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Weighted N- Factor Satisfactory Roommates Problem

KEYWORDS: Optimal matching, Preference Value, Satisfactory value matrix, Satisfactory Level, N - Factor

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ABSTRACT *As the extension of our previous work N-Factor SFRP, now we are extending the case in to weighted N-Factor SFRP, in this we are giving different weightage to the factors based on our needs, by considering all the factors finally a better matching can be carried out, the technique is described and illustrated with suitable examples.*

1.INTRODUCTION

In an instance of the Stable Roommates (SR) problem, first introduced by Gale and Shapley [1], there is a set of n members where n is even. Each member ranks the n -1 others in strict order of preference. A matching M is a partition of the set of members into disjoint pairs. A blocking pair for M is a pair of members {x, y} $\notin$ M such that x prefers y to M(x) and y prefers x to M(y) where M(q) denotes q's partner in M for any member q. A matching is stable if it admits no blocking pair.

It is well-known that the stable roommates' problem is the extension of stable marriage problem, since the set of stable matching is unchanged if we reduce an SM instance I into an SR instance by appending to the very end of each members preference list all the other members that are of the same sex in I [2]. Not all SR instances admit a stable matching [1], and Knuth [3] posed the question of whether the problem of determining the solvability of SR instances might be NP-complete. This question was answered by Irving [5], who gave an O (n^2) algorithm for finding a stable matching or reporting that no such matching exists. Alternative approaches for finding a stable matching if one exists, given an SR instance have since been described [6, 7, 8, 9, 10].

As the problem name suggests, an application of SR arises in the context of campus accommodation allocation, where we seek to assign students to share two-person rooms, based on their preferences over one another. Another application occurs in the context of forming pairings of players for chess tournaments [11]. Very recently, a more serious application of SR has been studied, involving pair wise kidney exchange between incompatible patient-donor pairs [12]. Here, preference lists can be constructed on the basis of compatibility profiles between patients and potential donors.

The classical satisfactory roommates' problem (SFRP) is closely related to the stable roommates' problem. In the satisfactory roommates problem each person in the set of even cardinality n ranks the n-1 others in order of preference. The objective is to find satisfactory matching of roommates' problem. This is the partition of the set into n/2 pairs of roommates based on the individual satisfactory level. It is known that some of the instances of the satisfactory roommates' problem are unsolvable.

2. Weighted N- factor Satisfactory Roommates problem

In the classical satisfactory roommates' problem (SFRP), we consider the single preference list and its respective preference values [4].After that we are considering N-factors [13] and every instance of the problem having dissimilar preference list. In this setting every member giving their own order of preference, as a preference list. For finding weighted N-Factor SFRP, we have collected all the details in a sports hostel, in that hostel different sports persons are accommodated, here we are considering the game interest of the players, such as Basket ball interest, Foot ball interest, Hockey interest, Volley ball interest, Cricket interest and so on. Like that they are giving their preference list based on game interest, by making use of this list we are finding the respective SVM, after that based on our needs we are giving different weightage to the SVM's and adding it to a single SVM, then applied Hungarian algorithm for finding the result.

Satisfactory Roommates' problem is solved by using proposed SMAR algorithm [4].Our objective is to determine a matching which satisfies all members in the group to the maximum possible extend equally likely. In this concept, in order to obtain optimal (satisfactory) matching in Weighted N – Factor SFRP we have applied assignment method. For the related definitions such as satisfactory value matrix, satisfactory level, satisfactory matching, assignment model refer [4].

3. Satisfactory Matching Algorithm for weighted N-Factor Roommates

- Get the preference lists of all members in each factor.
- Form a Satisfactory Value Matrix (SVM) for each factor.
- Form a SVM with different weightage for each factor.
- Summing all the SVM with different weightage that gives a single SVM.
- Apply Hungarian method to find optimal (satisfactory) matching for all members such that the total assignment value should be maximized. That indicates optimal satisfactory level.

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Example: 1 Consider the problem instance of size 4 based on order of preference, here all the students are considering the Basket ball interest factor of the remaining students.

1	2 4 3
2	3 1 4
3	1 4 2
4	2 3 1

The Satisfactory value Matrix for the Basket bal interest factor

$$\text{SVM}= \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} - & \frac{5}{3} & \frac{4}{3} & \frac{3}{3} \\ \frac{5}{3} & - & \frac{4}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{4}{3} & - & \frac{4}{3} \\ \frac{3}{3} & \frac{4}{3} & \frac{4}{3} & - \end{pmatrix} \end{array}$$

Here we are taking 50% of Basket ball interest factor, so that the SVM becomes

$$50\%\ \text{of SVM} = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} - & \frac{5}{6} & \frac{4}{6} & \frac{3}{6} \\ \frac{5}{6} & - & \frac{4}{6} & \frac{4}{6} \\ \frac{4}{6} & \frac{4}{6} & - & \frac{4}{6} \\ \frac{3}{6} & \frac{4}{6} & \frac{4}{6} & - \end{pmatrix} \end{array}$$

Consider the problem instance of size 4 based on order of preference, here all the students are considering the Football interest factor of the remaining students.

1	3 2 4
2	4 3 1
3	2 4 1
4	1 2 3

The Satisfactory value Matrix for the Foot ball interest factor

$$\text{SVM} = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} - & \frac{3}{3} & \frac{4}{3} & \frac{4}{3} \\ \frac{3}{3} & - & \frac{5}{3} & \frac{5}{3} \\ \frac{4}{3} & \frac{5}{3} & - & \frac{3}{3} \\ \frac{4}{3} & \frac{5}{3} & \frac{3}{3} & - \end{pmatrix} \end{array}$$

Here we are taking 20% of Foot ball interest factor, so that the SVM becomes

$$20\%\ \text{of SVM} = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} - & \frac{3}{15} & \frac{4}{15} & \frac{4}{15} \\ \frac{3}{15} & - & \frac{5}{15} & \frac{5}{15} \\ \frac{4}{15} & \frac{5}{15} & - & \frac{3}{15} \\ \frac{4}{15} & \frac{5}{15} & \frac{3}{15} & - \end{pmatrix} \end{array}$$

In the problem instance of size 4 based on order of preference, here all the students are considering the Hockey interest factor of the remaining students.

1	4 3 2
2	3 1 4
3	2 4 1
4	1 2 3

The Satisfactory value Matrix for the Hockey interest factor

$$\text{SVM}= \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} - & \frac{3}{3} & \frac{3}{3} & \frac{6}{3} \\ \frac{3}{3} & - & \frac{6}{3} & \frac{3}{3} \\ \frac{3}{3} & \frac{6}{3} & - & \frac{3}{3} \\ \frac{6}{3} & \frac{3}{3} & \frac{3}{3} & - \end{pmatrix} \end{array}$$

Here we are taking 10% of Hockey interest factor, so that the SVM becomes

$$10\%\ \text{SVM}= \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} - & \frac{3}{30} & \frac{3}{30} & \frac{6}{30} \\ \frac{3}{30} & - & \frac{6}{30} & \frac{3}{30} \\ \frac{3}{30} & \frac{6}{30} & - & \frac{3}{30} \\ \frac{6}{30} & \frac{3}{30} & \frac{3}{30} & - \end{pmatrix} \end{array}$$

In the problem instance of size 4 based on order of preference, here all the students are considering the Volley ball interest factor of the remaining students.

1	2 4 3
2	4 3 1
3	1 2 4
4	2 1 3

The Satisfactory value Matrix for the Volley ball interest factor

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$$\text{SVM}= \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} - & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} \\ \frac{4}{3} & - & \frac{4}{3} & \frac{6}{3} \\ \frac{4}{3} & \frac{4}{3} & - & \frac{2}{3} \\ \frac{4}{3} & \frac{6}{3} & \frac{2}{3} & - \end{pmatrix} \end{array}$$

Here we are taking 10% of Volley ball interest factor, so that the SVM becomes

$$10\ \%\ \text{of SVM}= \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} - & \frac{4}{30} & \frac{4}{30} & \frac{4}{30} \\ \frac{4}{30} & - & \frac{4}{30} & \frac{6}{30} \\ \frac{4}{30} & \frac{4}{30} & - & \frac{2}{30} \\ \frac{4}{30} & \frac{6}{30} & \frac{2}{30} & - \end{pmatrix} \end{array}$$

In the problem instance of size 4 based on order of preference, here all the students are considering the Cricket interest factor of the remaining students.

1	3 4 2
2	1 3 4
3	4 2 1
4	3 1 2

The Satisfactory value Matrix for the Cricket interest factor

$$\text{SVM}= \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} - & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} \\ \frac{4}{3} & - & \frac{4}{3} & \frac{2}{3} \\ \frac{4}{3} & \frac{4}{3} & - & \frac{6}{3} \\ \frac{4}{3} & \frac{2}{3} & \frac{6}{3} & - \end{pmatrix} \end{array}$$

Here we are taking 10% of Cricket interest factor, so that the SVM becomes

$$10\ \%\ \text{of SVM}= \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} - & \frac{4}{30} & \frac{4}{30} & \frac{4}{30} \\ \frac{4}{30} & - & \frac{4}{30} & \frac{2}{30} \\ \frac{4}{30} & \frac{4}{30} & - & \frac{6}{30} \\ \frac{4}{30} & \frac{2}{30} & \frac{6}{30} & - \end{pmatrix} \end{array}$$

The final SVM, ie, 50% of Basket ball interest factor + 20% of Foot ball interest factor +10% of Hockey interest factor +10% of Volley ball interest factor +10% of Cricket interest factor

$$\text{Final SVM}= \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} - & \frac{42}{30} & \frac{39}{30} & \frac{37}{30} \\ \frac{42}{30} & - & \frac{44}{30} & \frac{41}{30} \\ \frac{39}{30} & \frac{44}{30} & - & \frac{37}{30} \\ \frac{37}{30} & \frac{41}{30} & \frac{37}{30} & - \end{pmatrix} \end{array}$$

The resultant matching for the above instance is (1, 4) and (2, 3).This matching is obtained by using the above mentioned five factors. The above result shows that the average satisfactory level of matching (1, 4) is 61.6 % and for (2, 3) is 73.3 %.

CONCLUSION

In this paper we have described Weighted N-factor Satisfactory Roommates problem. This is the extension of our previous work N-factor Satisfactory Roommates problem. This paper explores different factors such as Basket ball-interest, Foot ball interest, Hockey interest, Volley ball interest, Cricket interest with different weightage; based on this interest, we get the preference list. In that list applying our algorithm, that gives the result whatever we need. Matching players in small no of objective criteria may help to reduce misunderstanding between the players in the hostel and help them to get more knowledge about the other games. We consider the overall satisfaction that will help them to make happiness among the roommates. The methodology, we propose here guarantee the maximum satisfaction between the pairs of the players.

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