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Time to Recruitment in a Two Grade Manpower System With Two Types of Policy Decisions and Two Sources of Depletion Using Univariate Max Policy of Recruitment

KEYWORDS: Two grade manpower system, two types of policy decisions, two sources of depletion of manpower and univariate MAX policy of recruitment

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ABSTRACT *Depletion of manpower due to attrition of personnel is a common phenomenon in any marketing organization when the management takes policy decisions regarding pay, perquisites and work targets. This depletion will lead to breakdown of the organization in due course of time if it is not compensated by recruitment. Since the depletion of manpower as a consequence of the inter-decision times is unpredictable and frequent recruitment which involves more cost is not advisable, the organization requires a suitable recruitment policy to plan for recruitment. A mathematical model is constructed and using a univariate MAX policy of recruitment based on shock model approach, analytical results for variance of time to recruitment is obtained. The results are numerically illustrated by assuming specific distributions and relevant findings are presented.*

Introduction

Many researchers [1],[8],[9]and[10] have studied the problem of time to recruitment for a marketing organization consisting of two grades by considering ***only one source*** (recurring) for depletion of man power which takes place due to attrition as the effect of policy decisions such as revision of pay, targets etc. taken by the organization, using univariate and bivariate policies of recruitment under different conditions. Apart from the above cited source of depletion which is recurrent there is another source of depletion due to transfer decisions which is non-recurrent. In the presence of these ***two different sources*** of depletion, Elangovan et.al [6] have initiated the problem of time to recruitment for an organization consisting of one grade and obtained the variance of the time to recruitment using a univariate CUM policy of recruitment when (i) the loss of man power in the organization due to the two sources of depletion and its threshold are independent and identically exponential random variables. (ii) inter-policy decision times and inter-transfer decision times form the same renewal process. In [2], Dhivya and Srinivasan have extended the work of Elangovan et.al [6] for a two grade manpower system for different forms of the thresholds for the cumulative loss of manpower in the organization when the inter-policy decisions and inter-transfer decisions form the same ordinary renewal process. In[3] they have study this problem when the renewal processes are different. In [4], they have studied their work in [2] & [3] using univariate **max** policy of recruitment. In all the above cited literature, it is assumed that all policy decisions have **same attrition rate**. This assumption on policy decisions is not realistic since policy decisions can have high or low intensity of attrition. In this context, Dhivya and Srinivasan[5] have studied their work in [2] when the policy decisions are classified into two types according to the intensity of attrition. The objective of the present paper is to study the problem in [5] using univariate **max** policy of recruitment.

Model Description:

Consider an organization with two grades taking policy and transfer decisions at random epochs in $(0,\infty)$.The policy decisions are classified into two types depending upon their intensity of attrition. Let p be proportion of policy decisions having high attrition rate μ_h and (1-p) be the proportion of policy decisions having low attrition rate μ_l. The most suitable distribution of inter-policy decision times is hyper exponential with distribution $F(.)$, probability density function $f(.)$. Let the inter-transfer decision times be independent and identically distributed exponential random variables with distribution $W(.)$, probability density function $w(.)$ and mean $\frac{1}{\mu_2}(\mu_2 > 0)$. It is assumed that the two sources of depletion are independent. Let $f_m(.)$ be the m-fold convolution of $f(.)$ with itself and $w_n(.)$ be the n-fold convolution of $w(.)$ with itself. At every policy decision making epoch a random number of persons quit the organizations and at every transfer decision making epoch a random number of persons are transferred to the sister organizations. There is an associated loss of manpower in each grade, if a person quits or transferred. It is assumed that the loss of manpower is linear and cumulative. For i=1,2,3…,let X_i be the continuous random variables representing the amount of depletion of manpower(loss of man hours) caused due to the i^{th} policy decision in the organization. It is assumed that X_i form a sequence of independent and identically distributed random variables with distribution $G(.)$. Let $\bar{X}_m$ be the maximum loss of manpower due to the first m policy decisions in the organization. For j=1,2,3…,let Y_j be the continuous random variables representing the amount of depletion of manpower in the organization caused due to the j^{th} transfer decision. It is assumed that Y_j form a sequence of independent and identically distributed random variables with probability distribution function $H(.)$. Let $\bar{Y}_n$ be the maximum loss of manpower in the organization due to the first n transfer decisions. For each i and j, X_i and Y_j are statistically independent. Let C $(C > 0)$ be the constant breakdown threshold level for the depletion of manpower in the organization. The univariate MAX policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the maximum loss of man hours in the organization exceeds the breakdown threshold.

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Let T be the random variable denoting the time to recruitment with distribution L(.),probability density function $l(.)$, Laplace transform $\bar{l}(.)$ for $l(.)$, mean E(T) and variance V(T).

Main Results:

$$P(T > t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left\{\begin{array}{c}\text{Probability that there are exactly } m \text{ policy decisions and } n \text{ transfer} \\ \text{decisions in } [0,t) \text{ and the maximum loss of manhours due to } m \text{ policy} \\ \text{decisions and } n \text{ transfer decisions does not exceed the threshold } C\end{array}\right\}$$

By using laws of probability and from renewal theory [7],

$$P(T > t) = \sum_{m=0}^{\infty}[F_m(t) - F_{m+1}(t)]\sum_{n=0}^{\infty}[W_m(t) - W_{m+1}(t)]P(\max(\bar{X}_m, \bar{Y}_n) \le C) \qquad (1)$$

where $F_0(t) = W_0(t) = 1$.

Since

$$P(\max(\bar{X}_m, \bar{Y}_n) \le C) = [G(C)]^m[H(C)]^n \qquad (2)$$

from (1), (2) and on simplification we get

$$L(t) = [1 - G(C)]\sum_{m=1}^{\infty}F_m(t)[G(C)]^{m-1} + [1 - H(C)]\sum_{n=1}^{\infty}W_n(t)[H(C)]^{n-1} - [1 - G(C)]\sum_{m=1}^{\infty}F_m(t)[G(C)]^{m-1}$$
$$[1 - H(C)]\sum_{n=1}^{\infty}W_n(t)[H(C)]^{n-1} \qquad (3)$$

From the hypothesis we note that $w_n(t) = \frac{{\mu_2}^n e^{-\mu_2 t} t^{n-1}}{(n-1)!}$. Therefore we find that

$$[1 - H(C)]\ \sum_{n=1}^{\infty}W_n(t)[H(C)]^{n-1} = 1 - e^{-\mu_2[1-H(C)]t}$$

and

$$[1 - H(C)]\ \sum_{n=1}^{\infty}w_n(t)[H(C)]^{n-1} = \mu_2[1 - H(C)]e^{-\mu_2[1-H(C)]t} \qquad (4)$$

From (3) and (4) we get

$$l(t) = \mu_2[1 - H(C)]e^{-\mu_2[1-H(C)]t} + [1 - G(C)]\sum_{m=1}^{\infty}e^{-\mu_2[1-H(C)]t}f_m(t)[G(C)]^{m-1} - [1 - G(C)]\mu_2$$
$$[1 - H(C)]\sum_{m=1}^{\infty}e^{-\mu_2[1-H(C)]t}F_m(t)[G(C)]^{m-1} \qquad (5)$$

The mean and variance of time to recruitment can be computed from (5) and from the result

$E(T) = -\left[\frac{d}{ds}[\bar{l}(s)]\right]_{s=0}$ and $E(T^2) = \left[\frac{d^2}{ds^2}[\bar{l}(s)]\right]_{s=0}$. Thus we get

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$$E(T)=\frac{1-\bar{f}[\mu_2(1-H(C))]}{\mu_2(1-H(C))[1-\bar{f}[\mu_2(1-H(C))]G(c)]} \tag{6}$$

and

$$E(T^2)=\frac{-2\mu_2(1-H(C))(1-G(C))\bar{f}'[\mu_2(1-H(C))]+2[1-\bar{f}[\mu_2(1-H(C))]G(c)][1-\bar{f}[\mu_2(1-H(C))]]}{[\mu_2(1-H(C))[1-\bar{f}[\mu_2(1-H(C))]G(c)]]^2} \tag{7}$$

where $\bar{f}\left(\mu_2(1-H(C))\right)=\frac{p\mu_h}{\mu_2(1-H(C))+\mu_h}+\frac{(1-p)\mu_l}{\mu_2(1-H(C))+\mu_l}$ and

$$\bar{f}'\left(\mu_2(1-H(C))\right)=\frac{-p\mu_h}{[\mu_2(1-H(C))+\mu_h]^2}-\frac{(1-p)\mu_l}{[\mu_2(1-H(C))+\mu_l]^2} \tag{8}$$

Special Case:

Suppose X_i *and* Y_j , $i.j=1,2,3 \ldots$,follow exponential distribution with parameters α_1 and α_2 respectively.

In this case, from (6),(7), (8) and on simplification we get

$$E(T)=\frac{1-\frac{p\mu_h}{\mu_2e^{-\alpha_2C}+\mu_h}-\frac{(1-p)\mu_l}{\mu_2e^{-\alpha_2C}+\mu_l}}{\mu_2e^{-\alpha_2C}\left[1-\frac{p\mu_h\left[1-e^{-\alpha_1C}\right]}{\mu_2e^{-\alpha_2C}+\mu_h}-\frac{(1-p)\mu_l\left[1-e^{-\alpha_1C}\right]}{\mu_2e^{-\alpha_2C}+\mu_l}\right]} \tag{9}$$

and

$$E(T^2)=\frac{2\mu_2e^{-(\alpha_1+\alpha_2)C}\left[-\frac{p\mu_h}{\left[\mu_2e^{-\alpha_2C}+\mu_h\right]^2}-\frac{(1-p)\mu_l}{\left[\mu_2e^{-\alpha_2C}+\mu_l\right]^2}\right]+2\left[1-\frac{p\mu_h\left[1-e^{-\alpha_1C}\right]}{\mu_2e^{-\alpha_2C}+\mu_h}-\frac{(1-p)\mu_l\left[1-e^{-\alpha_1C}\right]}{\mu_2e^{-\alpha_2C}+\mu_l}\right]\left[1-\frac{p\mu_h}{\mu_2e^{-\alpha_2C}+\mu_h}-\frac{(1-p)\mu_l}{\mu_2e^{-\alpha_2C}+\mu_l}\right]}{\left[\mu_2e^{-\alpha_2C}\left[1-\frac{p\mu_h\left[1-e^{-\alpha_1C}\right]}{\mu_2e^{-\alpha_2C}+\mu_h}-\frac{(1-p)\mu_l\left[1-e^{-\alpha_1C}\right]}{\mu_2e^{-\alpha_2C}+\mu_l}\right]\right]^2} \tag{10}$$

(9) together with (10) give the mean and variance of the time to recruitment for the present case.

Note 1:

Suppose $\bar{X}_{Am}=\max_{1\leq i\leq m}X_{Ai}$, $\bar{X}_{Bm}=\max_{1\leq i\leq m}X_{Bi}$ and $\bar{X}_m=max(\bar{X}_{Am},\bar{X}_{Bm})$

$\bar{Y}_{An}=\max_{1\leq j\leq n}Y_{Aj}$, $\bar{Y}_{Bn}=\max_{1\leq i\leq n}Y_{Bj}$ and $\bar{Y}_n=max(\bar{Y}_{An},\bar{Y}_{Bn})$

For this choice of $\bar{X}_m$ *and* $\bar{Y}_n$, it can be shown that

$$P(\max(\bar{X}_m,\bar{Y}_n)\leq C)=[G_A(C).G_B(C)]^m[H_A(C).H_B(C)]^n$$

$$E(T)=\frac{1-\frac{p\mu_h}{\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]+\mu_h}-\frac{(1-p)\mu_l}{\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]+\mu_l}}{\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]\left[1-\frac{p\mu_h\left[1-e^{-\alpha_{1A}C}-e^{-\alpha_{1B}C}+e^{-(\alpha_{1A}+\alpha_{1B})C}\right]}{\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]+\mu_h}-\frac{(1-p)\mu_l\left[1-e^{-\alpha_{1A}C}-e^{-\alpha_{1B}C}+e^{-(\alpha_{1A}+\alpha_{1B})C}\right]}{\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]+\mu_l}\right]} \quad (11)$$

and

$E(T^2)=$

$$\frac{\begin{array}{l}-2\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]\left[e^{-\alpha_{1A}C}+e^{-\alpha_{1B}C}-e^{-(\alpha_{1A}+\alpha_{1B})C}\right]\left[-\frac{p\mu_h}{\left[\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]+\mu_h\right]^2}-\frac{(1-p)\mu_l}{\left[\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]+\mu_l\right]^2}\right]\\ +2\left[1-\frac{p\mu_h\left[1-e^{-\alpha_{1A}C}-e^{-\alpha_{1B}C}+e^{-(\alpha_{1A}+\alpha_{1B})C}\right]}{\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]+\mu_h}-\frac{(1-p)\mu_l\left[1-e^{-\alpha_{1A}C}-e^{-\alpha_{1B}C}+e^{-(\alpha_{1A}+\alpha_{1B})C}\right]}{\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]+\mu_l}\right]\left[1-\frac{p\mu_h}{\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]+\mu_h}-\frac{(1-p)\mu_l}{\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]+\mu_l}\right]\end{array}}{\left[\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]\left[1-\frac{p\mu_h\left[1-e^{-\alpha_{1A}C}-e^{-\alpha_{1B}C}+e^{-(\alpha_{1A}+\alpha_{1B})C}\right]}{\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]+\mu_h}-\frac{(1-p)\mu_l\left[1-e^{-\alpha_{1A}C}-e^{-\alpha_{1B}C}+e^{-(\alpha_{1A}+\alpha_{1B})C}\right]}{\mu_2\left[e^{-\alpha_{2A}C}+e^{-\alpha_{2B}C}-e^{-(\alpha_{2A}+\alpha_{2B})C}\right]+\mu_l}\right]\right]^2} \quad (12)$$

While(11) gives mean time to recruitment, (11) together with (12) give variance of the time to recruitment for the present choice of $\bar{X}_m$ *and* $\bar{Y}_n$.

Note 2:

The results obtained in [5] when the inter-policy decisions times are independent and identically distributed exponential random variables can be deduced from the present results when p=1.

Numerical Illustration:

The mean and variance of time to recruitment for all the cases are numerically illustrated by varying one parameter and keeping all the other parameters fixed. The effect of the nodal parameters α_1, α_2, p,μ_h, μ_l and $\mu_2$on the mean and variance of time to recruitment are shown in the following table.

Table 1

(C=1.5)

α_1	α_2	p	μ_h	μ_l	μ_2	E(T)	V(T)
0.1	0.1	0.07	0.2	0.4	0.5	1.3173	10.3309
0.3	0.1	0.07	0.2	0.4	0.5	1.4839	14.1050
0.5	0.1	0.07	0.2	0.4	0.5	1.6373	19.4551
0.1	0.1	0.07	0.2	0.4	0.5	1.3173	10.3309
0.1	0.3	0.07	0.2	0.4	0.5	1.5460	23.7186
0.1	0.5	0.07	0.2	0.4	0.5	1.7748	59.6499
0.2	0.1	0.1	0.2	0.4	0.5	1.4129	0.2980
0.2	0.1	0.3	0.2	0.4	0.5	1.4854	0.9197
0.2	0.1	0.5	0.2	0.4	0.5	1.5565	1.5253
0.1	0.1	0.07	0.1	0.4	0.5	1.3379	1.5199
0.1	0.1	0.07	0.3	0.4	0.5	1.3023	1.2115
0.1	0.1	0.07	0.5	0.4	0.5	1.2820	1.0950
0.1	0.1	0.07	0.4	0.1	0.5	1.8930	5.9267
0.1	0.1	0.07	0.4	0.3	0.5	1.4411	2.0686
0.1	0.1	0.07	0.4	0.5	0.5	1.1709	0.5268
0.1	0.1	0.07	0.3	0.1	0.5	1.9035	5.9893
0.1	0.1	0.07	0.3	0.1	0.7	1.4325	3.5898
0.1	0.1	0.07	0.3	0.1	0.9	1.1486	2.3792

Findings:

From the above tables the following inference are presented which agree with reality,

1. When α_1, α_2 and p increase separately and keeping all the other parameters fixed, the mean and variance of time to recruitment increases.

2. When $\mu_h, \mu_l \ and \ \mu_2$ increase separately and keeping all the other parameters fixed, the mean and variance of time to recruitment decreases.

Conclusion:

The results of this paper will be very useful in planning recruitments in future for those marketing organizations with depletion of manpower due to attrition. The goodness of fit for the distributions assumed in this paper can be examined by collecting relevant data and using simulation the applicability of the developed model can be studied.

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