RESEARCH PAPER | PHYSICS | Volume : 5 | Issue : 1 | Jan 2015 | ISSN - 2249-555X

Modification of classical evolutionary equations of soliton

KEYWORDS soliton, KdV equation, Kerr effect.

Dr.Brijesh.N.chawda	Harshita chawda
Professor , Department of Humanities and sciences, Jayaprakash Narayan college of Engg, Mahaboobnagar, Andhra Pradesh	Assistant Professor, Diploma, Jayaprakash Narayan college of Engg, Mahaboobnagar, Andhra Pradesh, India.

ABSTRACT *A wave is represented by a wave equation mathematically. As matter wave is represented by Schrodinger wave equation which may be time dependent (or) time independent. Similarly, non linear Schrodinger equation (NLSE) is used to describe the propagation of solitons in optical fiber.*

1.1.1 INTRODUCTION:

The soliton wave concept was suggested by John Scott-Russell [7] which can travel rapidly and unattenuated over very long distance even thousands of kilometers maintaining its shape and size.

In 1834, John Scott Russell [7] describes his wave of translation. The discovery is described here in Russell's own words "I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed."

It was my first chance to interview with that singular and beautiful phenomenon which I have called the Wave of Translation" [7].

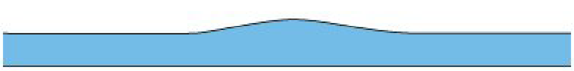

Fig: 1: Soliton
Sources: Wikipedia

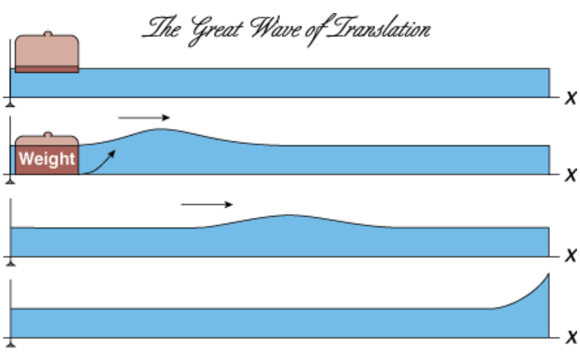

Fig: 2: J. Scott Russell experimented in the 30-foot tank which he built in his back garden in 1834:

Sources: Report on wave

The sight of this wave inspired J. Scott Russell to further investigate this "great wave of translation". He set up experiments where he would drop weights at one end of a long, but shallow water channel.

From this, he made two key discoveries:

1. The existence of a solitary wave, i.e, a long, shallow water wave of permanent form.
2. For these waves, Russell showed that the speed of the wave, v, was given by

$$v = \sqrt{g(h + ç)}$$

τ is the amplitude of the wave measured from the plane of the water, h is the depth of the channel and g is the measure of gravity (η/h ‹ 1 for Russell's experiments). Russell noted that there was not an appropriate mathematical theory which described his results.

The existence of solitary waves was verified mathematically, When Korteweg and de Vries derived an equation (the so-called KdV equation) which describes shallow water waves [5].

1.1.2 Origin of Korteweg deVries equation:

In the development of soliton concept, paper of Korteweg and deVries set a very important milestone. Debye predicted equipartition of energy among 64 modes but the behavior of KdV equation showed the exchange of energy as per Fermi, Pasta and Ulam Observation [3].

The Korteweg deVries equation is given by

$$u_t + 6u\,_x + u_{xxx} = 0,\ kdv \tag{2.13}$$

And modified Korteweg deVries equation is given by

$$v_t - 6v^2 v_x + v_{xxx} = 0,\ mkdv \tag{2.14}$$

Consider the equation:

$$u(x,t) = -(v^2 + v_x) \tag{2.15}$$

Volume : 5 | Issue : 1 | Jan 2015 | ISSN - 2249-555X

Substituting 2.15 in 2.13 and rewriting we get

$$-\left(2v+\frac{\partial}{\partial x}\right)(v_t-6v^2v_x+v_{xxx})=0. \tag{2.16}$$

This transformation was discovered by Miura [6] and is called the Miura transform.

By using inverse scattering transform (IST) method [1] solving Korteweg and deVries equation [5] is possible because of relation between KdV equation and time independent Schrödinger equation.

1.1.3: Results and Discussion:

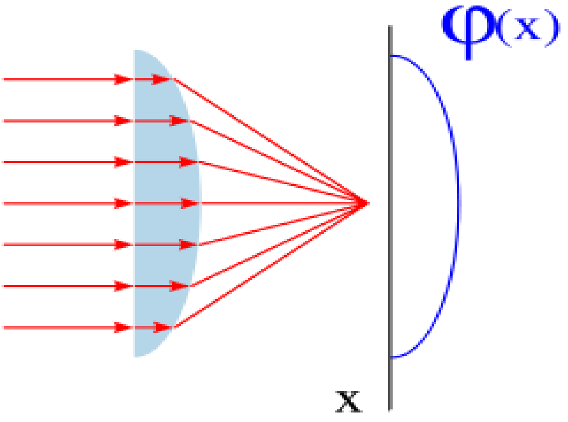

Fig. 3 : Working of lens

Consider a convex lens of length L(x), the optical held focused suffer a p$\varphi(x)$ange which is function of space represented with

It is given by $\ddot{o}(x) = k_0 \hbar \ (x)$

Where Ko and N are constants.

Thus for a focusing effect just introduce a phase change without changing the width. Spatial soliton follow Kerr effect principle [2] introduces

The relationship between intensity and electric field is give by

$$I=\left|\frac{E}{2n}\right|$$

Here held E can be expresses as

$$E(x,z,t) = A_m a(x,z)e^{i(konz-tw)}$$

Am is the amplitude.

Solving Helmholtz equation & substituting equation of electric field.

Weget

$$\frac{1}{2k_0 n}\frac{\partial^2 a}{\partial x^2} + i\frac{\partial a}{\partial z} + \frac{k_0 n_2 |A_m|^2}{2\varsigma_0}|a|^2 a = 0$$

Called as nonlinear Schrödinger equation.

The first term of NLSE equation gives the GVD effects in which dispersion tends to broaden pulse. The second term gives non-linear factor which shows the relationship between refractive index of the fiber and intensity of light. This leads to broadening of frequency spectrum of pulse through self phase modulation (SPM).

The third term gives the alternation or amplification in other words the loss or gain of energy.

CONCLUSIONS:

Soliton are narrow and high intensity pulses which can retain their shape by compensating the effects of SPM and GVD Mechanisms and are represented by as nonlinear Schrödinger equation.

Acknowledgments:

We are grateful to the chairman Shri K.S Ravi Kumar and principal of JPNCE institution for all the possible support extended to my work.

REFERENCE

1) Ablowitz M.J,Kaup, D.J., Newell, A.C. and H. Segur (1974) "The inverse scattering transform – Fourier analysis for nonlinear problems", Stud. Appl. Math, 53, 249. | 2) Biswasand.A and S. Konar (2005) "Soliton-solitons interaction with kerr law non-linearity," Journal of Electromagnetic Waves and Applications, Vol. 19, No. 11, 1443–1453. | 3) Fermi.E, Pasta, J. and Ulam, S(1955) "Studies of nonlinear problems", Los Alamos Report, LA1940,; reproduced in Nonlinear Wave Motion, A.C. Newell, ed., American Mathematical Society, Providence, RI. | 4) Hasegawa. A and F. Tappert (1973) "Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers.I.Anamalous dispersion. Appl. Phys. Lett. Volume 23, Issue 3, pp. 142-144. | 5) Gardner C.S et al (1974) "The Korteweg-deVries equation and generalizations. VI. Methods for exact solution", Comm. Pure Appl. Math, 27, 97. | 6) Gardner.C.S, Greene, J.M., Kruskal, M.D. and Miura, R.M., (1967) "Method For Solving The Korteweg deVries Equation", Phys. Rev. Lett., 19, 1095. | 7) Scott Russell. J (1844) "Report on waves", fourteenth meeting of the British Association for the Advancement of Science. |