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Some Results on Acyclic Chromatic Number of Mth Power of Paths

KEYWORDS: Acyclic colouring, acyclic chromatic number, graph power, adjacency matrix.

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ABSTRACT *In this paper we discuss the graph power of paths, and some of their structural properties are derived. The acyclic chromatic number of P_n^m and is derived both analytically and using their adjacency matrices. Also a relation between them is established.*

INTRODUCTION

Throughout this paper we are concerned only with finite, undirected simple graphs. Terms not defined here are used in the sense of Harary [3].

The notion of Graph power was introduced by Skiena [7] in 1990. The kth power G^k of an undirected graph G is another graph that has the same set of vertices, but in which two vertices are adjacent when their distance in G is at most k [2]. Powers of graphs are referred to using terminology similar to that for exponentiation of numbers: G^2 is called the square of G, G^3 is called the cube of G, etc. Since a path of length two between vertices u and v exists for every vertex w such that {u,w} and {w,v} are edges in G, the square of the adjacency matrix of G counts the number of such paths. Similarly, the (u,v)th element of the kth power of the adjacency matrix of G gives the number of paths of length k between vertices u and v. It has been proved that adj(G^k)= $\sum_{i=1}^{k}[adj(G)]^i$, , where adj(G) is the adjacency matrix.

Graph colouring on the square of a graph may be used to assign frequencies to the participants of wireless communication networks so that no two participants interfere with each other at any of their common neighbours, and to find graph drawings with high angular resolution.

PRELIMINARIES

The following basic definitions are taken from [3]. Graph colouring is an assignment of labels traditionally called "colours" to elements of a graph subject to certain constraints. The most common types of colourings are vertex colouring, edge colouring and face colouring. The vertex colouring is proper, if no two adjacent vertices are assigned the same colour. A proper vertex colouring of a graph is acyclic if every cycle uses at least three colours [4]. The acyclic chromatic number of G, denoted by a(G), is the minimum colours required for its acyclic colouring.

The diameter d of a graph is the maximum eccentricity of any vertex in the graph. That is, d it is the greatest distance between any pair of vertices. It has been proved that, if a graph has diameter d then its dth power is the complete graph [9].

1. ACYCLIC COLOURING OF mTH POWER OF AN n-PATH P_n

The mth power of an n-path P_n is a graph P_n^m with the same vertex set as P_n in which two vertices are joined by an edge if their distance in P_n is atmost m.

1.1 Structural properties of mth power of the path P_n for any n

The number of vertices in P_n^m is $p(P_n^m)=n$

The number of edges in P_n^m is $q(P_n^m)=mn-\frac{m(m+1)}{2}$, where $m,n\in N$ and $m<n-1$.

The minimum degree in P_n^m is $\delta(P_n^m)=m$, $n>1$ and $m<n$.

The maximum degree in P_n^m is $\Delta(P_n^m)=n-1$, $\left\lfloor\frac{n}{2}\right\rfloor\leq m<n$.

For m=2, P_n^m is planar for all n.

1.2 Theorem

$a(P_n^{n-1}))=n$, for any $n\in N$.

Proof

The eccentricity $\varepsilon(v)$ of the end vertices of P_n are n-1. That is the diameter, d=n-1. So by the theorem 2.2, for d=n-1, $P_n^{(n-1)}$ is isomorphic to K_n. Hence the theorem.

1.3 Theorem

$a(P_n^m)=m+1$, $n\geq m+1$

Proof

Case: 1 when m=1

Then the theorem is trivially true

Case: 2 when m>1

If m=n-1

by the theorem 2.3, $a(P_n^{n-1})=n$, for any $n\in N$, the result is true.

If m<n-1

Let $V=\{v_1v_2v_3,\ldots v_n\}$ is the vertex set of the given graph P_n^m. Consider the colour class $C=\{c_1,c_2,c_3,\ldots,c_{(m+1)}\}$. Since in P_n^m all the vertices with distance at most m in P_n are joined by an edge, we can find a complete sub graph K_{m+1} with vertices $v_1,v_2v_3,\ldots,v_{m+1}$, which are assigned the colours by the colour class C. Now the remaining n-m-1 vertices starting from $v_{(m+2)},v_{(m+3)},v_{(m+4)},\ldots$ are respectively assigned the colours $c_{-1},c_{-2},c_{-3},\ldots$ cyclically. Now the colouring is minimum as it contains K_{m+1}, minimum m+1 colours required for its

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proper colouring. The colouring is acyclic, because for all $i \neq j$ the subgraph induced by the colour class <ci,cj> satisfies the relation $\varepsilon = v-1$, which is the necessary and sufficient condition for a tree. Hence the theorem.

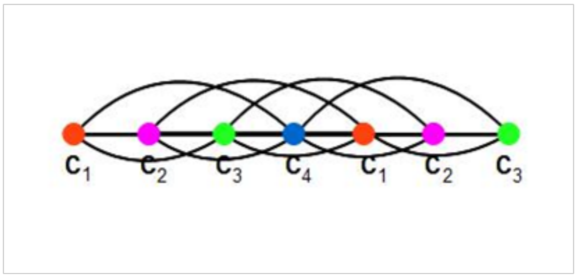

Figure 1: $a(P_7^3) = 4$

2. ADJACENCY MATRIX OF P_n^m

For $m<n$, the adjacency matrix of P_n^m is an $n\times n$ matrix $A=[a_{ij}]$, with $a_{ij} = \begin{cases} 1, \text{for } |i-j| \in (0,m] \\ 0, \text{ else} \end{cases}$

2.1 Theorem

$a(P_n^m) = \min_v d^m(v)+1$, $n \in N$, where $d^m(v)$ denote degree of vertex v in the m^th^power of P_n.

Proof:

For $m \le n-1$, the graph P_n^m is obtained by joining each vertex of P_n to nearby vertices which are at distance of at most m, so that the end vertices have degree m, which is the minimum degree. So by the theorem 1.3, $a(P_n^m) = \delta+1$. Hence in the adjacency matrix of P_n^m, $a(P_n^m) = \min_v d^m(v)+1$, $n \in N$.

Example

The adjacency matrix of P_{12}^5 is the 12×12 matrix $A=[a_{ij}]$,where a_{ij}={(1,for |i-j|∈(0,5] 0,else

$$A = \begin{bmatrix} 0&1&1&1&1&1&0&0&0&0&0&0 \\ 1&0&1&1&1&1&1&0&0&0&0&0 \\ 1&1&0&1&1&1&1&1&0&0&0&0 \\ 1&1&1&0&1&1&1&1&1&0&0&0 \\ 1&1&1&1&0&1&1&1&1&1&0&0 \\ 1&1&1&1&1&0&1&1&1&1&1&0 \\ 0&1&1&1&1&1&0&1&1&1&1&1 \\ 0&0&1&1&1&1&1&0&1&1&1&1 \\ 0&0&0&1&1&1&1&1&0&1&1&1 \\ 0&0&0&0&1&1&1&1&1&0&1&1 \\ 0&0&0&0&0&1&1&1&1&1&0&1 \\ 0&0&0&0&0&0&1&1&1&1&1&0 \end{bmatrix}$$

Now using the theorem 6.1, $a(P_n^m) = \min_v d^m(v)+1$

Therefore, we have $a(P_{12}^5) = \min_v d^5(v)+1 = 5+1 = 6$.

2.2 Corollary

$a(P_n^m)$ =(Number of integers in (0,m]) +1.

Example

In the previous example, $a(P_{12}^5)$ =(Number of integers in(0,5]) +1=5+1=6.

3. RELATION BETWEEN ACYCLIC CHROMATIC NUMBERS OF C_n^m AND P_n^m

3.1 Observations

$a(C_n^m) = 2\, a(P_n^m) - 2$ when $m = \left\lfloor \frac{n}{2} \right\rfloor$ and for even $n \ge 3$

$a(C_n^m) = 2\, a(P_n^m) - 1$ when $m < \left\lfloor \frac{n}{2} \right\rfloor$ and for all $n \ge 3$

CONCLUSIONS

In this paper, we propose some methods to determine the acyclic chromatic number of mth power of an n-path. We defined the acyclic chromatic number of this graphs in terms of adjacency matrices. This makes acyclic colouring of such graphs easy since the adjacency matrices of different graphs is a widely studied area in graph theory. A relation between the acyclic chromatic number of C_n^m and $P_{\left\lfloor \frac{n}{2} \right\rfloor}$ is also obtained.

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