the nonlinearities insignificant. Such procedures change the actual problem or lead to lose some important information. The variational iteration method(VIM) was successfully applied to autonomous ordinary and partial differential equation. The method provides a solution without linearization. Perturbation, or unjustified assumption for linear and nonlinear differential equation.

II. PRELIMINARIES

II.1 SOME FORMULAE of fractional derivatives

Let us first start with Liouville's first formula with the known result $D^n e^{ax} = a^n e^{ax}$ where $D = \frac{d}{dx}, n \in N$ and extended it at first in the particular case $\alpha = \frac{1}{2}, \alpha = 2$ a (rational, irrational or complex) by

$$D^{\alpha} e^{ax} = a^{\alpha} e^{ax} \quad (1)$$

He assumed the series representation for $f(x)$ as

$$f(x) = \sum_{k=0}^{\infty} c_k e^{a_k x}$$

and defined the derivative of arbitrary order α by

$$D^{\alpha} f(x) = \sum_{k=0}^{\infty} c_k a_k^{\alpha} e^{a_k x} \quad (2)$$

Secondly the above formula was applied to the explicit function $x^{-\alpha}$. He considered the integral

$$I = \int_0^{\infty} u^{\beta-1} e^{-xu} du \quad (3)$$

Substituting $xu = t$ gives t he result

$$I = x^{-\beta} \int_0^{\infty} t^{\beta-1} e^{-t} dt = x^{-\beta} \Gamma(\beta) , , , , \operatorname{Re} \alpha > 0 \quad (4)$$

Operating on both sides of $x^{-\beta} = \frac{1}{\Gamma(\beta)}$ whit D^{α} with respect to x He obtained

$$\Gamma(\beta) D^{\alpha} x^{-\beta} = \int_0^{\infty} u^{\beta-1} D^{\alpha} e^{-xu} du$$

$$D^{\alpha} (e^{-xu}) = (-1)^{\alpha} u^{\alpha} e^{-xu}$$

$$D^{\alpha} x^{-\beta} = (-1)^{\alpha} \frac{\Gamma \alpha + \beta}{\Gamma \beta} x^{-\alpha-\beta} \quad (5)$$

Liouville used the latter in his investigation of potential theory

11.11. The methodology

To give a clear overview of the basic concepts of the variational iteration method we consider the following

differential equation we consider the following differential equation

$$Lu + Nu = g(t), \quad (6)$$

Where L and N are linear and nonlinear operators respectively, and $g(t)$ is the source inhomogeneous term.

The variational iteration method presents a correction functional for Eq. (6) in the form

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(\xi)(Lu_n(\xi) + N\tilde{u}_n(\xi) - g(\xi))d\xi,$$

$$(7)$$

Where λ is a general Lagrange multiplier, which can be identified optimally via the variational theory, and $\tilde{u}$ is a restricted variation which means $\delta \tilde{u}_n = 0$.

It is obvious now that main steps of the He's variational iteration method require first the determination of the Lagrange multiplier $\lambda(\xi)$ that will be identified optimally [1,2]. Integration by parts is usually used for the determination of the Lagrange multiplier $\lambda(\xi)$. In other words we can use

$$\int \lambda(\xi) u_n'(\xi) d\xi = \lambda(\xi) u_n(\xi) - \int \lambda'(\xi) u_n(\xi) d\xi$$

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$$\int \lambda(\xi) u_n''(\xi) d\xi = \lambda(\xi) u_n'(\xi) - \lambda'(\xi) u_n(\xi) + \int \lambda''(\xi) u_n(\xi) d\xi$$

(8)

And so on. The last two identities can be obtained by integrating by parts.

Having determined the Lagrange multiplier $\lambda(\xi)$ the successive approximations $u_{n+1}, n \geq 0$, of the solution u will be readily obtained upon using any selective function u_0. Consequently, the solution

$u = \lim_{n \to \infty} u_n$. (9)

In other words, the correction functional (7) will give several approximations, and therefore the exact solution is obtained as the limit of the resulting successive approximations.

III. SEEPAGE FLOW DERIVATIVES IN POROUS MEDIA FINDINGS

We shall treat the above mention problem using variational iteration method (VIM). The problem is modelled by the fractional partial differential equation (FPDE):

$$\frac{\partial^\alpha p(x,y,t)}{\partial x^\alpha} + \frac{\partial^\alpha p(x,y,t)}{\partial y^\alpha} - \frac{1}{v}\frac{\partial p(x,y,t)}{\partial t} = 0$$

$$p(0,y,,t) = 1 + e^y + e^t$$
$$p(x,0,t) = 1 + e^x + e^t$$
$$p(x,y,0) = 1 + e^x + e^y$$
$$p_0(x,y,t) = 1 + e^y + e^x$$

(10)

Solution:

$$p_{n+1}(x,y,t) = p_n(x,y,t) + \lambda \int_0^x \left[\frac{\partial^\alpha p_n(\xi,y,t)}{\partial \xi^\alpha} + \frac{\partial^\alpha p_n(\xi,y,t)}{\partial y^\alpha} - \frac{1}{v}\frac{\partial p_n(\xi,y,t)}{\partial t} \right] d\xi$$

$n \geq 0$

(11)

The stationary conditions are

$$1 + \lambda|_{\xi = x} = 0$$
$$\lambda'|_{\xi = x} = 0 \quad (12)$$

This gives $\lambda = -1$

Substituting this value of the Lagrange multiplier $\lambda = -1$ in to the functional (12)

gives the iteration formula

$$p_{n+1}(x,y,t) = p_n(x,y,t) - \int_0^x \left[\frac{\partial^\alpha p_n(\xi,y,t)}{\partial \xi^\alpha} + \frac{\partial^\alpha p_n(\xi,y,t)}{\partial y^\alpha} - \frac{1}{v}\frac{\partial p_n(\xi,y,t)}{\partial t} \right] d\xi$$

$n \geq 0$

(13)

$$p_0(x,y,t) = 1 + e^y + e^x$$

$$p_1(x,y,t) = p_0(x,y,t) - \int_0^x \left[\frac{\partial^\alpha p_0(\xi,y,t)}{\partial \xi^\alpha} + \frac{\partial^\alpha p_0(\xi,y,t)}{\partial y^\alpha} - \frac{1}{v}\frac{\partial p_0(\xi,y,t)}{\partial t} \right] d\xi$$

$$= 1 + e^y + e^t - \int_0^x \left[\frac{\partial^\alpha (1+e^y+e^t)}{\partial \xi^\alpha} + \frac{\partial^\alpha (1+e^y+e^t)}{\partial y^\alpha} - \frac{1}{v}\frac{\partial (1+e^y+e^t)}{\partial t} \right] d\xi$$

$$= 1 + e^y + e^t - \int_0^x \left[0 + e^y - \frac{1}{v} e^t \right] d\xi$$

$$= 1 + e^y + e^t - xe^y + \frac{x}{v} e^t$$

$$= 1 + [1 - x]e^y + \left[1 + \frac{x}{v}\right] e^t$$

$$p_1(x,y,t) = 1 + \left[1 - \frac{x}{\Gamma}\right] e^y + \left[1 + \frac{x}{\Gamma v}\right] e^t$$

$$p_2(x,y,t) = p_1(x,y,t) - \int_0^x \left[\frac{\partial^\alpha p_1(\xi,y,t)}{\partial \xi^\alpha} + \frac{\partial^\alpha p_1(\xi,y,t)}{\partial y^\alpha} - \frac{1}{v}\frac{\partial p_1(\xi,y,t)}{\partial t} \right] d\xi$$

$$= 1 + [1 - x]e^y + \left[1 + \frac{x}{v}\right] e^t$$

$$- \int_0^x \left[\frac{\partial^\alpha \left[1 + [1-\xi]e^y + \left[1 + \frac{\xi}{v}\right] e^t\right]}{\partial \xi^\alpha} + \frac{\partial^\alpha \left[1 + [1-\xi]e^y + \left[1 + \frac{\xi}{v}\right] e^t\right]}{\partial y^\alpha} - \frac{1}{v} \frac{\partial \left[1 + [1-\xi]e^y + \left[1 + \frac{\xi}{v}\right] e^t\right]}{\partial t} \right] d\xi$$

$$= 1 + [1 - x]e^y + \left[1 + \frac{x}{v}\right] e^t$$

$$- \int_0^x \left[\frac{\xi^{1-\alpha}}{\Gamma(2-\alpha)} e^y + \frac{\xi^{1-\alpha}}{v\Gamma(2-\alpha)} e^t + [1-\xi]e^y - \frac{1}{v}\left[1 + \frac{\xi}{v}\right] e^t \right] d\xi$$

$$= 1 + [1 - x]e^y + \left[1 + \frac{x}{v}\right] e^t$$

$$- \left[\left[-\frac{\xi^{2-\alpha}}{(2-\alpha)\Gamma(2-\alpha)} e^y + \frac{\xi^{2-\alpha}}{v(2-\alpha)\Gamma(2-\alpha)} e^t + \xi e^y - \frac{\xi^2 e^y}{2} - \frac{\xi e^t}{v} - \frac{\xi^2 e^t}{v^2} \right]\right]$$

$$= 1 + [1 - x]e^y + \left[1 + \frac{x}{v}\right] e^t$$

$$- \left[\frac{x^{2-\alpha}}{\Gamma(3-\alpha)} e^y + \frac{x^{2-\alpha}}{v\Gamma(3-\alpha)} e^t + xe^y - \frac{x^2 e^y}{2} - \frac{x e^t}{v} - \frac{x^2 e^t}{v^2} \right]$$

$$p_2(x,y,t) = 1 + \left[1 - 2x + \frac{x^2}{2} + \frac{x^{2-\alpha}}{\Gamma(3-\alpha)}\right] e^y$$

$$+ \left[1 + \frac{2}{v} x + \frac{1}{2v^2} x^2 - \frac{x^{2-\alpha}}{v\Gamma(3-\alpha)}\right] e^t$$

$$p_2(x,y,t) = \left[1 - \frac{x}{1} + \frac{x^2}{2} + \left[-x + \frac{x^{2-\alpha}}{\Gamma(3-\alpha)}\right] e^y + \left[1 + \frac{x}{1v} + \frac{x^2}{2v^2} + \left[\frac{x}{v} - \frac{x^{2-\alpha}}{v\Gamma(3-\alpha)}\right] e^t\right]\right]$$

$$p_3(x,y,t) = p_2(x,y,t) - \int_0^x \left[\frac{\partial^\alpha p_2(\xi,y,t)}{\partial \xi^\alpha} + \frac{\partial^\alpha p_2(\xi,y,t)}{\partial y^\alpha} - \frac{1}{v}\frac{\partial p_2(\xi,y,t)}{\partial t} \right] d\xi$$

$$= 1 + \left[1 - 2x + \frac{x^2}{2} + \frac{x^{2-\alpha}}{\Gamma(3-\alpha)}\right] e^y + \left[1 + \frac{2}{v} x + \frac{x^2}{2v^2} - \frac{x^{2-\alpha}}{v\Gamma(3-\alpha)}\right] e^t$$

$$- \int_0^x \frac{\partial^\alpha \left[1 + \left[1 - 2\xi + \frac{\xi^2}{2} + \frac{\xi^{2-\alpha}}{\Gamma(3-\alpha)}\right] e^y + \left[1 + \frac{2}{v}\xi + \frac{\xi^2}{2v^2} - \frac{\xi^{2-\alpha}}{v\Gamma(3-\alpha)}\right] e^t\right]}{\partial \xi^\alpha}$$

$$+ \frac{\partial^\alpha \left[1 + \left[1 - 2\xi + \frac{\xi^2}{2} + \frac{\xi^{2-\alpha}}{\Gamma(3-\alpha)}\right] e^y + \left[1 + \frac{2}{v}\xi + \frac{\xi^2}{2v^2} - \frac{\xi^{2-\alpha}}{v\Gamma(3-\alpha)}\right] e^t\right]}{\partial y^\alpha}$$

$$- \frac{1}{v} \frac{\partial \left[1 + \left[1 - 2\xi + \frac{\xi^2}{2} + \frac{\xi^{2-\alpha}}{\Gamma(3-\alpha)}\right] e^y + \left[1 + \frac{2}{v}\xi + \frac{\xi^2}{2v^2} - \frac{\xi^{2-\alpha}}{v\Gamma(3-\alpha)}\right] e^t\right]}{\partial t} d\xi$$

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$p_2(x,y,t)=$

$$=1+\left[1-2x+\frac{x^2}{2}+\frac{x^{2-\alpha}}{\Gamma(3-\alpha)}\right]e^{y}+\left[1+\frac{2}{v}x+\frac{x^2}{2v^2}-\frac{x^{2-\alpha}}{v\Gamma(3-\alpha)}\right]e^{t}$$

$$-\int_0^t\left[\frac{-2\xi^{1-\alpha}}{\Gamma(2-\alpha)}+\frac{\Gamma(3)\xi^{2-\alpha}}{2\Gamma(3-\alpha)}+\frac{\Gamma(3-\alpha)\xi^{2-2\alpha}}{\Gamma(3-\alpha)\Gamma(3-2\alpha)}\right]e^{y}+\left[\frac{2\xi^{1-\alpha}}{v\Gamma(2-\alpha)}+\frac{\Gamma(3)\xi^{2-\alpha}}{2v^2\Gamma(3-\alpha)}-\frac{\Gamma(3)\xi^{2-2\alpha}}{v\Gamma(3-\alpha)\Gamma(3-2\alpha)}\right]e^{t}$$

$$+\left[1-\xi+\frac{\xi^2}{2}+\frac{\xi^{2-\alpha}}{\Gamma(3-\alpha)}\right]e^{y}-\frac{1}{v}\left[1+\frac{1}{v}\xi+\frac{\xi^2}{2v^2}-\frac{\xi^{2-\alpha}}{v\Gamma(3-\alpha)}\right]e^{t}d\xi$$

$p_3(x,y,t)=$

$$=1+\left[1-2x+\frac{x^2}{2}+\frac{x^{2-\alpha}}{\Gamma(3-\alpha)}\right]e^{y}+\left[1+\frac{2}{v}x+\frac{x^2}{2v^2}-\frac{x^{2-\alpha}}{v\Gamma(3-\alpha)}\right]e^{t}$$

$$-\left[\begin{array}{l}\left[-\frac{2\xi^{2-\alpha}}{(2-\alpha)\Gamma(2-\alpha)}+\frac{\xi^{3-\alpha}}{(3-\alpha)\Gamma(3-\alpha)}+\frac{\xi^{3-2\alpha}}{(3-2\alpha)\Gamma(3-2\alpha)}\right]e^{y}\\+\left[\frac{2\xi^{2-\alpha}}{v(2-\alpha)\Gamma(2-\alpha)}+\frac{\xi^{3-\alpha}}{v^2(3-\alpha)\Gamma(3-\alpha)}-\frac{2\xi^{3-2\alpha}}{v(3-2\alpha)\Gamma(3-2\alpha)}\right]e^{t}\\+\left[\xi-\frac{\xi^2}{2}+\frac{\xi^3}{3}+\frac{\xi^{3-\alpha}}{(3-\alpha)\Gamma(3-\alpha)}\right]e^{y}\\-\frac{1}{v}\left[\xi+\frac{\xi^2}{v}+\frac{\xi^3}{3v^2}-\frac{\xi^{3-\alpha}}{v(3-\alpha)\Gamma(3-\alpha)}\right]e^{t}\end{array}\right]$$

$p_3(x,y,t)=$

$$=1+\left[1-2x+\frac{x^2}{2}+\frac{x^{2-\alpha}}{\Gamma(3-\alpha)}\right]e^{y}+\left[1+\frac{2}{v}x+\frac{x^2}{2v^2}-\frac{x^{2-\alpha}}{v\Gamma(3-\alpha)}\right]e^{t}$$

$$\left[\begin{array}{l}\left[\frac{2x^{2-\alpha}}{\Gamma(3-\alpha)}-\frac{x^{3-\alpha}}{\Gamma(4-\alpha)}-\frac{x^{3-2\alpha}}{\Gamma(4-2\alpha)}\right]e^{y}\\+\left[-\frac{2x^{2-\alpha}}{v\Gamma(3-\alpha)}-\frac{x^{3-\alpha}}{v^2\Gamma(4-\alpha)}+\frac{2x^{3-2\alpha}}{v\Gamma(4-2\alpha)}\right]e^{t}\\+\left[-x+\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^{3-\alpha}}{\Gamma(4-\alpha)}\right]e^{y}\\+\left[\frac{x}{v}+\frac{x^2}{v^2}+\frac{x^3}{3v^3}-\frac{x^{3-\alpha}}{v^2\Gamma(4-\alpha)}\right]e^{t}\end{array}\right]$$

$p_3(x,y,t)=$

$$=1+\left[1-3x+\frac{3x^2}{2}-\frac{x^3}{3}+\frac{3x^{2-\alpha}}{\Gamma(3-\alpha)}-\frac{2x^{3-\alpha}}{\Gamma(4-\alpha)}-\frac{x^{3-2\alpha}}{\Gamma(4-2\alpha)}\right]e^{y}$$

$$+\left[1+\frac{3x}{2v}+\frac{x^2}{2v^2}+\frac{x^2}{v^2}+\frac{x^3}{3v^3}-\frac{3x^{2-\alpha}}{v\Gamma(3-\alpha)}-\frac{3x^{3-\alpha}}{v^2\Gamma(4-\alpha)}+\frac{2x^{3-2\alpha}}{v\Gamma(4-2\alpha)}-\frac{x^{3-\alpha}}{v^2\Gamma(4-\alpha)}\right]e^{t}$$

$$p_3(x,y,t)=1+\left[1-\frac{x}{1}+\frac{x^2}{2}-\frac{x^3}{3}+\left[-2x+\frac{3x^{2-\alpha}}{\Gamma(3-\alpha)}-\frac{x^{3-2\alpha}}{\Gamma(4-2\alpha)}\right]+\left[x^2-\frac{2x^{3-\alpha}}{\Gamma(4-\alpha)}\right]\right]e^{y}$$

$$+\left[1+\frac{x}{2v}+\frac{x^2}{2v^2}+\frac{x^3}{3v^3}+\left[\frac{x}{v}-\frac{x^{2-\alpha}}{v\Gamma(3-\alpha)}\right]+\left[\frac{-2x^{2-\alpha}}{v\Gamma(3-\alpha)}+\frac{2x^{3-2\alpha}}{v\Gamma(4-2\alpha)}\right]+\left[\frac{x^{3-\alpha}}{v^2\Gamma(4-\alpha)}-\frac{x^{3-\alpha}}{v^2\Gamma(4-\alpha)}\right]\right]$$

$$p_n(x,y,t)=1+\left[1-\frac{x}{1}+\frac{x^2}{2}-\frac{x^3}{3}+\ldots+\frac{x^n}{n!}+\left[-x+\frac{x^{2-\alpha}}{\Gamma(3-\alpha)}\right]+\left[-2x+\frac{3x^{2-\alpha}}{\Gamma(3-\alpha)}-\frac{x^{3-2\alpha}}{\Gamma(4-\alpha)}\right]+\left[x^2-\frac{2x^{3-\alpha}}{\Gamma(4-\alpha)}\right]\right]e^{y}$$

$$+\left[1+\frac{x}{2v}+\frac{x^2}{2v^2}+\frac{x^3}{3v^3}+\ldots+\frac{x^n}{n!v^n}+\left[\frac{x}{v}-\frac{x^{2-\alpha}}{v\Gamma(3-\alpha)}\right]+\left[\frac{-2x^{2-\alpha}}{v\Gamma(3-\alpha)}+\frac{2x^{3-2\alpha}}{v\Gamma(4-2\alpha)}\right]+\left[\frac{x^{3-\alpha}}{v^2\Gamma(4-\alpha)}-\frac{x^{3-\alpha}}{v^2\Gamma(4-\alpha)}\right]\right]e^{t}$$

$+\ldots$

The exact solution my obtained by using

$$p(x,y,t)=\lim_{n\to\infty}p_n(x,y,t)$$

Then the exact solution is

$$\lim_{n\to\infty}p_n(x,y,t)=1+\left[1-\frac{x}{1}+\frac{x^2}{2}-\frac{x^3}{3}+\ldots+\frac{x^n}{n!}+\left[-x+\frac{x^{2-\alpha}}{\Gamma(3-\alpha)}\right]+\left[-2x+\frac{3x^{2-\alpha}}{\Gamma(3-\alpha)}-\frac{x^{3-2\alpha}}{\Gamma(4-\alpha)}\right]+\left[x^2-\frac{2x^{3-\alpha}}{\Gamma(4-\alpha)}\right]\right]e^{y}$$

$$+\left[1+\frac{x}{2v}+\frac{x^2}{2v^2}+\frac{x^3}{3v^3}+\ldots+\frac{x^n}{n!v^n}+\left[\frac{x}{v}-\frac{x^{2-\alpha}}{v\Gamma(3-\alpha)}\right]+\left[\frac{-2x^{2-\alpha}}{v\Gamma(3-\alpha)}+\frac{2x^{3-2\alpha}}{v\Gamma(4-2\alpha)}\right]+\left[\frac{x^{3-\alpha}}{v^2\Gamma(4-\alpha)}-\frac{x^{3-\alpha}}{v^2\Gamma(4-\alpha)}\right]\right]e^{t}$$

$$p(x,y,t)=\lim_{n\to\infty}\left[1+\left[\left[-x+\frac{x^{2-\alpha}}{\Gamma(3-\alpha)}\right]+\left[-2x+\frac{3x^{2-\alpha}}{\Gamma(3-\alpha)}-\frac{x^{3-2\alpha}}{\Gamma(4-\alpha)}\right]+\left[x^2-\frac{2x^{3-\alpha}}{\Gamma(4-\alpha)}\right]+\sum_{n=0}^{n}\frac{(-1)^n x^n}{n!}\right]e^{y}\right.$$

$$\left.+\left[\left[\frac{x}{v}-\frac{x^{2-\alpha}}{v\Gamma(3-\alpha)}\right]+\left[\frac{-2x^{2-\alpha}}{v\Gamma(3-\alpha)}+\frac{2x^{3-2\alpha}}{v\Gamma(4-2\alpha)}\right]+\left[\frac{x^{3-\alpha}}{v^2\Gamma(4-\alpha)}-\frac{x^{3-\alpha}}{v^2\Gamma(4-\alpha)}\right]+\sum_{n=0}^{n}\frac{x^n}{n!v^n}\right]e^{t}\right]$$

$+\ldots$

$$p(x,y,t)=\left[\left[-x+\frac{x^{2-\alpha}}{\Gamma(3-\alpha)}\right]+\left[-2x+\frac{3x^{2-\alpha}}{\Gamma(3-\alpha)}-\frac{x^{3-2\alpha}}{\Gamma(4-\alpha)}\right]+\left[x^2-\frac{2x^{3-\alpha}}{\Gamma(4-\alpha)}\right]\right]e^{y}$$

$$+\left[\left[\frac{x}{v}-\frac{x^{2-\alpha}}{v\Gamma(3-\alpha)}\right]+\left[\frac{-2x^{2-\alpha}}{v\Gamma(3-\alpha)}+\frac{2x^{3-2\alpha}}{v\Gamma(4-2\alpha)}\right]+\left[\frac{x^{3-\alpha}}{v^2\Gamma(4-\alpha)}-\frac{x^{3-\alpha}}{v^2\Gamma(4-\alpha)}\right]\right]e^{t}$$

$$+\ldots+1+e^{-x}e^{y}+e^{t}e^{\frac{x}{v}}$$

$+\ldots$

(14)

When $\alpha=1$ then the exact solution is

$$p(x,y,t)=1+e^{y-x}+e^{t+\frac{x}{v}}$$

IV. CONCLUSION

The fundamental goal of this work has been to construct an approximate solution of seepage flow derivatives in porous media. The goal has been achieved by using the (VIM). The method was used in a direct way without using linearization, perturbation or restrictive assumptions. Comparing this method with others, such as Adomian Decomposition Method, we consider this method to be more effective.

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