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Variance of Time to Recruitment for a Single Grade Manpower System with Two Thresholds having Different epochs for Decisions and Exits

KEYWORDS: Single grade marketing organization, decision and exit epochs, univariate policy of recruitment with two thresholds, renewal process and variance of the time to recruitment.

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ABSTRACT *In this paper, the problem of time to recruitment is studied for a single grade marketing organization with attrition generated by its policy decisions using a univariate policy of recruitment involving two thresholds. Assuming that the policy decisions and exits occur at different epochs, a stochastic model is constructed and the variance of the time to recruitment is obtained when the inter-policy decision times and inter- exit times form two different renewal processes. The analytical results are numerically illustrated with relevant findings by assuming specific distributions.*

Introduction

Attrition is a common phenomenon in many organizations. This leads to the depletion of manpower. Recruitment on every occasion of depletion of manpower is not advisable since every recruitment involves cost. Hence the cumulative depletion of manpower is permitted till it reaches a level, called the threshold. If the total loss of manpower exceeds this threshold, the activities in the organization will be affected and hence recruitment becomes necessary. In [1] the author has initiated the study on the problem of time to recruitment for a single and multi-grade manpower system using Markovian and renewal theoretic approach. This problem is studied in [2] using shock model approach in reliability theory. In [3] the author has considered a single grade manpower system and obtained system characteristics when the loss of manpower process and inter-decision time process form a correlated pair of renewal sequence by employing a univariate policy of recruitment. The optimum cost of recruitment for a single grade manpower system using several univariate and bivariate policies of recruitment is discussed in [6].The authors in [4], [5], [7], [8], [9], [10], [11], [12],[13],[14] have studied the problem of time to recruitment in single grade manpower system with attrition under different policies of recruitment. In [15] the author has studied this problem by incorporating alertness in the event of cumulative loss of manpower due to attrition crossing the threshold, by considering optional and mandatory thresholds for the cumulative loss of manpower in this manpower system. In all the above cited work, it is assumed that attrition takes place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decision points. This aspect is taken into account for the first time in the present study. In this paper, for a single grade manpower system, a mathematical model is constructed in which it is assumed that attrition due to policy decisions takes place at exit points and there is an optional and a mandatory threshold as control limits for cumulative loss of manpower. A univariate policy of recruitment based on shock model approach is used to determine the variance of time to recruitment when the policy decisions and exits which occur at different epochs, form two different renewal processes. The present work extends the work of Devi and Srinivasan in [16] for the single grade manpower system with two control limits for the cumulative loss of manpower.

Model Description

Consider an organization taking decisions at random epochs in (0, ∞) and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. Let be the continuous random variable representing the amount of depletion of manpower (loss of man hours) caused at the exit point and S_k be the total loss of manpower up to the first k exit points. It is assumed that's are independent and identically distributed random variables with probability density function m(.), distribution function M(.) and mean (α>0). Let U_j be the continuous random variable representing the time between the (j-1)th and jth policy decisions. It is assumed that U_j's are independent and identically distributed random variables with probability density function *f(.)*distribution function F(.) and mean $\frac{1}{\alpha}(\alpha>0)$. Let u_j be the continuous random variable representing the time between the (i-1)th and ith exit times. It is assumed that W_i's are independent and identically distributed random variables with probability density function g(.), probability distribution function G(.) and mean $\frac{1}{\delta}(\delta>0)$. Let $N_e(t)$ be the number of exit points lying in (0,t]. Let Y be the optional threshold level and Z the mandatory threshold level for the cumulative depletion of manpower in the organization with probability density function h(.) and distribution function H(.). Let p be the probability that the organization is not going for recruitment when optional threshold is exceeded by the cumulative loss of manpower. Let q be the probability that every policy decision has exit of personnel. As q=0 corresponds to the case where exits are impossible, it is assumed that q ≠ 0. Let T be the random variable denoting the time to recruitment with probability distribution function L(.), density function l (.), mean E(T) and variance V(T). Let $A^*(.)$ and $\bar{a}(.)$ be the Laplace - Stieltjes transform and the Laplace transform of A(.) and a(.) respectively. The univariate CUM policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the cumulative loss of manpower in the organization exceeds the mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of manpower exceeds the optional threshold.

Main result

Numerical Illustrations:

P(T > t) = P {Total loss of manpower at the exit points in (0, t] does not exceed the optional threshold Y or the total loss of manpower at the exit points in (0, t] exceeds Y but lies below the mandatory level Z and the organization is not making recruitment }

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ie $P(T > t) = P\{ (S_{N_e(t)} \le Y) \cup (Y < S_{N_e(t)} \le Z) \cap (the\ \text{organization is not making recruitment}) \}$

$$P(T > t) = P(S_{N_e(t)} \le Y) + P(Y < S_{N_e(t)} \le Z)p$$

$$P(T > t) = \sum_{k=0}^{\infty} P[N_e(t) = k]\, P(S_k \le Y) + p \sum_{k=0}^{\infty} P[N_e(t) = k]\, P(Y < S_k \le Z)$$

$$\text{P(T > t)} = \sum_{k=0}^{\infty} P[N_e(t) = k]\, P(S_k \le Y) + p \sum_{k=0}^{\infty} P[N_e(t) = k]\, P(S_k > Y)\, P(S_k \le Z) \qquad (1)$$

We now determine the variance of the time to recruitment for different forms of the distribution of the thresholds by assuming $M(x) = 1 - e^{-\alpha x}$, $\text{F}(x) = 1 - e^{-\lambda x}$, $G(x) = 1 - e^{-\delta x}$

Case (i):

$H(y) = 1 - e^{-\theta_1 y}$ and $\text{H(z)} = 1 - e^{-\theta_2 z}$

In this case write $a = E[e^{-\theta_1 X}]$, $b = E[e^{-\theta_2 X}]$.

From (1), we get

$$\text{P(T > t)} = \sum_{k=0}^{\infty} P[N_e(t) = k]\, a^k + p\sum_{k=0}^{\infty} P[N_e(t) = k][1 - a^k]b^k \qquad (2)$$

From Renewal theory [J. Medhi], we have

$P[N_e(t) = k] = G_k(t) - G_{k+1}(t)$ and $G_0(t) = 1$

Substituting the above in (2) & on simplification, we get

$$\text{L(t)} = \bar{a} \sum_{k=1}^{\infty} G_k(t)\, a^{k-1} + p\left\{ \bar{b} \sum_{k=1}^{\infty} G_k(t)\, b^{k-1} - \overline{ab} \sum_{k=1}^{\infty} G_k(t)\, (ab)^{k-1} \right\} \qquad (3)$$

where $\bar{a} = 1 - a, \bar{b} = 1 - b, \overline{ab} = 1 - ab$. (4)

From (3), we get

$$\bar{l}(s) = \frac{\bar{a}\,\bar{g}(s)}{1 - a\,\bar{g}(s)} + p\left\{ \frac{\bar{b}\,\bar{g}(s)}{1 - b\,\bar{g}(s)} - \frac{\overline{ab}\,\bar{g}(s)}{1 - ab\,\bar{g}(s)} \right. \qquad (5)$$

It can be shown that the distribution function G(.) of the inter-exit times satisfy the relation

$$\text{G(x)} = \text{q} \sum_{n=1}^{\infty} (1 - \text{q})^{n-1} F_n(\text{x}).$$

Therefore $$\bar{g}(s) = \frac{q\bar{f}(s)}{1 - (1 - q)\bar{f}(s)} \qquad (6)$$

Substituting (6) in (5) & on simplification, we get

$$\bar{l}(s) = \frac{\bar{a}q\bar{f}(s)}{1 + (\bar{a}q - 1)\bar{f}(s)} + p\left\{ \frac{\bar{b}q\bar{f}(s)}{1 + (\bar{b}q - 1)\bar{f}(s)} - \frac{\overline{ab}q\bar{f}(s)}{1 + (\overline{ab}q - 1)\bar{f}(s)} \right\} \qquad (7)$$

From (7), it can be shown that

$$E(T) = \frac{1}{\lambda q}\left\{ \frac{1}{\bar{a}} + \frac{p}{\bar{b}} - \frac{p}{\overline{ab}} \right\},$$

$$E(T^2) = \frac{2}{(\lambda q)^2}\left\{ \frac{1}{(\bar{a})^2} + \frac{p}{(\bar{b})^2} - \frac{p}{(\overline{ab})^2} \right\} \qquad (8)$$

and $V(T) = E(T^2) - [E(T)]^2$ (9)

where $\bar{a}, \bar{b}, \overline{ab}$ are given by (4).

Eqns.(8) & (9) give the mean and variance of the time to recruitment for case(i).

Case (ii):

$H(y) = [1 - e^{-\theta_1 y}]^2$ and $H(z) = [1 - e^{-\theta_2 z}]^2$

which are the extended exponential

distributions with scale parameter θ and shape parameter 2.

In this case, it can be shown that

$$L(t) = 2\bar{a}\sum_{k=1}^{\infty} G_k(t)\, a^{k-1} - \bar{b}\sum_{k=1}^{\infty} G_k(t)\, b^{k-1} + p\Big\{2\bar{a}_1\sum_{k=1}^{\infty} G_k(t)\, a_1^{k-1} - \bar{b}_1\sum_{k=1}^{\infty} G_k(t)\, b_1^{k-1} - 4\,\overline{aa_1}\sum_{k=1}^{\infty} G_k(t)\, aa_1^{k-1} + 2\overline{ab_1}\sum_{k=1}^{\infty} G_k(t)\, ab_1^{k-1} + 2\overline{a_1 b}\sum_{k=1}^{\infty} G_k(t)\, a_1 b^{\,k-1} - \overline{bb_1}\sum_{k=1}^{\infty} G_k(t)\, bb_1^{k-1}\Big\} \quad (10)$$

where $a = a(\theta_1) = E[e^{-\theta_1 X}]$, $b = a(2\theta_1)$,

$a_1 = a(\theta_2)$, $b_1 = a(2\theta_2)$ and $\bar{a} = 1 - a$,

$\bar{b} = 1 - b, \bar{a}_1 = 1 - a_1, \bar{b}_1 = 1 - b_1, \overline{aa_1} = 1 - aa_1,$

$\overline{ab_1} = 1 - ab_1, \overline{a_1 b} = 1 - a_1 b, \overline{bb_1} = 1 - bb_1$ (11)

$$E(T) = \frac{1}{\lambda q}\left\{\frac{2}{\bar{a}} - \frac{1}{\bar{b}} + \frac{2p}{\bar{a}_1} - \frac{p}{\bar{b}_1} - \frac{4p}{\overline{aa_1}} + \frac{2p}{\overline{ab_1}} + \frac{2p}{\overline{a_1 b}} - \frac{p}{\overline{bb_1}}\right\}$$

$$E(T^2) = \frac{2}{(\lambda q)^2}\left\{\frac{2}{(\bar{a})^2} - \frac{1}{(\bar{b})^2} + \frac{2p}{(\bar{a}_1)^2} - \frac{p}{(\bar{b}_1)^2} - \frac{4p}{(\overline{aa_1})^2} + \frac{2p}{(\overline{ab_1})^2} + \frac{2p}{(\overline{a_1 b})^2} - \frac{p}{(\overline{bb_1})^2}\right\} \quad (12)$$

and $V(T) = E(T^2) - [E(T)]^2$ (13)

where $\bar{a}$, $\bar{b}$, $\bar{a}_1$, $\bar{b}_1$, $\overline{aa_1}$, $\overline{ab_1}$, $\overline{a_1 b}$, $\overline{bb_1}$ *are* given by (11).

Eqns. (12) & (13) give the mean and variance of the time to recruitment for case (ii).

Case (iii): $H(y) = p_1 e^{-(\theta_3+\mu_1)y} + q_1 e^{-\theta_4 y}$,

where $p_1 = \dfrac{\theta_3 - \theta_4}{\mu_1 + \theta_3 - \theta_4}, q_1 = 1 - p_1$ and

$H(z) = p_2 e^{-(\theta_5+\mu_2)z} + q_2 e^{-\theta_6 z}$, where

$p_2 = \dfrac{\theta_5 - \theta_6}{\mu_2 + \theta_5 - \theta_6}, q_2 = 1 - p_2$ which are the distribution functions with SCBZ property.

In this case, it can be shown that

$$L(t) = p_1\bar{a}\sum_{k=1}^{\infty} G_k(t)\, a^{k-1} + q_1\bar{b}\sum_{k=1}^{\infty} G_k(t)\, b^{k-1} + p\Big\{p_2\bar{a}_1\sum_{k=1}^{\infty} G_k(t)\, a_1^{k-1} + q_2\bar{b}_1\sum_{k=1}^{\infty} G_k(t)\, b_1^{k-1} - p_1 p_2\,\overline{aa_1}\sum_{k=1}^{\infty} G_k(t)\, aa_1^{k-1} - p_1 q_2\,\overline{ab_1}\sum_{k=1}^{\infty} G_k(t)\, ab_1^{k-1} - p_2 q_1\,\overline{a_1 b}\sum_{k=1}^{\infty} G_k(t)\, a_1 b^{\,k-1} - q_1 q_2\,\overline{bb_1}\sum_{k=1}^{\infty} G_k(t)\, bb_1^{k-1}\Big\} \quad (14)$$

where $a = a(\theta_3 + \mu_1) = E[e^{-(\theta_3+\mu_1)X}]$,

$b = b(\theta_4) = E(e^{-\theta_4 x})$, $a_1 = a(\theta_5 + \mu_2)$,

$b_1 = b(\theta_6)$ and

$\bar{a} = 1 - a, \bar{b} = 1 - b, \bar{a}_1 = 1 - a_1, \bar{b}_1 = 1 - b_1, \overline{aa_1} = 1 - aa_1,$

$\overline{ab_1} = 1 - ab_1, \overline{a_1 b} = 1 - a_1 b, \overline{bb_1} = 1 - bb_1$ (15)

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$$E(T)=\frac{1}{\lambda q}\left\{\frac{p_1}{\bar{a}}+\frac{q_1}{\bar{b}}+\frac{pp_2}{\bar{a}_1}+\frac{pq_2}{\bar{b}_1}-\frac{pp_1p_2}{\overline{aa_1}}-\frac{pp_1q_2}{\overline{ab_1}}-\frac{pp_2q_2}{\overline{a_1b}}-\frac{pq_1q_2}{\overline{bb_1}}\right\},$$

$$E(T^2)=\frac{2}{(\lambda q)^2}$$

$$\left\{\frac{p_1}{(\bar{a})^2}+\frac{q_1}{(\bar{b})^2}+\frac{pp_2}{(\bar{a}_1)^2}+\frac{pq_2}{(\bar{b}_1)^2}-\frac{pp_1p_2}{(\overline{aa_1})^2}-\frac{pp_1q_2}{(\overline{ab_1})^2}-\frac{pp_2q_1}{(\overline{a_1b})^2}-\frac{pq_1q_2}{(\overline{bb_1})^2}\right\}$$

(16)

$$V(T)=E(T^2)-[E(T)]^2 \qquad (17)$$

where $\bar{a}$, $\bar{b}$, $\bar{a}_1$, $\bar{b}_1$, $\overline{aa_1}$, $\overline{a_1b}$, $\overline{ab_1}$, $\overline{bb_1}$ *are* given by (15).

Note:

(i)When p=0, our results for cases (i), (ii) & (iii) agree with results in [16] for the manpower system having only one threshold which is the mandatory threshold.

(ii) When q=1, our results for cases (i),(ii) & (iii) agree with results in [15] for the manpower system with two thresholds having only the decision epochs.

Numerical Illustrations:

The mean and variance of time to recruitment for the cases(i),(ii) and (iii) are numerically illustrated by varying the three nodal parameters λ, and p one at a time. The effect of the nodal parameters on the mean and variance is shown in the following table.

Effect of nodal parameters λ, a and P on performance measure

$(\theta_1 = 0.06,\ \theta_2 = 0.003,\ q = 0.4;\ \theta_3 = 0.008,\ \theta_4 = 0.012, \theta_5 = 0.0042,\ \theta_6 = 0.009,\ \mu_1 = 0.012,\ \mu_2 = 0.0092)$

Table 1:

λ	α	p	Case(i)	
			E(T)	V(T)
0.1	0.05	0.5	244.7823	1.3744×10^5
0.125	0.05	0.5	195.8258	0.8796×10^5
0.1667	0.05	0.5	146.8694	0.4948×10^5
0.25	0.05	0.5	97.9129	0.2199×10^5
0.2	0.05	0.5	122.3911	0.3436×10^5
0.2	0.0667	0.5	158.9358	0.5976×10^5
0.2	0.1	0.5	232.0216	1.3156×10^5
0.2	0.2	0.5	451.2715	5.1499×10^5
0.2	0.05	0.2	62.7065	1.6434×10^4
0.2	0.05	0.3	89.2330	2.5280×10^4
0.2	0.05	0.4	102.4962	2.9176×10^4
0.2	0.05	0.5	122.3911	3.4359×10^4

Table 2:

λ	α	p	Case(ii)	
			E(T)	V(T)
0.1	0.05	0.5	505.3928	1.4024×10^5
0.125	0.05	0.5	404.3142	0.8976×10^5
0.1667	0.05	0.5	303.2357	0.5049×10^5
0.25	0.05	0.5	202.1571	0.2244×10^5
0.2	0.05	0.5	252.6964	0.3506×10^5
0.2	0.0667	0.5	326.4827	0.6372×10^5
0.2	0.1	0.5	474.0608	1.4641×10^5
0.2	0.2	0.5	916.8076	5.9739×10^5
0.2	0.05	0.2	128.5403	2.4304×10^4
0.2	0.05	0.3	183.7208	3.2891×10^4
0.2	0.05	0.4	211.3110	3.4901×10^4
0.2	0.05	0.5	252.6964	3.5061×10^4

Table 3:

λ	α	p	Case(iii)	
			E(T)	V(T)
0.1	0.05	0.5	206.6691	3.1147×10^4
0.125	0.05	0.5	165.3353	1.9934×10^4
0.1667	0.05	0.5	124.0015	1.1213×10^4
0.25	0.05	0.5	82.6676	0.4984×10^4
0.2	0.05	0.5	103.3346	0.7787×10^4
0.2	0.0667	0.5	133.1239	1.2725×10^4
0.2	0.1	0.5	192.6751	2.6205×10^4
0.2	0.2	0.5	371.2709	9.5486×10^4
0.2	0.05	0.2	86.3338	6.2918×10^3

0.2	0.05	0.3	93.8897	7.0276×10^3
0.2	0.05	0.4	97.6676	7.3527×10^3
0.2	0.05	0.5	103.3346	7.7869×10^3

Findings:

From the above table, the following observations are presented which agree with reality.

1. When λ increases and keeping all the other parameters fixed, the mean and variance of time to recruitment

decreases for all the three cases. Infact, increase in λ implies that decisions are taken frequently on the average and consequently, the time to recruitment is shortened.

2. When α increases and keeping all the other parameters fixed, the mean and variance of time to recruitment

increases for all the three cases. Infact, decrease in α increases the loss of manpower on the average which inturn prepone the time to recruitment.

3. As p increases, the mean and variance of time to recruitment increases for all the three cases when the other parameters are fixed.

Conclusion:

The models discussed in this paper are found to be more realistic and new in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs and (ii) associating a probability for any decision to have exit points. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

REFERENCE [1] Bartholomew, D.J (1982), The Stochastic | model for social processes, 3rd ed., John | Wiley and Sons, New York | [2] Sathyamoorthi, R & Elangovan, R (1998), | Shock model approach to determine the | expected time to recruitment, Journal of | Decision and Mathematika Sciences, | 3(1-3), 67-78. | [3] Mariappan, P (2002), Study of some | manpower models and their analysis, | Ph.D Thesis, Bharathidasan University, | Tiruchirappalli. | [4] Parthasarathy. S (2003), On some | stochastic models for manpower planning | using SCBZ property, Ph.D Thesis, | Annamalai University. | [5] Sathiyamoothi. R & Parthasarathi. S | (2003), On the expected time to | Recruitment when threshold distribution | has SCBZ property, International Journal | of Management and systems, 19(3), 233- | 240. | [6] Saavithri, V (2003), Cost analysis on | some policies of recruitment in manpower | models, Ph.D Thesis, Bharathidasan | University, Tiruchirappalli. | [7] Kasturi. K (2007), Expected time to | recruitment for correlated inter-decision | time of exits when threshold distribution | has SCBZ property, Doctoral diss., | Bharathidasan University, | Tiruchirappalli. | [8] Parthasarathy. S & Vinoth. R (2009), | Determination of expected time to | recruitment in manpower planning, | Recent Research in Science and | Technolgy,1(3), 147-150. | [9] Parthasarathy. S & Vinoth. R (2009), | Determination of expected time to | recruitment in manpower planning, | Recent Research in Science and | Technology,1(3),147-150. | [10] Muthaiyan. A & Sathiyamoorthi. R | (2010), Stochastic model using | geometric process for interarrival times | between wastages, Acta Ciencia Indica, | 36 M(4),479-486. | [11] Uma. K.P (2010), A Study on manpower | models with univariate and bivariat | policy of recruitment, Doctoral diss., | Avinashilingam University for Women, | Coimbatore. | [12] Srinivasan . A , Mariappan. P & | Ishwarya. G (2011), Mean time to | recruitment in a single graded | manpower system with exponentiated | exponential threshold, Antarctica | Journal of Mathematics,8(3), 213- 218. | | | [13] Elangovan. R, Sathiyamoorthi. R & | Susiganeshkumar. E (2011), Estimation | of expected time to recruitment under two | sources of depletion manpowerProceeding | of the International Conference on | Stochastic Modelling and Simulation, | Allied Publishers Pvt. Ltd., 276-283. | [14] Sridharan. J, Saivarajan. A & Srinivasan. | A (2014), Expected time to recruitment in | a single graded manpower system with | two thresholds, Journal of Advances in | Mathematics, 7(1), 1147-1157. | [15] Esther Clara, J.B (2012), Contributions | to the study on some stochastic models in | manpower planning, Ph.D Thesis, | Bharathidasan University, Tiruchirappalli. | [16] Devi. A & Srinivasan. A (2014), | Variance of time to recruitment for a | Single grade manpower system with | different epochs for decisions and exits, | International Journal of Research in | Mathematics &Computation, Volume 2, | Issue 1, January- June, pp. 23-27. |