Target Setting - Free Disposable Hull (Fdh)

KEYWORDS: Data Envelopment Analysis, Free Disposable Hull, Directional Distance Function, Shortest Targets

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ABSTRACT *Data Envelopment Analysis provides both short-run and long-run targets to the inefficient firms. It also identifies the efficient peer of the inefficient firm which is a convex combination of more than one efficient firms. If bench marked targets suggest extensively restructure organization, the inefficient of firm hesitates persuasion, inspite of the fact that the available bench marking information is useful. The Free Disposable Hull provides shorter targets then the convex production possibility set. This study provides targets to inefficient firms projecting the inefficient production plan onto FDH frontier choosing directional distance function input approach. The closed form solutions derived for FDH technology are implemented in a study of 16 total manufacturing sectors of Indian states.*

INTRODUCTION:

Let T be the technology set that envelopes all feasible input and output combinations, constructed exploiting information provided by input and output vectors of decision making units, imposing a priori specified structure. T may be a convex technology or non-convex technology. A popularly used non-convex technology for efficiency measurement is Free Disposable Hull (FDH) technology, whose technology set appears to be a step function in two dimensional space, where scalar input and output are measured along horizontal and vertical axes, respectively. Under all returns to scale the FDH production possibility set is a proper subset of the convex production possibility set. Consequently, the targets assigned to inefficient Decision Making Unit (DMU) by an FDH efficient frontier are smaller than those located on the convex frontier.

REVIEW OF LITERATURE:

Larger targets may frighten the production manager and there is necessity to explore shorter paths to reach the efficient frontier. Frei and Harker (1999) demonstrated that closest targets do not correspond the extreme points of the convex set of all feasible input and output vectors. He proved that closest targets are the input and output vector of a virtual DMU. Under convex technology virtual DMU is an unobservable DMU whose input and output vector are convex combinations of extreme points of DMUs determining the full dimension efficient facet on to which the input and output vector of the DMU in assessment is projected.

Among orientation models of efficiency measurement, the radial measures of CCR (1978) and BCC (1984) models were extensively used in empirical research to identify target coordinates on the convex efficient frontier. These models seek largest equal proportional reduction of inputs and augmentation of outputs. For these models it is likely that the projection coordinates fall on the weak efficient frontier, which implies the necessity of slacks minimization to move from weak efficient frontier to strong efficient frontier which assumes inputs and outputs are freely disposable. For such a shift a 'stage two' optimization problem is solved, in which the sums of slacks are exclusively minimized.

The CCR (1978) problem assumes that returns to scale are constant for every decision making unit, there by it assigns larger targets to the DMU in assessment if its true returns to scale are either increasing or decreasing. The BCC (1984) problem allows returns to scale variable. Consequently it provides smaller targets to the inefficient DMU for which returns to scale are either increasing or decreasing.

A merit associated with radial efficiency measures is that they seek input reduction or output expansion along a ray. In input orientation the input vectors that fall on a ray possess the same input mix, similarly, under output orientation the output vectors along the ray satisfy the same output mix. Thus, movements along rays allow techniques which use the same input or output mix. It is reasonable to compare an inefficient DMU with an efficient DMU that allows the same input or output mix and the same returns to scale.

Radial measures based comparison are appropriate in short run during which input substitution or output transformation are not possible since such substitution needs change of technique.

If input prices are known, factor minimal cost can be evaluated. Cost minimizing coordinates serve as long run bench marks for he DMU being assessed. A shift from radial path to cost minimizing bench marks requires input substitution, which is possible only in long run.

Non-radial measures can also be used to project the input and output vector of a firm operating interior to the production possibility set onto the frontier of the technology set. The hyperbolic efficiency measure proposed by Fare et.al (1985)* simultaneously contract inputs and expands outputs. If input contraction takes place at the rate of λ, output expansion occurs at the rate of λ^{-1}. The bench marks provided by this model are

$$\left(\lambda x_0, \lambda^{-1} y_0\right).$$

Fare et.al (1985) proposed Russell measure which contracts different inputs or outputs at different rates. The Russell measure based projections under input orientation and output orientation land on efficient subset of the production possibility set. There is no need to augment either input slacks or output slacks with the Russell measure of efficiency. The targets provided by the Russell measure are closer to the input and output vector of the DMU under assessment than those by the radial measures.

Non-oriented efficiency measures were very widely used for targets setting. A common feature of non-oriented measures is that they involve slacks in optimization. The additive model of Cooper (1999), the slack based efficiency measure of Tone (2001) the range adjusted measure (RAM) of Cooper and the BRWZ measure of Brockett et.al (1997) were very widely used non-oriented measures to locate bench marks of inefficient decision making units on the frontier of the technology set. The merits of BRWZ measure over other non-oriented measures can be found in Portela et.al (2003). The additive model optimizes L_1-measure. Frei and Harker (1999) used the dual form of additive model to optimize L_2 measure, to locate frontier targets of inefficient DMUs.

A generalized class of distance function to measure efficiency in production was formulated by Chambers et.al (1996, 1998), called directional distance functions. An inefficient firm's input and output vector can be projected onto the frontier in any feasible direction. It allows input contraction and output expansion simultaneously to land on efficient frontier. It can be shown that the radial measures are special cases of the directional distance functions. These problems may be oriented or non-oriented. Fare et.al (1985) show that Tone's (2001) slack based efficiency measure is a special case of directional distance measure. This directional function handles both convex and non-convex technologies with comfortable case. In two directional case if horizontal direction is choosen we can contract input leaving output unaffected. If a vertical direction is considered output expansion takes place, leaving input unaffected. At will, the input and output vector of interior production possibility set can be projected onto the frontier choosing desirable and feasible direction.

The closest targets provided by the convex technologies are the input and output vectors of virtual decision making units. These firms are unobservable. In farming activities a farmer who is inefficient, always prefers to visit a farm similar in operation, that is in existence. He cannot visit hypothetical firm. His closest peer activities should be similar to the activities of the follower. Both the farms shall be equal in size, application of technique as far as possible. Both the farms shall be facing the same returns to scale.

EFFICIENT PEER – FDH TECHNOLOGY:

For a firm or a farm identification of observed peer is more desirable than identification of an unobservable virtual peer for comparison and following. The follower and the efficient peer shall face the same returns to scale and employ the same technique in short run comparisons.

A virtual DMU's input and output vectors are convex combination of input and output vectors of a subset of extremely efficient DMUs. It leaves the farmer or firm manager in confusion to visit which of the farm or firm participated in the convex combination of virtual targets. If we appeal to FDH technology, we can find a single observable peer providing the coordinates of target point on the efficient frontier.

DIRECTIONAL DISTANCE FUNCTIONS – EFFICIENT TARGETS:

Chambers et.al (1996, 1998) proposed directional distance functions which encompass a wide variety of distance functions in efficiency literature. Let T be the technology set appropriately structured by the input and output vectors of observed decision making units.

$$\vec{D}_T\left(x_0, y_0; g_x, g_y\right) = Max\ \beta$$

such that $\left(x_0 - \beta g_x,\ y_0 + \beta g_y\right) \in T$

The efficient targets set by the directional distance function are,

$$\left(x_{i0} - \vec{D}_T\left(x_0, y_0; g_x, g_y\right) g_{x_i}, y_{r0} + \vec{D}_T\left(x_0, y_0; g_x, g_y\right) g_{y_r}\right) i \in M, r \in S$$

LONG RUN & SHORT RUN PRODUCTION POSSIBILITY SETS:

Attainment of technical efficiency (TE) is short run objective, since it involves mainly eradication of managerial inefficiency. To achieve technical efficiency by ray contraction or ray expansion the input or output isoquants are reached. Movements along rays hold input mix or output mix unaltered so that change of technique is not allowed. Further, the ray based projections land on input or output isoquant, that refers to short run. The appropriate isoquant to identify referent points belongs to the level set that admits variable returns to scale and strong disposability of inputs and outputs. By radial input contraction or output expansion a firm can reach the variable returns to scale input or output set respectively. Thus, the first stage for efficiency enhancement is to reach the short run frontier.

Divergence between actual and ideal size of production leads to scale inefficiency. A firm that operates on VRS frontier is only short run technical efficient, but its local returns to scale are either increasing or decreasing. To attain scale efficiency the firm under evaluation should scale down its inputs or scale up its outputs, depending upon the orientation. Attainment of scale efficiency is long run objective. The movements are along a ray and the projections land on isoquant of $L(y)$ or $P(x)$ that refers to long run, admits strong disposability of inputs and outputs.

Allocative efficiency can be achieved by moving the point reached on long run isoquant by radial contraction of inputs or expansion of outputs to a point where input cost is minimized or output revenue is maximized. This requires to bring a change in input mix or output mix, which is possible through a change in technique. Attainment of allocative efficiency too is a long run objective, the task of the third stage.

RETURNS TO SCALE:

Returns to scale are surface property of the production possibility set. If (x_0, y_0) is the input and output vector of a firm operating interior to the technology set, depending on the flexibility of its production plan it needs to adjust its input and / or outputs to reach the surface of the technology set convex or non-convex. Returns to scale at input and output orientation projection points need not be the same.

In DEA frame work for taxonomy of returns to scale, the observed inefficient firm's input and / or output vectors are projected on to frontiers of technology set admitting constant, non-increasing and non-decreasing returns to scale.

$$T^{DEA-VRS} = T^{DEA-NDRS} \cap T^{DEA-NIRS}$$

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Clearly, $T^{DEA-VRS} \subseteq T^{DEA-NDRS}$

$T^{DEA-VRS} \subseteq T^{DEA-NIRS}$

Tulkens (1993) is the closest inner approximation of the true, strongly disposable technology. The FDH production possibility set satisfies the axioms of (i) Inclusion (ii) Free Disposability and (iii) Minimum extrapolation. The FDH production possibility set is a proper subset of DEA technology set. Consequently, the FDH targets are smaller than the convex targets.

Briec et.al (2004) formulated a non-linear integer programming problem that separates returns to scale from convexity. To classify a production plan according to its returns to scale an index set of better observations is constructed, rescaling input and output vectors under returns to scale specification. They named such a set as scaled better set, as follows:

$$B(x_0, y_0, \delta \in \Gamma) = \{(x_k, y_k) : \delta x_k \le x_0,\ \delta y_k \ge y_0,\ \delta \in \Gamma\}$$

(i) $\delta \in \Gamma \Rightarrow \delta > 0 \Rightarrow CRS$

(ii) $\delta \in \Gamma \Rightarrow 0 < \delta \le 1 \Rightarrow NIRS$

(iii) $\delta \in \Gamma \Rightarrow \delta \ge 1 \Rightarrow NDRS$

(iv) $\delta \in \Gamma \Rightarrow \delta = 1 \Rightarrow VRS$

(v) The scaled vectors of $B(x_0, y_0, \Gamma)$ generate the economic region of the input level set $L^{FDH}(y_0)$ denoted by , $L^{FDH}(y_0 / \delta x_k \le x_0)$. The scaled vectors δx_k span the isoquant of the conditional FDH input level set. $\{\delta x_k\}$ constitute the efficient subset of $L^{FDH}(y_0 / \delta x_k \le x_0)$, where $\delta \in \Gamma$.

For the construction of the scaled better set, one need not experiment with all the δ values belonging to the ranges postulated for different returns to scale. It is sufficient if one experiments with critical values of δ.

NEW METHOD:

This method is based on the concept of scaled vector dominance. The distance function chosen for efficiency measurement is the directional distance function. The directions are provided by either naturally or scaled vector dominating firms. The critical values of δ are the critical values suggested by Briec et.al (2004). The orientation of measuring directional distance efficiency is input orientation.

DIRECTIONAL DISTANCE FUNCTION – CLOSEST TARGET IDENTIFICATION IN FDH TECHNOLOGY:

(i) Increasing Returns to Scale: $\delta \ge 1$

Critical value of $\delta : \delta_k = Max\left\{1,\ \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)\right\}$

Let $(x_k, y_k) \in B(x_0, y_0,\ IRS)$

$$\Rightarrow \delta x_k \le x_0$$

$$\delta y_k \ge y_0$$

By choosing suitable directional vector any point that belongs to the isoquant of economic region can be reached. However, we are interested in reaching either naturally dominating or scaled vector dominating decision making units spanning the isoquant of $L^{FDH}(y_0, IRS)$. For (x_k, y_k) the following optimization problem is proposed:

$$\beta_k = Max\ \beta$$

s.t

$$\delta x_k \le x_0 - \beta \delta x_k$$

$$\delta y_k \ge y_0$$

From the second constraint we obtain the critical value for δ, viz., δ_1

$$\beta_k = Max\ \beta$$

s.t

$$\delta_k x_k \le x_0 - \beta \delta_k$$

$$\delta_k (1+\beta) x_k \le x_0$$

$$\delta_k (1+\beta) x_{ik} \le x_{i0},\ i \in M$$

$$\delta_k (1+\beta) \le \frac{x_{i0}}{x_{ik}}$$

$$(1+\beta)\delta_k \le \underset{i}{Min}\frac{x_{i0}}{x_{ik}}$$

$$1+\beta \le \delta_k^{-1}\ \underset{i}{Min}\frac{x_{i0}}{x_{ik}}$$

$$\beta \le \delta_k^{-1}\ \underset{i}{Min}\frac{x_{i0}}{x_{ik}} - 1$$

$$\beta_k = \delta_k^{-1}\ \underset{i}{Min}\left(\frac{x_{i0}}{x_{ik}}\right) - 1$$

$$\beta_k = \frac{\underset{i}{Min}\left(\frac{x_{i0}}{x_{ik}}\right) - Max\left\{1,\ \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)\right\}}{Max\left\{1,\ \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)\right\}}$$

If minimum occurs for $k = k_0$, then the directional distance evaluated is β_{k_0} and the closet input and output targets are,

$$\left(x_0 - \beta_{k_0} x_{k_0} \delta_{k_0},\ y_0\right)$$

(ii) Decreasing Returns to Scale: $\delta \le 1$

Let $\left(x_k, y_k\right) \in B\left(x_0, y_0,\ DRS\right)$

Critical value: $\delta_k = \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)$

$$\beta_k = \frac{\underset{i}{Min}\frac{x_{i0}}{x_{ik}} - \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)}{\underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)}$$

Find $\underset{k}{Min}\ \beta_k$

If minimum occurs for $k = k_0$, then the closet input and output targets are

$$\left(x_0 - \beta_{k_0} x_{k_0} \delta_{k_0},\ y_0\right)$$

(iii) Constant Returns to Scale: $\delta \ge 0$

Critical value of $\delta : \delta_k = \underset{r}{Max}\frac{y_{r0}}{y_{rk}}$

Let $\left(x_k, y_k\right) \in B\left(x_0, y_0,\ CRS\right)$

$$\beta_k = \frac{\underset{i}{Min}\frac{x_{i0}}{x_{ik}} - \underset{r}{Max}\frac{y_{r0}}{y_{rk}}}{\underset{r}{Max}\frac{y_{r0}}{y_{rk}}}$$

Let β_k attain minimum for $k = k_0$, in which case,

$$\beta_{k_0} = \underset{k}{Min}\ \frac{\left[\underset{i}{Min}\left(\frac{x_{i0}}{x_{ik}}\right) - \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)\right]}{\underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)},\ (x_k, y_k) \in B\left(x_0, y_0, \Gamma\right)$$

Furthest targets are given by

$$\beta_{k_0} = \underset{k}{Max}\ \frac{\left[\underset{i}{Min}\left(\frac{x_{i0}}{x_{ik}}\right) - \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)\right]}{\underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)},\ (x_k, y_k) \in B\left(x_0, y_0, \Gamma\right)$$

The closest targets are given by,

$$\left(x_0 - \beta_{k_0} \delta_{k_0} x_{k_0},\ y_0\right)$$

$$\beta_k = \begin{cases} \dfrac{\underset{i}{Min}\left(\frac{x_{i0}}{x_{ik}}\right) - \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)}{\underset{i}{Min}\frac{x_{i0}}{x_{ik}}}, & \beta_k \ge 0,\ \left(x_k, y_k\right) \in B\left(y_0, x_0, CRS\right) \\ \dfrac{Min\left[1, \underset{i}{Min}\left(\frac{x_{i0}}{x_{ik}}\right)\right] - \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)}{Min\left[1, \underset{i}{Min}\left(\frac{x_{i0}}{x_{ik}}\right)\right]}, & \beta_k \ge 0,\ \left(x_k, y_k\right) \in B\left(y_0, x_0, NIRS\right) \\ \dfrac{\underset{i}{Min}\left(\frac{x_{i0}}{x_{ik}}\right) - \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)}{\underset{i}{Min}\left(\frac{x_{i0}}{x_{ik}}\right)}, & \beta_k \ge 0,\ \underset{r}{Min}\left(\frac{x_{i0}}{x_{ik}}\right) \ge 1,\ \left(x_k, y_k\right) \in B\left(y_0, x_0, NDRS\right) \end{cases}$$

Combining all above expressions we propose the following

The above expression can easily extended to Output Orientation

EMPIRICAL ANALYSIS:

DATA:

The numerical example worked out refers to the data collected from the bulletins of Annual Survey of Industries (ASI), for 2012-13. The variables of the study are (i) Net value Added, (ii) No. of Employees, (iii) Fixed Capital, (iv) Total Emoluments.

The total manufacturing 16 Indian states arranged in descending order of their contribution to the Indias total Value Added are the decision making units of the present study.

TABLE- (4.1)
FDH DIRECTIONAL INPUT EFFICIENCY -SHORTEST DISTANCE.

S.No	Total Manufacturing Sector	Efficiency Score	Returns to Scale	Efficient Peer
1	Maharastra (MH)	0	CRS	-
2	Gujarat (GUJ)	0.1486	DRS	Maharastra
3	Tamilnadu (TN)	0.0056	IRS	Rajasthan
4	Karnataka (KA)	0.0968	IRS	Rajasthan
5	Uttar Pradesh (UP)	0.2259	IRS	Rajasthan
6	Haryana (HA)	0	IRS	-
7	Uttarakhund (UK)	0	CRS	-
8	Rajasthan (RA)	0	IRS	-

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9	Telangana (TEL)	0	IRS	-
10	Andhra Pradesh (AP)	0.1368	IRS	Rajasthan
11	West Bengal (WB)	0.0015	IRS	Punjab
12	Himachal Pradesh (HP)	0	CRS	-
13	Madhya Pradesh (MP)	0.41	DRS	Maharastra
14	Jharkhand (JHA)	0	IRS	-
15	Punjab (PUN)	0	IRS	-
16	Odisha (ODI)	0	IRS	-

(a) If a directional distance efficiency score is zero for a firm, then it is said to be directional efficient.
(b) Further the score is away from zero, greater is the inefficiency. Out of 16 total manufacturing sectors 9 are directional efficient. Three out of the nine attained constant returns to scale, and the remaining operate at increasing returns to scale.
(c) The total manufacturing sectors of Gujarat and Madhya Pradesh admit decreasing returns to scale.

The input targets of the various decision making units are as follows

VARIABLE RETURNS TO SCALE
TABLE - (4.2)
FDH – INPUT TARGETS (SHORT RUN)

S.No	Total Manufacturing Sector	x_1	Target x_1	x_2	Target x_2	RTS	PEER
1	Maharastra (MH)	34492959	34492959	1784909	1784909	DRS	--
2	Gujarat (GUJ)	32612528	29211522	1363628	1187636	DRS	MH
3	Tamilnadu (TN)	18724233	18619202	1965020	1957444	IRS	RA
4	Karnataka (KA)	14515109	13461728	862203	786091	IRS	RA
5	Uttar Pradesh (UP)	10271141	8378127	825537	688996	IRS	HA
6	Haryana (HA)	7839623	7839623	566595	566595	IRS	--
7	Uttarakhund (UK)	5299803	5299803	335300	335300	CRS	--
8	Rajasthan (RA)	6142162	6142162	443027	443027	IRS	--
9	Telangana (TEL)	5847297	5847297	701110	701110	IRS	--
10	Andhra Pradesh (AP)	13081848	12241600	503615	434720	IRS	RA
11	West Bengal (WB)	8206693	4002518	656123	655131	IRS	PUN
12	Himachal Pradesh(HP)	4009160	4009160	184833	184833	CRS	--
13	Madhya Pradesh (MP)	13657975	11264164	302209	302209	DRS	MH
14	Jharkhand (JHA)	6728469	6728469	188046	188046	IRS	--
15	Punjab (PUN)	3906929	3906929	583520	583520	IRS	--
16	Odisha (ODI)	16377525	16377525	263651	263651	IRS	--

Among the above 16 total Manufacturing sectors only two are scale efficient, they being the total manufacturing sectors of Uttarakhund and Himachal Pradesh. The total manufacturing sector of Maharastra admits decreasing returns to scale and remains to be efficient under the same efficiency classification. The total manufacturing sectors of Haryana, Telangana, Jharkhand, Punjab and Odisha are efficient under Increasing returns to scale

CONSTANT REURNS TO SCALE
TABLE- (4.3)
FDH-INPUT TARGETS (LONG-RUN)

S.No	Total Manufacturing Sector	Target x_1	Target x_2	RTS	PEER
1	Maharastra (MH)	32209341	1679628	CRS	HP
2	Gujarat (GUJ)	28415614	1098104	CRS	UK
3	Tamilnadu (TN)	16465071	1860866	CRS	HP
4	Karnataka (KA)	9637565	637335	CRS	HP
5	Uttar Pradesh (UP)	7409002	693585	CRS	HP
6	Haryana (HA)	6998100	527799	CRS	HP
7	Uttarakhund (UK)	5299803	335300	CRS	--
8	Rajasthan (RA)	5431739	410275	CRS	HP
9	Telangana (TEL)	5075133	665511	CRS	HP
10	Andhra Pradesh (AP)	9691301	220052	CRS	UK

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11	West Bengal (WB)	4015517	462899	CRS	HP
12	Himachal Pradesh (HP)	4009160	184833	CRS	--
13	Madhya Pradesh (MP)	12014506	198232	CRS	UK
14	Jharkhand (JHA)	6418561	173758	CRS	HP
15	Punjab (PUN)	3469695	568271	CRS	HP
16	Odisha (ODI)	14846412	166783	CRS	UK

The total manufacturing sectors of Uttarakund and Himachal Pradesh are scale efficient. Uttarakund served as peer to four states while Himachal Pradesh is peer of the states.

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