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On Coc-Convergence of Nets and Filters

KEYWORDS: coc-open,coc-closed,coc-convergent, coc-limit, coc-cluster and cocE _f set.

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ABSTRACT *In this paper we introduce and study another types of convergence in a topological spaces namely ,cocompact convergence(coc-convergence)of nets and filters by using the concept of coc-open sets. Also we investigate some properties of these concepts*

Introduction: The notion of convergence is one of the basic notion in analysis. There are two different convergence theories used in general topology that lead to equivalent results . One of them based on the notion of a net in 1922 due to Moore and Smith [5]; another one, which goes back to workof Cartan [3] in 1937, is based on the notion of a filter. Al Ghour S.and Samarah. S in [1] introduce the definition of coc-open set . *Al-Hussaini* F.H.[2] introduce coc-continuity as a generalization of continuity.The family of all coc-open sets of a space X is denoted by τ^k[1]. It is the purpose of this paper to offer some more characterization of coc-compact spaces in [2] by the concept of coc-convergence coc-cluster) of nets and filters respectively .Also, we give some properties of the coc-proper functions by using the concept of cocompact exceptional ($cocE_f$) set. For a subset$A$ of$X$, the closure and the interior of Ain $X$are denoted by$coc-cl(A)$and $coc-Int(A)$respectively [1],[2].Now, Throughout this paper(X,T) and(Y,σ) (or simply$X$and$Y$) represent non-emptytopological spaceon which no separation axiom areassumed unless otherwise mentioned .

1. Basic definitions and notations:

We introduce some elementary concept which we need in our work.

1.1. Definition : [1]

A subset A of a space (X,τ) is calledcocompact open set (notation:coc-open set) if for every $x \in A$ there exists an open set $U \subseteq \mathrm{X}$ and a compact subset $\mathrm{K} \in \mathrm{C}(X,\tau)$ such that $x \in U - \mathrm{K} \subseteq \mathrm{A}$. the complement of coc-open set is called coc-closed set.

The family of all coc-open subset of a space (X,τ) is denoted by τ^k

1.2. Example:

Let$X = \{1,2,3\}$ with $T = T_{\text{dis}}$, then$A = \{1\}$ is coc-open set.

1.3. Example:

Let $X = R$ with$T = T_U$, then $A = [a,b]$ is not coc-open.

<u>1.4.Remark : [1][2]</u>

i. Every open set is an coc-open set .
ii. Every closed set is an coc-closed set.
The converse of (i, ii) is not true in general as the following example shows:
Let $X = \mathbb{N}$,$T = T_{fin}$ .The set A= $\{1,5,6,7, \ldots\}$ is coc-open set , but it's not an open set and B= {5,6} is an coc- closed set, but it's not an closed set.

<u>1.5 . Theorem :[1]</u>

Let (X,T)be a topological space Then
i.the collection T^kforms a topology on$X$.
ii.the collection $\beta^k(\tau)$ forms a base for$T^k$where $\beta^k(\tau) = \{U - K : U \epsilon \tau and K \in C(X,\tau)\}$.
iii. $T \subseteq T^k$.

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The converse of (iii) is not true as the following example shows: Let $= \mathbb{N}$,$T = T_{ind}$ then T^k $=T_{dis}$ and then $T^k \nsubseteq T$ the space (X, T) is an example to compact space for which $(X, T^k)$is not compact .

1.6.Definition:[1]

A space X is called CC if every compact set in X is closed.

1.7. Theorem :[1]

Let (X, τ) be a space. Then the following statements are equivalent:

i. (X, τ) is CC.

ii. $\tau = \tau^k$.

1.8.Corollary :[1]

Let (X, τ) be a T_2-space, then $\tau = \tau^k$.

1.9.Definition:[1]

Let$X$ be a space and $A \subseteq X$. The intersection of all coc-closed sets of X containing A, is called coc- closure of A and is denoted by $\overline{A}^{coc}$ or coc- $Cl_\tau(A)$.

coc-$Cl_\tau(A)$ $= \cap \{B: B is \text{coc} - \text{closed in X and A} \subseteq \text{B}\}$

1.10. Definition: [2]

Let$X$ be a space and $A \subseteq X$. The union of all coc-open sets of X contained in A is called coc- Interior of A and denoted by $\text{A}^{\circ\text{coc}}$ or coc-$In_\tau(A)$.

coc-$In_\tau(A)$ $= \cup \{B: B is \text{coc} - \text{open in X and B} \subseteq \text{A}\}$.

<u>1.11.Remark:</u>

It is clear that $\text{A}^\circ \subseteq \text{coc} - In_\tau(A)$. and coc-$Cl_\tau(A) \subseteq \overline{A}$, but the converse is not true in general as the following example shows:

Let $X = \{1,2,3\}$, and $T = T_{ind}$ and $A = \{2\}$. Then $\text{A}^\circ = \emptyset$, $\text{A}^{\circ\text{coc}} = \{2\}$, $\overline{A}^{coc} = \{2\} and \overline{A} = X$.

<u>1.12.Proposition: [2]</u>

Let X be a space and $A, B \subseteq X$. Then:

i. if $A \subseteq B$ then. $\overline{A}^{coc} \subseteq \overline{B}^{\text{coc}}$.

ii.$x \in \overline{A}^{coc}$ iff for each coc-open set$U$ in X containa point$x$ we have $U \cap A \neq \phi$.

iii.A is an coc-closed set if and only if $A = \bar{A}^{coc}$.

iv.A is an coc-open set if and only if $A = A^{\circ coc}$.

1.13. Definition: [2]

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Let X be a space and $B \subseteq X$. An coc-neighborhood of B is any subset of X which contains an coc-open set containing B.The coc-neighborhood of a subset $\{x\}$ is also called coc-neighborhood of the point x.

1.14. Corollary: [2]

Let X be a space and Y be any nonempty closed in X.If B is an coc-closed set in X then $B \cap Y$ is an coc-closed set in Y.

1.15. Definition: [2]

A topological space $X$is called coc- Hausdorff ($coc - T_2$) if for any two distinct points $x, y \in X$there are disjoint coc-open sets U ,$V$of Xsuch that $x \in U$ and $y \in V$.

1.16. Definition: [2]

Let $f: X \to Y$ be a function of a space$X$ into a space$Y$ then$f$ is called:

i. coc-continuous function if $f^{-1}(A)$ is an coc-open set in$X$ for every open set A in$Y$.

ii. coc-irresolute function if $f^{-1}(A)$ is an coc-open set in$X$ for every coc-open set A in$Y$.

iii.strongly coc-closed function if $f(A)$ is an coc-closed set in$Y$ for every coc-closed set A in$X$.

Now, we review some basic definitions, theorems and remarks about a net

1.17. Definition [6]:

A set $D$is called a directed if there is a relation $\leq$on$D$satisfying:

(i)$d \leq d$for each$d \in D$.
(ii) If $d_1 \leq d_2$and $d_2 \leq d_3$then$d_1 \leq d_3$.
(iii) If $d_1, d_2 \in D$, there is some$d_3 \in D$ with $d_1 \leq d_3$and$d_2 \leq d_3$.

1.18. Definition [6]:
A net in a set $X$is a function $\chi: D \to X$, where$D$ is directed set.Thepoint$\chi(d)$is usually denoted by χ_d.

1.19. Definition [6]:
A subnet of a net$\chi: D \to X$is thecomposition$\chi o \varphi$,where$\varphi: M \to D$and M is directed set, such that :
(i)$\varphi(m_1) \leq \varphi(m_2)$, where $m_1 \leq m_2$.
(ii) For each $d \in D$there is some $m \in M$such that$d \leq \varphi(m)$. For$m \in M$the point $\chi o \varphi(m)$ is often written χ_{dm} .

1.20. Definition [7] [5]:
Let$(\chi_d)_{d \in D}$be a net in a topological space $X$and $A \subseteq X$,$x \in X$ then:

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(i)$(\chi_d)_{d\in D}$is called eventually in Aif there is$d_0 \in D$such that$\chi_d \in A$for all $d \geq d_0$.
(ii)$(\chi_d)_{d\in D}$is called frequently in$A$for each$d \in D$there is $d_0 \in D$with $d_0 \geq d$ such that $\chi_{d_0} \in A$.
(iii)$(\chi_d)_{d\in D}$is said to be convergence to$x$if $(\chi_d)_{d\in D}$eventually ineach neighborhoodof$x$(written$\chi_d \to x$) .The point$x$is called a limit pointof$(\chi_d)_{d\in D}$.
(iv)$(\chi_d)_{d\in D}$is called have$x$ as a cluster point if$(\chi_d)_{d\in D}$is frequently in eachneighborhood of$x$(written$\chi_d \propto x$) .

2. COC- Convergence of Nets:

In this section , we introduce and study other types of convergence in a topological spaces namely cocompact(coc- convergence) of net and study some properties of the concept of coc- limit point and coc- cluster point of the net in a given space .Also , we give some theorems, remarks and examples about this subject.

2.1. Definition:

Let $(\chi_d)_{d\in D}$be a net in a topological space X , and $x \in X$. Then$(\chi_d)_{d\in D}$is a coc-convergence to a point$x$ if $(\chi_d)_{d\in D}$is eventuallyin everycoc-neighborhood of$x$,(written $\chi_d \xrightarrow{coc} x$) .The point $x$is called coc- limit point of $(\chi_d)_{d\in D}$.
A net $(\chi_d)_{d\in D}$in$X$ is said to be have no coc-convergent subnet in X, (written$\chi_d \xrightarrow{coc} \infty$) , if and only if everysubnet of $(\chi_d)_{d\in D}$has no coc-limit point .

2.2. Definition:

Let $(\chi_d)_{d\in D}$be a net in a topological space $X$and $x \in X$is said to have $x \in X$as coc- cluster point if $(\chi_d)_{d\in D}$is frequently in every coc-neighborhood of x, (written $\chi_d \overset{coc}{\propto} x$) .

2.3. Remark:

Let(X, T) be aspace and let $A \subseteq X$ then :
i.If (χ_d) is a net in X , $x \in X$ .Then $(\chi_d \xrightarrow{coc} x)$in (X, T) if and only if $\chi_d \longrightarrow x$ in (X, T^k)
ii. If (χ_d) is a net in X, $x \in X$ .Then $(\chi_d \overset{coc}{\propto} x)$in (X, T) if and only if $\chi_d \overset{coc}{\propto} x$ in (X, T^k)
iii. if (χ_d) is a net in X, $x \in X$ .Then $(\chi_d \overset{coc}{\propto} x)$in (X, T) if and only ifthere exist asubnet$(\chi_{d_m})_{\mathrm{dm} \in \mathrm{D}}$ of $(\chi_d)_{d\in D}$ such that $\chi_{d_m} \xrightarrow{coc} x$.
iv. if (χ_d) is a net in X, $x \in X$ such that$(\chi_d \xrightarrow{coc} x)$ then $\chi_d \longrightarrow x$.

Note that if(χ_d) is a net in X , $x \in X$such that $\chi_d \longrightarrow x$ then(χ_d) is not necessary be ($\chi_d \xrightarrow{coc} x$) as the following example shows:

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2.4. Example:

Let $X = \{-1,1\}$with $T = T_{ind}$ and let $\{(-1)^n\}$ be a net in X ,then $\{(-1)^n\}$ is eventually in every neighborhood of 1 . But $\{(-1)^n\}$is not eventually in every coc-neighborhood of 1 .i.e. since $\{1\}$iscoc-neighborhood of 1 , but $\{(-1)^n\}$ is not eventually in$\{1\}$.

2.5 Theorem:

Let X be a topological space and $A \subseteq X$, $x \in X$. Then $x \in coc-cl(A)$if and only if there is a net $(\chi_d)_{d\in D}$ in A such that $\chi_d \xrightarrow{coc} x$.

Proof

Suppose that there isanet $(\chi_d)_{d\in D}$ in Asuch that$\chi_d \xrightarrow{coc} x$. To prove that $x \in coc-cl(A)$. Let$U \in \mathcal{N}_{coc}(x)$, since $\chi_d \xrightarrow{coc} x$, then there is $d_0 \in D$ such that $\chi_d \in U$ for all$d \geq d_0$. But $\chi_d \in U$ for all $d \in D$. Thus $A \cap U \neq \emptyset$ for all $U \in \mathcal{N}_{coc}(x)$. Hence by theorem (1.12.i) , $x \in coc-cl(A)$.**Conversely:**Suppose that $x \in coc-cl(A)$ .To prove that there is a net $(\chi_d)_{d\in D}$ in$A$such that$\chi_d \xrightarrow{coc} x$. Since $x \in coc-cl(A)$, then by theorem (1.12.i) we get $A \cap U \neq \emptyset$ for all$U \in \mathcal{N}_{coc}(x)$ .Then $D = \mathcal{N}_{coc}(x)$is directed set by inclusion . Since $A \cap U \neq \emptyset$ for all $U \in \mathcal{N}_{coc}(x)$,there is $\chi_U \in A \cap U$. Define $\chi: \mathcal{N}_{coc}(x) \to A$ by $\chi(U) = \chi_U$, for all $U \in \mathcal{N}_{coc}(x)$. $(\chi_U)_{U\in\mathcal{N}_{coc}(x)}$ is a net in A . To prove $\chi_U \xrightarrow{coc} x$. Let $U \in \mathcal{N}_{coc}(x)$ to find $d_0 \in D$ such that $\chi_d \in U$ for all $d \geq d_0$. Let $d_0 = U$ then for all $\geq d_0$ $d = V \in \mathcal{N}_{coc}(x)$. i.e. , $V \geq U \Leftrightarrow V \subseteq U$ .Then $\chi_d = \chi(d) = \chi(V) = \chi_V \in V \cap A \subseteq V \subseteq U \Rightarrow \chi_V \in U$ for all $d \geq d_0$. Thus $\chi_U \xrightarrow{coc} x$.

2.6. Corollary:

Let $(\chi_d)_{d\in D}$be a net in a topological space X and $x \in X$, then $\chi_d \overset{coc}{\propto} x$if and only if there is a subnet of$(\chi_d)_{d\in D}$ coc-convergence to x .

2.7. Corollary:

Let$X$ be a topological space and$A \subseteq X$, $x \in X$. Then$x \in \bar{A}^{coc}$ if and only if there is a net $(\chi_d)_{d\in D}$ in A such that $\chi_d \overset{coc}{\propto} x$.

2.8.Theorem:

A topological space X is $coc-T_2$-space if and only if every coc-convergent net in X has a unique coc-limit point .

Proof:

Let X be $coc-T_2$-space and $(\chi_d)_{d\in D}$ is a net in X suchthat $\chi_d \xrightarrow{coc} x$, $\chi_d \xrightarrow{coc} y$ and $x \neq y$. Since Xbe a $cocT_2$-space . There are $U \in N_{coc}(x)$ and $V \in N_{coc}(y)$ such that $U \cap V = \phi$. Since $\chi_d \xrightarrow{coc} x$, there is $d_0 \in D$ such that $\chi_d \in U$ for all $d \geq d_0$. Since $\chi_d \xrightarrow{coc} y$, there is $d_1 \in D$ such that $\chi_{d_1} \in V$for all $d \geq d_1$. Since

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D is directed set and d_0 , $d_1 \in D$, then there is $d_2 \in D$ such that $d_2 \geq d_0$ and$d_2 \geq d_1$. Then $\chi_d \in U$ for all $d \geq d_2$ and $\chi_d \in V$ for all $d \geq d_2$, thus $\cap V \neq \phi$, this is a contradiction .So $x = y$.**Conversely:** Suppose that X is not $coc - T_2$-space , there are $x, y \in X$ and $x \neq y$, for all $U \in N_{coc}(x), V \in N_{coc}(y)$ such that $U \cap V \neq \phi$. Put$N_x^y = \{U \cap V : U \in N_{coc}(x)$and $V \in N_{coc}(y)\}$, where N_x^y is directed set . Thus for all $D \in N_x^y$,there is $\chi_D \in D$ then $(\chi_D)_{D\in N_x^y}$ is a net in X .To prove$\chi_D \xrightarrow{coc} x$ and $\chi_D \xrightarrow{coc} y$, let $G \in N_{coc}(x)$ then$G \in N_x^y$, $G \cap X \neq \emptyset$ .Thus $\chi_D \in G$ for all $D \geq G$, so $\chi_D \xrightarrow{coc} x$. Also , let $H \in N_{coc}(y)$then $H \in N_x^y$, $H \cap X \neq \emptyset$. Thus $\chi_D \in H$ for all $D \geq G$, so $\chi_D \xrightarrow{coc} y$. This is a contradiction.

2.9. Theorem:

Let X be an topological space and $A \subseteq X$, then :

i. A point $x \in X$ is coc-limit point of A if and only if there is a net in A-$\{x\}$coc-convergence to x .

ii.A set A is coc-closed in X if and only if no net in A coc-convergence to a point in A^c.

iii. A set A is coc-open in X if and only if no net in A^c coc-convergence to a point in A .

Proof:

i. Let $x$is coc-limit point of$A$. To prove that there is a net $(\chi_d)_{d\in D}$in A-$\{x\}$ such that $\chi_d \xrightarrow{coc} x$. Since $x$is coc-limit point of A, for all $U \in \mathcal{N}_{coc}(x)$, $U \cap A$-$\{x\} \neq \phi$. Then $(\mathcal{N}_{coc}(x), \subseteq)$is directed set by inclusion . Since$U \cap A$-$\{x\} \neq \phi$, for all $U \in \mathcal{N}_{coc}(x)$then there is $\chi_U \in U \cap A$-$\{x\}$. Define $\chi : \mathcal{N}_{coc}(x) \to A$-$\{x\}by(U) = \chi_U$ for all $U \in \mathcal{N}_{coc}(x)$, then $(\chi_U)_{U\in\mathcal{N}_{coc}(x)}$ is a net in A-$\{x\}$. Toprove that$\chi_U \xrightarrow{coc} x$, let$U \in \mathcal{N}_{coc}(x)$ to find $d_0 \in D$ such that $\chi_d \in U$ for all $d \geq d_0$. Let $d_0 = U$then for all $d \geq d_0$,$d = V \in \mathcal{N}_{coc}(x)$, i.e. , $V \geq U \Leftrightarrow V \subseteq U$. Then$\chi_d = \chi(d) = \chi(V) = \chi_V \in V \cap A$-$\{x\} \subseteq V \subseteq U \Rightarrow \chi_V \in U$for all$d \geq d_0$. Thus$\chi_U \xrightarrow{coc} x$.**Conversely:** Suppose that there is a net $(\chi_d)_{d\in D}$in A-$\{x\}$such that$\chi_d \xrightarrow{coc} x$. To prove$x$ is coc-limit point of$A$ .Let $U \in \mathcal{N}_{coc}(x)$, since$\chi_d \xrightarrow{coc} x$, then there is $d_0 \in D$ such that $\chi_d \in U$ for all $d \geq d_0$. But $\chi_d \in A$-$\{x\}$ for all $\in D$, then $U \cap A$-$\{x\} \neq \emptyset$ for all$U \in \mathcal{N}_{coc}(x)$. Thus $x$is coc-limit point of$A$.

ii. Suppose that A is coc-closed set in X and there is a net $(\chi_d)_{d\in D}$ in A such that$\chi_d \xrightarrow{coc} x$ and $x \in A^c$. Then $x \in \bar{A}^{coc}$, since A is coc-closed set , then$A = \bar{A}^{coc}$, hence $x \in A$, then $A \cap A^c \neq \emptyset$, this is a contradiction . Thus no net in A coc-convergence to a point inA^c.**Conversely**: Suppose that no net in A coc-convergent to a point in A^c. Let $x \in \bar{A}^{coc}$by theorem (2.5) there is a net in$A$ such that $\chi_d \xrightarrow{coc} x$. By hypotheses , we get every net in A coc-convergence to a point in A . Thus$x \in A$, so $A = \bar{A}^{coc}$ implies that A is coc-closed.

ii. By using (**i**) .

2.10. Remark:

Let $(\chi_d)_{d\in D}$be a net in a topological space X , then:

(i) If $\chi_d \xrightarrow{coc} x$, then every subnet of $(\chi_d)_{d\in D}$is coc-convergence to x .

(ii) If every subnet of $(\chi_d)_{d\in D}$has a subnet coc-convergence to x then $\chi_d \overset{coc}{\longrightarrow} x$.

(iii) If $\chi_d = x$ for all $d \in D$, then$\chi_d \overset{coc}{\longrightarrow} x$.

2.11. Remark:

i. Let $f: X \to Y$be a function from a topological space X into a topological space Y. if $(\chi_d)_{d\in D}$ is a net in X, then $\{f(\chi_d)\}_{d\in D}$is a net in Y .

ii. Let $f: X \to Y$be a function from a topological space Xonto a topological space $Y$and $(y_d)_{d\in D}$be a net in Y. Then there is a net $(\chi_d)_{d\in D}$in X such that $f(\chi_d) = y_d$for all$d \in D$.

2.12. Theorem:

Let X and Y be topological spaces . A function $f: X \to Y$is coc irresolute continuous if and only if whenever $(\chi_d)_{d\in D}$is a net in X such that$\chi_d \overset{coc}{\longrightarrow} x$, then$f(\chi_d) \overset{coc}{\longrightarrow} f(x)$.

Proof:

Suppose that $f: X \to Y$is iscoc irresolute -continuous and $(\chi_d)_{d\in D}$ is a net in X such that $\chi_d \overset{coc}{\longrightarrow} x$.To prove $f(\chi_d) \overset{coc}{\longrightarrow} f(x)$. Let $V \in \mathcal{N}_{coc}(f(x))inY$. Then $f^{-1}(\mathrm{V}) \in \mathcal{N}_{coc}(x)$, for some$d_0 \in D$,$d \geq d_0$ implies that $\chi_d \in f^{-1}(V)$. Thus , showing that $f(\chi_d) \overset{coc}{\longrightarrow} f(x)$, since $(\chi_d)_{d\in D}$is eventually in each coc-neighborhood of x , then $(f(\chi_d))_{d\in D}$is a net in Y which is eventually in each coc-neighborhood of $f(x)$. Therefore $f(\chi_d) \overset{coc}{\longrightarrow} f(x)$.**Conversely:**To prove $f: X \to Y$is iscoc irresolute -continuous , suppose not . Then there is $V \in \mathcal{N}_{coc}(f(x))$ such that $f(U) \not\subset V$ for any$U \in \mathcal{N}_{coc}(x)$. Thus for all $U \in \mathcal{N}_{coc}(x)$ we can $\chi_U \in U$ such that $f(\chi_U) \notin V$. But $(\chi_U)_{U\in\mathcal{N}_{coc}(x)}$ is a net in$X$ with $\chi_U \overset{coc}{\longrightarrow} x$ while $(f(\chi_U))_{U\in\mathcal{N}_{coc}(x)}$does not coc-convergent to$f(x)$. This is a contradiction, then $f$is coc irresolute -continuous function.The following theorem shows that the condition on a topological space X (or Y) to be$f$ is coc-continuous (strongly coc-continuous) function respectively.

2.13. Theorem:

Let $(\chi_d)_{d\in D}$be a net in a topological space X and for each $d_0 \in D$, $A_{d_0} = \{\chi_d : d \geq d_0\}$, $x \in X$ is coc-cluster point of $(\chi_d)_{d\in D}$if and only if $x \in \overline{A_{d_0}}^{coc}$ for each $d_0 \in D$.

Proof:

If $x$is coc-cluster point of $(\chi_d)_{d\in D}$, then for each $d_0 \in D$, A_{d_0}intersects each coc-neighborhood of $x$because $(\chi_d)_{d\in D}$is frequently in each coc-neighborhood of$x$. There fore $x \in \overline{A_{d_0}}^{coc}$.**Conversely:**If $x$is not coc-cluster point of $(\chi_d)_{d\in D}$, then there is$U \in N_{coc}(x)$such that $(\chi_d)_{d\in D}$ is not frequently in$U$. Hence for some $d_0 \in D$ if $d \geq d_0$, then$\chi_d \notin U$, so that$A_{d_0} \cap U = \emptyset$, consequently $x \notin \overline{A_{d_0}}^{coc}$.

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2.14. Definition [2]:

A space X is said to be coc-compact if every coc-open cover of X has finite sub cover.

2.15.Remark:[2]

The space(X,τ) is coc-compact if and only if the space$(X,\tau^k)$is compact.

2.16. Theorem:

i. Every coc-compact space is compact , the converse is not true in general .
ii. Every coc-closed subset of a coc-compact space is coc-compact.
iii.coc irresolute -continuous image of coc-compact space is coc-compact.

Proof :

(i.ii.)see in [2]

iv. let f be coc-irresoult continuous function from a space X in to a space Y and suppose B is coc-compact set in X . To show that B is also coc-compact , let $\{U_\alpha\}_{\alpha\in\Lambda}$be coc-open cover of f(B), that is f(B)=$\bigcup_{\alpha\in\Lambda} u_\alpha$, so $B\subset f^{-1}f(B) = f^{-1}(\bigcup_{\alpha\in\Lambda} U_\alpha) = \bigcup_{\alpha\in\Lambda} f^{-1}(U_\alpha)$, then $\{f^{-1}(U_\alpha)\}$ is a coc-open cover of B , which is coc-compact , then $B\subset \bigcup_{i=1}^{n} f^{-1}(U_{\propto i})$.
But $f(B) f \bigcup_{i=1}^{n} f^{-1}(U_{\propto i}) = \bigcup_{i=1}^{n} f f^{-1}(U_{\propto i}) \subset \bigcup_{i=1}^{n}(U_{\propto i})$. There fore f(B)

is coc-compact set.

2.17.Proposition :[2]

For any space X the following statement are equivalent:

i. X is coc-compact

ii. Every family of coc-closed sets$\{F_\alpha : \alpha\in\Lambda\}$ of X such that$\bigcap_{\alpha\in\Lambda} F_\alpha = \phi$, then there exist a finite subset $\Lambda_o\subseteq\Lambda$such that $\bigcap_{\alpha\in\Lambda_o} F_\alpha = \phi$.

2.18. Theorem:[7]

A spaces X is compact if and only if every net in X has a cluster point in X.

2.19. Theorem :

Let Xbe a topological space , then $X$is coc-compact ifand only if every net in Xhas a coc-cluster point in X.

Proof :

Let (X,T) be an coc-compact space and $(\chi_d)_{d\in D}$ be a net in X ,then ,(X,T^k) is a compact space . Then by Theorem (2.18), the net $(\chi_d)_{d\in D}$ has cluster point x in (X,T^k) then x is coc-cluster point of the net $(\chi_d)_{d\in D}$.(i.e $\chi_d \propto^{coc} x$) .Hence every net in X has coc-cluster point in X . Conversely : Let every net in X has coc-cluster point in (X,T) ,then , every net in X has cluster point in (X,T^k) . Then by Theorem (2.18) , (X,T^k) is a compact space , then , (X,T) is coc-compact space.

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2.20. Corollary:

Let Xbe topological space. Then $X$is coc-compact if and only if every net in Xhas a sub net which coc-convergence to a point in X.

2.21. Theorem:

A net $(\chi_d)_{d\in D}$in a product topological cc-space $\prod X_\lambda$, $\lambda \in \Lambda$ is coc-convergence to $x \in \prod X_\lambda$, if and only if $Pr_\lambda\ (\chi_d) \overset{coc}{\longrightarrow} Pr_\lambda(x)$for all $\lambda \in \Lambda$(wherePr_λ is theλ -th projection function) .

Proof:

If $\chi_d \overset{coc}{\longrightarrow} x$ in $\prod \chi_\lambda$, since $Pr_\lambda$are coc irresolute -continuous function , then by the theorem (2.12) we have $Pr_\lambda\ (\chi_d) \overset{coc}{\longrightarrow} Pr_\lambda(x)$.**Conversely:** Suppose that Pr_λ $(\chi_d) \overset{coc}{\longrightarrow} Pr_\lambda(x)$for all $\lambda \in \Lambda$. Let $Pr_{\lambda_1}^{-1}(\mathrm{U}_{\lambda_1}) \cap Pr_{\lambda_2}^{-1}(U_{\lambda_2}) \cap \dots .\cap Pr_{\lambda_n}^{-1}(U_{\lambda_n})$ be a basis coc-neighborhood of$x$in $\prod X_\lambda$. Then for all $i = 1\,, 2, \dots .,\ n,$ there is d_i such that whenever $d \geq d_i$, $Pr_{\lambda_i} \in \mathrm{U}_{\lambda_i}$. Then d_0 greater than for all d_i , $i = 1,2, \dots . , n$, we have $Pr_{\lambda_i} \in U_{\lambda_i}$ for all $d \geq d_0$. It follows that for all $d \geq d_0$, $\chi_d \in \cap Pr_{\lambda_i}^{-1}(U_{\lambda_i})$, $i = 1\,, 2, . . , n$. So $\chi_d \overset{coc}{\longrightarrow} x$.

2.22. Corollary:

If $(\chi_d)_{d\in D}$ is a net in $\prod X_\lambda$ having $x \in \prod X_\lambda$ as coc-cluster point , then for each $\lambda \in \Lambda$, $(Pr_\lambda(\chi_d))_{d\in D}$ has $Pr_\lambda(x)$ for coc-cluster point .

Now, we give the definition of coc-proper functions and some results which are related to this concept .

2.23. Definition: [2]

Let f be a function from a topological space X into a topological space Y. Then f is called coc-closed function, if $f(B)$is coc-closed set in Y for every closed set B in X .

2.24. Definition:

Let f be a function from a topological space X into a topological space Y . Then f is called coc- proper function if :

i. f is coc-continuous function .

ii. The function $f \times I_Z : X \times Z \to Y \times Z$is coc-closed for every space Z.

Recall that a subset $E_f$of $f(X)$ is called exceptional set of f which defined by:$E_f = \{\ y \in f(X) :\ \text{there is a net}\ (\chi_d)_{\mathrm{d}\in\mathrm{D}}\ \text{in}\ X\ \text{with}\ \chi_d \to \infty\ \text{and} f(\chi_d) \to y\,\}$, where f is a function from a topological space X into a topological space Y [3].We shall introduce a new characterization, which is very useful for coc-proper function by using a special set namely, coc- exceptional (for brief $cocE_f$) set of f .

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2.25. Definition:

Let f be a function from a topological space X into a topological space Y , the coc- exceptional set of f (for brief $cocE_f$) set is a subset of $f(X)$ which defined by :

$$cocE_f = \left\{ \begin{matrix} y \in f(X) : \text{there is a net} (\chi_d)_{\mathrm{d}\in\mathrm{D}} \text{in } X \text{ with } \chi_d \overset{coc}{\longrightarrow} \infty \\ \text{and } f(\chi_d) \overset{coc}{\longrightarrow} y \end{matrix} \right\}.$$

Now, we shall use $cocE_f$ to characterize coc-proper functions.

2.26. Theorem:

Let $f: X \to Y$ be a coc-continuous function ,where X is a coc-compact , X and Y be a Hausdorff spaces . Then the following statements are equivalent :

i.f is coc-proper function .

ii. If $(\chi_d)_{d\in D}$ is a net in X and $y \in Y$ is coc-cluster point of $f\{(\chi_d)\}$, then there is coc-cluster point $x \in X$ of $(\chi_d)_{d\in D}$ such that $f(x) = y$.

Proof:

$(\mathbf{i} \to \mathbf{ii})$ Since f be an coc-proper function . Then f is an coc- closed function anc $f^{-1}\{y\}$ is an coc-compact , $\forall y \in Y$.Let $(\chi_d)_{d\in D}$ be a net in X and $y \in$ be an coc-cluster point of a net $f(\chi_d)_{d\in D}$ in Y .Claim $f^{-1}\{y\} \neq \emptyset$, i $f^{-1}\{y\} = \emptyset$, then $y \notin f(X) \Rightarrow y \in (f(X))^c$ since X is a closed set ir and f is an coc-proper (coc-closed) , then $f(X)$ is an coc-closed set ir Thus $(f(X))^c$ is an coc-open set in Y.Therefore $f(\chi_d)_{d\in D}$ is frequen in $(f(X))^c$. But $f(\chi_d) \in f(X)$, for each d$\in D$. Then $f(X) \cap (f(X))^c$ $\emptyset$, and this is a contradiction.Thus $f^{-1}\{y\} \neq \emptyset$,is not frequently.

Now,suppose that the statements (ii) is not true , that means for $x \in f^{-1}\{y\}$ there exists coc-open set U_x in X contains x such that $(\chi_d)_{d\in D}$ is not frequently in U_x . Notice that$f^{-1}\{y\} = \bigcup_{x\in f^{-1}\{y\}}\{x\}$. Therefore the family $\{U_x : x \in f^{-1}\{y\}\}$ is coc-open cover of $f^{-1}\{y\}$, but $f^{-1}\{y\}$ is coc-compact set . There are $x_1, x_2, \ldots\ldots, x_n$ such that$f^{-1}\{y\} \subseteq \bigcup_{i=1}^{n} U_{x_i}$, then$f^{-1}\{y\} \cap (\bigcup_{i=1}^{n} U_{x_i})^c = \emptyset$. Then $f^{-1}\{y\} \cap (\bigcap_{i=1}^{n} U_{x_i}^c) = \emptyset$. But $(\chi_d)_{d\in D}$ is not frequently in $U_{x_i}$for each $i = 1 , 2 , \ldots , n$. Thus is not frequently in $\bigcup_{i=1}^{n} U_{x_i}$, but $\bigcup_{i=1}^{n} U_{x_i}$is coc-open set in X , so $(\bigcap_{i=1}^{n} U_{x_i}^c)$ iscoc-closed (closed) set in X . Thus by assumption $f(\bigcap_{i=1}^{n} U_{x_i}^c)$ is coc-closed set in Y .

Claim $y \notin f(\bigcap_{i=1}^{n} U_{x_i}^c)$, if $y \in f(\bigcap_{i=1}^{n} U_{x_i}^c)$ then there is $x \in \bigcap_{i=1}^{n} U_{x_i}^c$ such that $(x) = y$, thus $x \notin \bigcup_{i=1}^{n} U_{x_i}$ but $x \in f^{-1}\{y\}$, hereforef$f^{-1}\{y\}$ is not subset of $\bigcup_{i=1}^{n} U_{x_i}$, this is a contradiction .Then there is coc-open

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