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Fuzzy Inventory Model for Deteriorating Item by Using Signed Distance Method in which Inventory Parameters are Treated as Pfn

KEYWORDS: Inventory, Deterioration, Fuzzy model, Shortages, Pentagonal Fuzzy Number [PFN], Signeddistance method.

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ABSTRACT *While the development and application of fuzzy inventory control models of deteriorating item is one of the main concerns of researchers and practitioners, most studies done in this field till trapezoidal fuzzy numbers by using different method. In this paper at first we study researches done on PFN by signed distance method to defuzzify the total cost function .In which demand increase with time and shortage are fully backlogged. In previous paper we have already discussed about PFN by using graded mean representation method to defuzzify.Then by, comparison of developed models some theoretical and practical results are derived and various directions are suggested for future research.*

I: Introduction:

The fuzzy set theory is developed for solving the phenomenon of fuzziness prevalent in the real world. Different types of fuzzy sets are defined in order to clear the vagueness of the existing problems. Membership function of these sets, which have the form A: R → [0, 1] and it has a quantitative meaning and viewed as fuzzy numbers. A fuzzy number [6], is a quantity whose values are imprecise, rather than exact as in the case with single-valued function. So far, fuzzy numbers like triangular fuzzy numbers [3], trapezoidal fuzzy numbers [2] [8], and pentagonal fuzzy numbers are introduced with its membership functions.

Deteriorating items are common in our daily life; Accordingto the definition, deteriorating items can be classifiedinto two categories. The first category refers to theitems that become decayed, damaged, evaporative, orexpired through time, like meat, vegetables, fruit, medicine,flowers, film and so on; the other category refers tothe items that lose part or total value through time becauseof new technology or the introduction of alternatives, like computer chips, mobile phones, fashion andseasonal goods, and so on. Both of the two categorieshave the characteristic of short life cycle.

This paper is organized as follows: In section II, we give some necessary definitions. In section III, we describe in brief the notations and assumptions used in the developed model. In section IV we present the mathematical model and in section V we present the corresponding Fuzzy model & solution procedure. In section VI, a numerical example is given to illustrate the model. Finally, conclusions are given in section VII.

II. Definitions and Preliminaries

[a]:-In order to treat fuzzy inventory model by using graded mean representation method to defuzzify, we need the following definitions

Definition 2.1: (By Pu and Liu [11])A fuzzy set ã on R= (-∞, ∞) is called a fuzzy point if its membership function is

$$\mu_{\tilde{a}}(x) = \begin{Bmatrix} 1, x = a \\ 0, x \neq a \end{Bmatrix} \qquad (1)$$

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Where the point a is called the support of fuzzy set $\tilde{a}$.

Definition 2.2-A fuzzy set $[a_\alpha, b_\alpha]$ where $0 \le \alpha \le 1$ and $a < b$ defined on R, is called a level of a fuzzy interval if its membership function is

$$\mu_{[a_\alpha,b_\alpha]}(x) = \begin{Bmatrix} \alpha, a \le x \le b \\ 0, otherwise \end{Bmatrix} \quad (2)$$

Definition 2.3 A fuzzy number $\tilde{A} = (\mathrm{a,b,c})$ where a$< b < c$ and defined on R, is called a triangular fuzzy number [27] if its membership function is

$$\mu_A = \begin{Bmatrix} \frac{x-a}{b-a}, a \le x \le b \\ \frac{c-x}{c-b}, b \le x \le c \\ 0, \ otherwise \end{Bmatrix} \quad (3)$$

When $a = b = c$, we have fuzzy point $(c, c, c) = \tilde{c}$ The family of all triangular fuzzy numbers on R is denoted as

$$F_N = \{(a, b, c), a < b < c, \forall a, b, c \in R\}$$

The α -cut of $\tilde{A} = (a, b, c) \in F_N, 0 \le \alpha \le 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$

Where $A_L(\alpha) = a + (b - a)\alpha$ and $A_R(\alpha) = c - (c - b)\alpha$ are the left and right endpoints of $A(\alpha)$.

Definition 2.4: A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is represented [18]with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(x) = \begin{Bmatrix} L(x) = \frac{x-a}{b-a}, a \le x \le b \\ 1 \qquad , b \le x \le c \\ R(x) = \frac{d-x}{d-c}, c \le x \le d \\ 0 \qquad , \ otherwise \end{Bmatrix} \quad (4)$$

The α -cut of $\tilde{A} = (a, b, c, d), 0 \le \alpha \le 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$

Where $A_L(\alpha) = a + (b - a)\alpha$ and $A_R(\alpha) = d - (d - c)\alpha$ are the left and right endpoints of $A(\alpha)$.

Definition 2.5: A pentagonal fuzzy number(PFN)[9] $\tilde{A} = (a, b, c, d, e)$ is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(x) = \begin{Bmatrix} L_1(x) = \dfrac{x-a}{b-a}, a \le x \le b \\ L_2(x) = \dfrac{x-b}{c-b}, b \le x \le c \\ 1 \qquad , x = c \\ R_1(x) = \dfrac{d-x}{d-c}, c \le x \le d \\ R_2(x) = \dfrac{e-x}{e-d}, d \le x \le e \\ 0 \qquad , \ otherwise \end{Bmatrix} \quad (5)$$

Theα -cut of $\tilde{A} = (a, b, c, d), 0 \le \alpha \le 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$

Where $A_L(\alpha) = a + (b - a)\alpha$ and $A_R(\alpha) = d - (d - c)\alpha$ are the left and right endpoints of$A(\alpha)$.

Definition 2.5: A pentagonal fuzzy number(PFN)[9] $\tilde{A} = (a, b, c, d, e)$ is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} L_1(x) = \dfrac{x-a}{b-a}, a \le x \le b \\ L_2(x) = \dfrac{x-b}{c-b}, b \le x \le c \\ 1 \qquad , x = c \\ R_1(x) = \dfrac{d-x}{d-c}, c \le x \le d \\ R_2(x) = \dfrac{e-x}{e-d}, d \le x \le e \\ 0 \qquad , \ otherwise \end{cases} \qquad (5)$$

Theα -cut of $\tilde{A} = (a, b, c, d, e), 0 \le \alpha \le 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$

Where$A_{L_1}(\alpha) = a + (b - a)\alpha = {L_1}^{-1}(\alpha)$, $A_{L_2}(\alpha) = b + (c - b)\alpha = {L_2}^{-1}(\alpha)$ and

$A_{R_1}(\alpha) = d - (d - c)\alpha = {R_1}^{-1}(\alpha)$,$A_{R_2}(\alpha) = e - (e - d)\alpha = {R_2}^{-1}(\alpha)$

So

$$L^{-1}(\alpha) = \frac{{L_1}^{-1}(\alpha) + {L_2}^{-1}(\alpha)}{2} = \frac{a + (b-a)\alpha + b + (c-b)\alpha}{2}$$
$$= \frac{a + b + (b - a + c - b)\alpha}{2} = \frac{a + b + (c-a)\alpha}{2}$$
$$= \frac{d + e - (d - c + e - d)\alpha}{2} = \frac{d + e - (e-c)\alpha}{2}$$

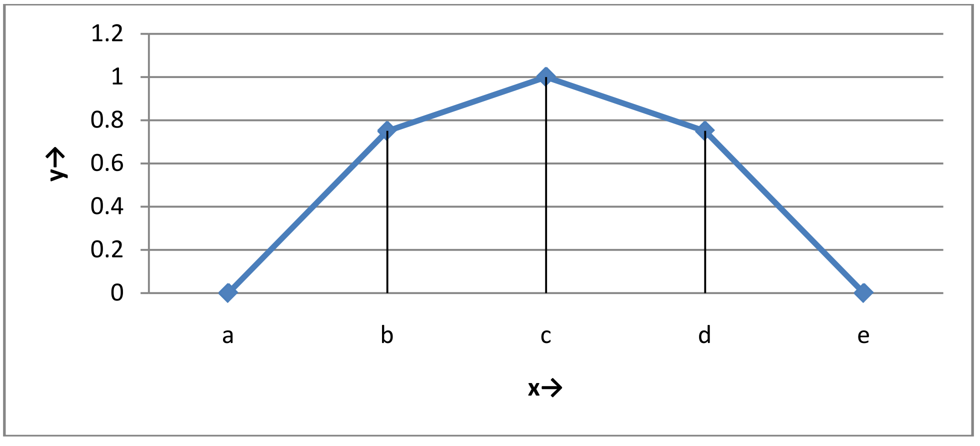

Fig (1): Graphical representation of Pentagonal Fuzzy Number (PFN)

Definition 2.6: If $\tilde{A} = (a, b, c, d, e)$ is a pentagonal fuzzy number then the signed distance method of $\tilde{A}$is defined as

$$d(\tilde{A}, \tilde{0}) = \int_0^1 d([A_L(\alpha)_\alpha, A_R(\alpha)_\alpha], \tilde{0}) = \frac{1}{8}(a + 2b + 2c + 2d + e) \qquad (6)$$

[b] Conditions on Pentagonal Fuzzy Number[PFN]:-

A Pentagonal Fuzzy Number $P(\tilde{A})$ should satisfy the following conditions[9];

1. $\mu_{\tilde{A}}(x)$is a continuous function in the interval [0,1].
2. $\mu_{\tilde{A}}(x)$is strictly increasing and continuous function on [a, b] and [b, c].
3. $\mu_{\tilde{A}}(x)$is strictly decreasing and continuous function on [c, d] and [d, e].

III. Notationsand Assumptions:

The mathematical model in this paper is developed on the basis of the following assumptions and notations[15].

3.1 Notations

1. D (t) is the demand rate at any time t per unit time.
2. A is the ordering cost per order.
3. $\theta$is the deterioration rate,$0 < \theta < 1$.
4. T is the length of the Cycle.
5. Q is the ordering Quantity per unit.
6. h is the holding cost per unit per unit time
7. S is the shortage Cost per unit time.
8. C is the unit Cost per unit time.
9. $K(t_1, T)$is the total inventory cost per unit time.
10. $\tilde{D}$is the fuzzy demand.
11. $\tilde{\theta}$is the fuzzy deterioration rate.
12. $\tilde{h}$is the fuzzy holding cost per unit per unit time.
13. $\tilde{S}$is the fuzzy shortage Cost per unit time.
14. $\tilde{C}$is the fuzzy unit Cost per unit time.
15. $\tilde{K}(t_1, T)$isthe total fuzzy inventory cost per unit time.
16. $K_{dS}(t_1, T)$ is the defuzzify value of $\tilde{K}(t_1, T)$ by applying Signed Distance Method.

3.2 Assumptions

1. Demand$D(t) = a(1 + bt)$ is assumed to be an increasing function of time i.e. where a and $b$are positive constants and $a > 0, 0 < b < 1$.
2. Replenishment is instantaneous and lead time is zero.
3. Shortages are allowed and fully backlogged.

IV. MATHEMATICAL MODEL

Let Q be the total amount of inventory purchased or produced at the beginning of each period and after fulfilling backorders. Due to reasons of market demand and deterioration of the items, the inventory level gradually diminishes during the period $[0, t_1]$ andultimately falls to zero at $t = t_1$.The period $[t_1, T]$ is the period of shortages, which are fully backlogged. Let $I[t]$ be the on-hand inventory level at any time t, which is governed by the following two differential equations:

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Crisp Model:

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t), \quad 0 \le t \le t_1 \qquad (4.1)$$

with $I(0) = Q$, $I(t_1) = 0$

$$\frac{dI(t)}{dt} = -D(t), \quad t_1 \le t \le T \qquad (4.2)$$

, with $I(t_1) = 0$

The solution of equation (4.1) and (4.2) is given by

$$I(t) = Qe^{-\theta t} + \left(\frac{a}{\theta} - \frac{ab}{\theta^2}\right)e^{-\theta t} + \frac{ab}{\theta^2} - \frac{a}{\theta}(1 + bt) \qquad (4.3)$$

And

$$I(t) = a(t_1 - T) + \frac{ab}{2}({t_1}^2 - T^2) \qquad (4.4)$$

By using $I(t_1) = 0$,put t= t_1 in equation (4.3),we get

$$Q = \left\{\frac{a}{\theta}(1 + bt_1) - \frac{ab}{\theta^2}\right\}e^{\theta t_1} - \left(\frac{a}{\theta} - \frac{ab}{\theta^2}\right) \qquad (4.5)$$

Now (4.3) becomes

$$I(t) = a\left\{(t_1 - t) + \frac{\theta}{2}(t_1 - t)^2\right\} + ab\left\{t_1(t_1 - t) - \frac{(t_1 - t)^2}{2} + \frac{\theta}{2}t_1(t_1 - t)^2 - \frac{\theta}{6}(t_1 - t)^3\right\} \qquad (4.6)$$

(Neglecting higher powers ofθ).

Total average no. of holding units (I_H) during period [0, T] is given by

$$I_H = \int_0^{t_1} I(t)\,dt = a\left\{\frac{{t_1}^2}{2} + \frac{\theta}{6}{t_1}^3\right\} + ab\left\{\frac{{t_1}^3}{3} + \frac{\theta}{8}{t_1}^4\right\} \qquad (4.7)$$

Total no. of deteriorated units (I_D) during period [0, T] is given by

$I_D = Q$ −Total demand= $Q - \int_0^{t_1} a(1 + bt)\,dt$

$$I_D = \frac{a\theta {t_1}^2}{2} + \frac{ab\theta}{3}{t_1}^3 \qquad (4.8)$$

Total average no. of shortage units (I_S) during period [0, T] is given by

$$I_S = -\int_{t_1}^{T} I(t)\,dt = \frac{a}{2}(t_1 - T)^2 - \frac{ab}{2}\left\{{t_1}^2 T - \frac{T^3}{3} - \frac{2}{3}{t_1}^3\right\} \qquad (4.9)$$

Total cost of the system per unit time is given by

$$K(t_1, T) = \frac{1}{T}[A + hI_H + CI_D + SI_S]$$

$$K(t_1, T) = \frac{1}{T}\left[A + ha\left\{\frac{{t_1}^2}{2} + \frac{\theta}{6}{t_1}^3\right\} + hab\left\{\frac{{t_1}^3}{3} + \frac{\theta}{8}{t_1}^4\right\} + C\left\{\frac{a\theta {t_1}^2}{2} + \frac{ab\theta}{3}{t_1}^3\right\} + S\left\{\frac{a}{2}(t_1 - T)^2 - \frac{ab}{2}\left({t_1}^2 T - \frac{T^3}{3} - \frac{2}{3}{t_1}^3\right)\right\}\right] \qquad (4.10)$$

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V. Fuzzy Model

Due to uncertainly in the environment it is not easy to define all the parameters precisely, accordingly we assume some of these parameters namely $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{S}, \tilde{\theta}, \tilde{h}$ may change within some limits.

Let$\tilde{a} = (a_1, a_2, a_3, a_4, a_5)$,$\tilde{b} = (b_1, b_2, b_3, b_4, b_5)$

$\tilde{C} = (C_1, C_2, C_3, C_4, C_5)$,$\tilde{S} = (S_1, S_2, S_3, S_4, S_5)$

$\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$, $\tilde{h} = (h_1, h_2, h_3, h_4, h_5)$

are as pentagonal fuzzy numbers.

Total cost of the system per unit time in fuzzy sense is given by

$$\widetilde{K}(t_1, T) = \frac{1}{T}\left[A + \tilde{h}\tilde{a}\left\{\frac{t_1^2}{2} + \frac{\tilde{\theta}}{6}t_1^3\right\} + \tilde{h}\tilde{a}\tilde{b}\left\{\frac{t_1^3}{3} + \frac{\tilde{\theta}}{8}t_1^4\right\} + \tilde{C}\left\{\frac{\tilde{a}\tilde{\theta}t_1^2}{2} + \frac{\tilde{a}\tilde{b}\tilde{\theta}}{3}t_1^3\right\} + \tilde{S}\left\{\frac{\tilde{a}}{2}(t_1 - T)^2 - \frac{\tilde{a}\tilde{b}}{2}\left(t_1^2 T - \frac{T^3}{3} - \frac{2}{3}t_1^3\right)\right\}\right] \tag{4.11}$$

We defuzzify the fuzzy total cost $\widetilde{K}(t_1, T)$ by signed distance method.

Bysigned distance method, Total Cost is given by.

$$K_{dS}(t_1, T) = \frac{1}{8}\left[K_{dS_1}(t_1, T), K_{dS_2}(t_1, T), K_{dS_3}(t_1, T), K_{dS_4}(t_1, T), K_{dS_5}(t_1, T)\right]$$

Where

$$K_{dS_1}(t_1, T) = \frac{1}{T}\left[A + h_1 a_1\left\{\frac{t_1^2}{2} + \frac{\theta_1}{6}t_1^3\right\} + h_1 a_1 b_1\left\{\frac{t_1^3}{3} + \frac{\theta_1}{8}t_1^4\right\} + C_1\left\{\frac{a_1\theta_1 t_1^2}{2} + \frac{a_1 b_1\theta_1}{3}t_1^3\right\} + S_1\left\{\frac{a_1}{2}(t_1 - T)^2 - \frac{a_1 b_1}{2}\left(t_1^2 T - \frac{T^3}{3} - \frac{2}{3}t_1^3\right)\right\}\right]$$

$$K_{dS_2}(t_1, T) = \frac{1}{T}\left[A + h_2 a_2\left\{\frac{t_1^2}{2} + \frac{\theta_2}{6}t_1^3\right\} + h_2 a_2 b_2\left\{\frac{t_1^3}{3} + \frac{\theta_2}{8}t_1^4\right\} + C_2\left\{\frac{a_2\theta_2 t_1^2}{2} + \frac{a_2 b_2\theta_2}{3}t_1^3\right\} + S_2\left\{\frac{a_2}{2}(t_1 - T)^2 - \frac{a_2 b_2}{2}\left(t_1^2 T - \frac{T^3}{3} - \frac{2}{3}t_1^3\right)\right\}\right]$$

$$K_{dS_3}(t_1, T) = \frac{1}{T}\left[A + h_3 a_3\left\{\frac{t_1^2}{2} + \frac{\theta_3}{6}t_1^3\right\} + h_3 a_3 b_3\left\{\frac{t_1^3}{3} + \frac{\theta_3}{8}t_1^4\right\} + C_3\left\{\frac{a_3\theta_3 t_1^2}{2} + \frac{a_3 b_3\theta_3}{3}t_1^3\right\} + S_3\left\{\frac{a_3}{2}(t_1 - T)^2 - \frac{a_3 b_3}{2}\left(t_1^2 T - \frac{T^3}{3} - \frac{2}{3}t_1^3\right)\right\}\right]$$

$$K_{dS_4}(t_1, T) = \frac{1}{T}\left[A + h_4 a_4\left\{\frac{t_1^2}{2} + \frac{\theta_4}{6}t_1^3\right\} + h_4 a_4 b_4\left\{\frac{t_1^3}{3} + \frac{\theta_4}{8}t_1^4\right\}\right.$$

$$+ C_4\left\{\frac{a_4\theta_4 {t_1}^2}{2} + \frac{a_4 b_4 \theta_4}{3}{t_1}^3\right\}$$
$$+ S_4\left\{\frac{a_4}{2}(t_1 - T)^2 - \frac{a_4 b_4}{2}\left({t_1}^2 T - \frac{T^3}{3} - \frac{2}{3}{t_1}^3\right)\right\}\Bigg]$$

$$K_{dS_5}(t_1,T) = \frac{1}{T}\Bigg[A + h_5 a_5\left\{\frac{{t_1}^2}{2} + \frac{\theta_5}{6}{t_1}^3\right\} + h_5 a_5 b_5\left\{\frac{{t_1}^3}{3} + \frac{\theta_5}{8}{t_1}^4\right\}$$
$$+ C_5\left\{\frac{a_5\theta_5 {t_1}^2}{2} + \frac{a_5 b_5 \theta_5}{3}{t_1}^3\right\}$$
$$+ S_5\left\{\frac{a_5}{2}(t_1 - T)^2 - \frac{a_5 b_5}{2}\left({t_1}^2 T - \frac{T^3}{3} - \frac{2}{3}{t_1}^3\right)\right\}\Bigg] \qquad (4.12)$$

$$K_{dS}(t_1,T) = \frac{1}{8}\left[K_{dS_1}(t_1,T) + 2K_{dS_2}(t_1,T) + 2K_{dS_3}(t_1,T) + 2K_{dS_4}(t_1,T) + K_{dS_5}(t_1,T)\right]$$

To minimize total cost function per unit time $K_{dS}(t_1,T)$, the optimal value of t_1 and T can be obtained by solving the following equations:

$$\frac{\partial K_{dS}(t_1,T)}{\partial t_1} = 0$$

and

$$\frac{\partial K_{dS}(t_1,T)}{\partial T} = 0 \qquad (4.13)$$

Equation (4.13) is equivalent to

$$\frac{1}{8T}\Bigg[h_1 a_1\left\{t_1 + \frac{\theta_1}{2}{t_1}^2\right\} + h_1 a_1 b_1\left\{{t_1}^2 + \frac{\theta_1}{2}{t_1}^3\right\} + C_1\{a_1\theta_1 t_1 + a_1 b_1 \theta_1 {t_1}^2\}$$
$$+ S_1\{a_1(t_1 - T) - a_1 b_1(t_1 T - {t_1}^2)\}$$
$$+ 2\Bigg\{h_2 a_2\left\{t_1 + \frac{\theta_2}{2}{t_1}^2\right\} + h_2 a_2 b_2\left\{{t_1}^2 + \frac{\theta_2}{2}{t_1}^3\right\} + C_2\{a_2\theta_2 t_1 + a_2 b_2 \theta_2 {t_1}^2\}$$
$$+ S_2\{a_2(t_1 - T) - a_2 b_2(t_1 T - {t_1}^2)\}\Bigg\}$$
$$+ 2\Bigg\{h_3 a_3\left\{t_1 + \frac{\theta_3}{2}{t_1}^2\right\} + h_3 a_3 b_3\left\{{t_1}^2 + \frac{\theta_3}{2}{t_1}^3\right\} + C_3\{a_3\theta_3 t_1 + a_3 b_3 \theta_3 {t_1}^2\}$$
$$+ S_3\{a_3(t_1 - T) - a_3 b_3(t_1 T - {t_1}^2)\}\Bigg\}$$
$$+ 2\Bigg\{h_4 a_4\left\{t_1 + \frac{\theta_4}{2}{t_1}^2\right\} + h_4 a_4 b_4\left\{{t_1}^2 + \frac{\theta_4}{2}{t_1}^3\right\} + C_4\{a_4\theta_4 t_1 + a_4 b_4 \theta_4 {t_1}^2\}$$
$$+ S_4\{a_4(t_1 - T) - a_4 b_4(t_1 T - {t_1}^2)\}\Bigg\} + h_5 a_5\left\{t_1 + \frac{\theta_5}{2}{t_1}^2\right\} + h_5 a_5 b_5\left\{{t_1}^2 + \frac{\theta_5}{2}{t_1}^3\right\}$$
$$+ C_5\{a_5\theta_5 t_1 + a_5 b_5 \theta_5 {t_1}^2\} + S_5\{a_5(t_1 - T) - a_5 b_5(t_1 T - {t_1}^2)\}\Bigg]$$
$$= 0 \qquad (4.14)$$

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$$And \left[\frac{1}{8T}\left\{S_1\left\{-a_1(t_1-T)-\frac{a_1b_1}{2}({t_1}^2-T^2)\right\}\right.\right.$$
$$+2S_2\left\{-a_2(t_1-T)-\frac{a_2b_2}{2}({t_1}^2-T^2)\right\}$$
$$+2S_3\left\{-a_3(t_1-T)-\frac{a_3b_3}{2}({t_1}^2-T^2)\right\}$$
$$+2S_4\left\{-a_4(t_1-T)-\frac{a_4b_4}{2}({t_1}^2-T^2)\right\}$$
$$\left.+S_5\left\{-a_5(t_1-T)-\frac{a_5b_5}{2}({t_1}^2-T^2)\right\}\right\}$$
$$-\frac{1}{8T^2}\left\{8A+h_1a_1\left\{\frac{{t_1}^2}{2}+\frac{\theta_1}{6}{t_1}^3\right\}+h_1a_1b_1\left\{\frac{{t_1}^3}{3}+\frac{\theta_1}{8}{t_1}^4\right\}\right.$$
$$+C_1\left\{\frac{a_1\theta_1{t_1}^2}{2}+\frac{a_1b_1\theta_1}{3}{t_1}^3\right\}$$
$$\left.+S_1\left\{\frac{a_1}{2}(t_1-T)^2-\frac{a_1b_1}{2}\left({t_1}^2T-\frac{T^3}{3}-\frac{2}{3}{t_1}^3\right)\right\}\right\}$$
$$+2\left\{h_2a_2\left\{\frac{{t_1}^2}{2}+\frac{\theta_2}{6}{t_1}^3\right\}+h_2a_2b_2\left\{\frac{{t_1}^3}{3}+\frac{\theta_2}{8}{t_1}^4\right\}\right.$$
$$+C_2\left\{\frac{a_2\theta_2{t_1}^2}{2}+\frac{a_2b_2\theta_2}{3}{t_1}^3\right\}$$
$$\left.+S_2\left\{\frac{a_2}{2}(t_1-T)^2-\frac{a_2b_2}{2}\left({t_1}^2T-\frac{T^3}{3}-\frac{2}{3}{t_1}^3\right)\right\}\right\}$$
$$+2\left\{h_3a_3\left\{\frac{{t_1}^2}{2}+\frac{\theta_3}{6}{t_1}^3\right\}+h_3a_3b_3\left\{\frac{{t_1}^3}{3}+\frac{\theta_3}{8}{t_1}^4\right\}\right.$$
$$+C_3\left\{\frac{a_3\theta_3{t_1}^2}{2}+\frac{a_3b_3\theta_3}{3}{t_1}^3\right\}$$
$$\left.+S_3\left\{\frac{a_3}{2}(t_1-T)^2-\frac{a_3b_3}{2}\left({t_1}^2T-\frac{T^3}{3}-\frac{2}{3}{t_1}^3\right)\right\}\right\}$$
$$+2\left\{h_4a_4\left\{\frac{{t_1}^2}{2}+\frac{\theta_4}{6}{t_1}^3\right\}+h_4a_4b_4\left\{\frac{{t_1}^3}{3}+\frac{\theta_4}{8}{t_1}^4\right\}\right.$$
$$+C_4\left\{\frac{a_4\theta_4{t_1}^2}{2}+\frac{a_4b_4\theta_4}{3}{t_1}^3\right\}$$
$$\left.+S_4\left\{\frac{a_4}{2}(t_1-T)^2-\frac{a_4b_4}{2}\left({t_1}^2T-\frac{T^3}{3}-\frac{2}{3}{t_1}^3\right)\right\}\right\}$$
$$+h_5a_5\left\{\frac{{t_1}^2}{2}+\frac{\theta_5}{6}{t_1}^3\right\}+h_5a_5b_5\left\{\frac{{t_1}^3}{3}+\frac{\theta_5}{8}{t_1}^4\right\}$$
$$+C_5\left\{\frac{a_5\theta_5{t_1}^2}{2}+\frac{a_5b_5\theta_5}{3}{t_1}^3\right\}$$
$$\left.+S_5\left\{\frac{a_5}{2}(t_1-T)^2-\frac{a_5b_5}{2}\left({t_1}^2T-\frac{T^3}{3}-\frac{2}{3}{t_1}^3\right)\right\}\right]=0$$

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Further, for the total cost function $K_{dS}(t_1, T)$ to be convex, the following conditions must be satisfied

$$\frac{\partial^2 K_{dS}(t_1,T)}{\partial {t_1}^2} > 0\,, \frac{\partial^2 K_{dS}(t_1,T)}{\partial T^2} > 0 \qquad (4.16)$$

And

$$\left(\frac{\partial^2 K_{dS}(t_1,T)}{\partial {t_1}^2}\right)\left(\frac{\partial^2 K_{dS}(t_1,T)}{\partial T^2}\right) - \left(\frac{\partial^2 K_{dS}(t_1,T)}{\partial t_1 \partial T}\right) > 0 \qquad (4.17)$$

The second derivatives of the total cost function $K_{dS}(t_1, T)$ are complicated and it is very difficult to prove the convexity mathematically. Thus, the convexity of total cost function has been established graphically. (Figure A)

VI. Numerical Example

Consider an inventory system with following parametric values.

Crisp Model, A=Rs.200/order, C=Rs.20/unit, h=Rs. 5/unit/year,a=100 units/year, b=0.1units/year, $\theta = 0.01$/year, S=Rs 15 /unit/year.
The solution of crisp model is $K(t_1, T)$ = Rs 404.3429, t_1 =0.7149 year, T = .9639 year.

Fuzzy model,

$\tilde{a} = (60,80,100,120,140)$, $\tilde{b} = (0.06,0.08,0.10,0.12,0.14)$ $\tilde{C} = (16,18,20,22,24)$, $\tilde{S} = (11,13,15,17,19)$ $\tilde{\theta} = (0.006,0.008,0.010,0.012,0.014)$, $\tilde{h} = (1,3,5,7,9)$

The solution of fuzzy model can be determined by following Signed Distance Method.

1. When $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{S}, \tilde{\theta}, \tilde{h}$ all are pentagonal fuzzy numbers.
   $K_{dS}(t_1,T) = Rs.419.6059$, $t_1 = 0.6797 year,$ $T = 0.9266 year$

2. When $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{S}, \tilde{\theta}$ all are pentagonal fuzzy numbers
   $K_{dS}(t_1,T) = Rs.408.2810$, $t_1 = 0.7135 year,$ $T = 0.9523 year$

3. When $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{\theta}$ all are pentagonal fuzzy numbers.
   $K_{dS}(t_1,T) = Rs.406.1163$, $t_1 = 0.7093 year,$ $T = 0.9576 year$

4. When $\tilde{a}, \tilde{b}, \tilde{\theta}$ all are pentagonal fuzzy numbers.
   $K_{dS}(t_1,T) = Rs.405.6640$, $t_1 = 0.7106 year,$ $T = 0.9587 year$

5. When $\tilde{a}$ and $\tilde{b}$ all are pentagonal fuzzy numbers.
   $K_{dS}(t_1,T) = Rs.405.1742$, $t_1 = 0.7122 year,$ $T = 0.9599 year$

If we plot the total cost function $K_{dS}(t_1, T)$with somevalues of $t_1$and Ts.t.t_1= .65 to 2 with equal interval T= .84 to 1, then we get strictly convex graph of total costfunction $K_{dS}(t_1, T)$given below.

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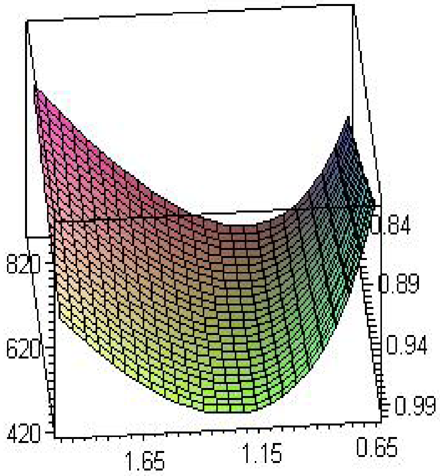

(Figure A) Total fuzzy cost $K_{dG}(t_1, T)$Vs.$t_1$and$T$.

VII. CONCLUSIONS:

This paper presents a fuzzy inventory model for deteriorating items with shortages under fully backlogged condition in which demand is an increasing function of time. Shortages and deterioration are natural in any inventory control system. The proposed model is developed in both the crisp and fuzzy environments. In fuzzy environment, all related inventory parameters are assumed to be pentagonal fuzzy numbers. For defuzzification, signed distance method is employed to evaluate the optimal time period of positive stock $t_1$and total cycle length T which minimizes the total cost. By given numerical example it has been tested that signed Distance method gives minimum cost.

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