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Probabilistic Analysis on Time to Recruitment for a Single Grade Man power System when the Breakdown Threshold has two Components using a Different Policy of Recruitment

KEYWORDS Single grade manpower system, exchangeable and constantly correlated exponential random variables, thresholds with two components, shock model approach, univariate Max policy of recruitment, different probabilistic analysis and variance of recruitment.

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ABSTRACT *In this paper, the problem of time to recruitment is analyzed for a single grade manpower system using an univariate Max policy of recruitment based on shock model approach. Two mathematical models are constructed and the variance of time to recruitment is obtained when the loss of manpower form a sequence of independent and identically distributed exponential random variables and the breakdown threshold for the maximum loss of manpower has two components. While in Model I, the inter-decision times are independent and identically distributed exponential random variables, in Model II, they are exchangeable and constantly correlated exponential random variables. A different probabilistic analysis is used to derive the analytical result.*

1.Introduction

Wastage of personnel due to retirement, death and resignation is a common phenomenon in administrative as well as production oriented organizations. There are certain special problems associated with the organization engaged in sales and marketing. Frequent exits and recruitments are very common in such organizations. Whenever the organization announces revised polices regarding sales target, revision of wages, incentives and perquisites the exodus is possible. Reduction in the total strength of marketing personnel adversely affects the sales turnover of the organization. Frequent recruitments may also be expensive due to the cost of recruitments and training. As the loss of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. The univariate max policy of recruitment is based on the replacement policy associated with the shock model approach in reliability theory and is stated as follows: *Recruitment is made whenever the maximum loss of man hours exceeds the break down threshold for the maximum loss of man hours*. Several researchers have studied the problem of time to recruitment for a single grade man power system using shock model approach. In [8] , the authors have obtained the variance of time to recruitment when the loss of man power and inter-decision times are independent and identically distributed random variables with constant breakdown threshold using univariate MAX policy of recruitment and Laplace transform technique. In [9] and [10], the authors have studied the work of [8] with exponential and SCBZ breakdown thresholds respectively. While in [5], the authors have studied the work in [8] for geometric loss of man power and exchangeable and constantly correlated exponential inter-policy decisions , the authors in [4] have studied the work of [5] with Poisson loss of man power and geometric threshold. in [6],[3] and [7] ,the authors have studied the work of [9]and [10] for exchangeable and constantly correlated exponential inter-policy decisions

In [1] ,the authors have studied the work in [8] for Erlang and extended exponential breakdown thresholds also using a different probabilistic analysis. In [2],the authors have analyzed the work of [1] with exchangeable and constantly correlated exponential random variables inter-decision times . In [11], the authors have studied the work in [1] when the breakdown threshold level for the cumulative loss of manpower is the sum of the exponential breakdown threshold levels of wastage and backup resource for man power using Laplace transform technique and univariate CUM policy. In the present paper, an attempt has been made to study the work in [1] with the help of two mathematical models using a different probabilistic analysis when (i) the inter - decision times are either independent and identically distributed exponential random variables or exchangeable and constantly correlated exponential random variables and (ii) the breakdown threshold for the maximum loss of manpower has two components .

2.Model Description and Analysis for Model– I

Consider an organization with single grade, taking policy decisions at random epochs in $(0,\infty)$. at

every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower, if a person quits. It is assumed that the loss of manpower is linear and cumulative. For i=1,2,3…, let X_i be the independent and identically distributed continuous random variables representing the amount of depletion of manpower(loss of man hours)due to i^{th} policy decision with distribution G(.), Z_i be the maximum loss of manpower in the first i decisions, U_i, the time between $(i-1)^{th}$ and i^{th} decisions, be independent and identically distributed exponential random variables. Let T be a continuous random variable representing the threshold for the maximum loss of manpower with probability density functions h(.). It is assumed that threshold T is the sum of the exponential breakdown threshold level T_1 of wastage with mean $(\frac{1}{\theta_1}>0)$ and exponential threshold T_2 of frequent breaks of existing workers with mean$(\frac{1}{\theta_2}>0)$. Let $\chi(A)$ be the indicator function of the event A. Let W be the time to recruitment for the organization with mean E(W) and variance V(W). Recruitment is done whenever the maximum loss of manpower exceeds the break down threshold.

3.Main Results

By the recruitment policy, recruitment is done whenever the maximum loss of manpower exceeds the threshold

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T. When the first decision is taken, recruitment would not have been done for U_1 units of time. If the loss of manpower $X_1(=Z_1)$ due to the the non recruitment period will continue till the next policy decision is taken. If the cumulative Z_2 of the first policy decision is greater than T, then recruitment is done and in this case $W=U_1=R_1$. However, if $Z_1\leq T$, loss of manpower in the first two decisions exceeds T, then recruitment is done and $W=U_1+U_2=R_2$. If $Z_2\leq T$, then the non recruitment period will continue till the next policy decision is taken and depending on $Z_3>T$ or $Z_2\leq T$, recruitment is done or the non-recruitment period continues and so on. Hence

$$W=\sum_{i=0}^{\infty} R_{i+1}\,\chi(Z_i\leq T<Z_{i+1}) \quad (1)$$

and

$$E(W)=\sum_{i=0}^{\infty} E(R_{i+1})\,P(Z_i\leq T<Z_{i+1}) \quad (2)$$

From (2) and from the definition of R_{i+1}, we get

$$E(W)=E(U)\sum_{i=0}^{\infty}(i+1)\,P(Z_i\leq T<Z_{i+1}) \quad (3)$$

By the law of total probability, we get

$$P(Z_i\leq T<Z_{i+1}) = \int_0^{\infty}[(\overline{G}(t))]^{i}h(t)dt \int_0^{\infty}\overline{G}(t)h(t)dt \quad (4)$$

Using (4) in (3) we get

$$E(W)=E(U)\int_0^{\infty}\overline{G}(t)h(t)dt \int_0^{\infty}[(\overline{G}(t))]^{-2}h(t)dt \quad (5)$$

where

$$\int_0^{\infty}\overline{G}(t)h(t)dt = \frac{\theta_1\theta_2}{(\alpha+\theta_1)(\alpha+\theta_2)} \quad (6)$$

and

$$\int_0^{\infty}[(\overline{G}(t))]^{-2}h(t)dt = \frac{\theta_1\theta_2}{(\theta_1-2\alpha)(\theta_2-2\alpha)} \quad (7)$$

Using (6) and (7) in (5), we get

$$E(W) = E(U)\left[\frac{(\theta_1\theta_2)^2}{(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)}\right] \quad (8)$$

(8) represents mean of time to recruitment for this model.

From(1),we get

$$W^2=[\sum_{i=0}^{\infty} R_{i+1}\,\chi(Z_i\leq T<Z_{i+1})]^2 \quad (9)$$

and

$$E(W^2)=\sum_{i=0}^{\infty} E(R_{i+1}^{\,2})\,P(Z_i\leq T<Z_{i+1}) \quad (10)$$

Since

$$E(R_{i+1}^{\,2})=V(R_{i+1})+E(R_{i+1})^2 \quad (11)$$

$$V(R_{i+1})=(i+1)V(U) \quad (12)$$

and

$$E(R_{i+1})^2=(i+1)^2[E(U)]^2 \quad (13)$$

from (4),(9),(11),(12) and (13) we get

$$E(W^2)=\left[\frac{(\theta_1\theta_2)^2}{(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)}\right]\left\{V(U)+(E(U))^2\frac{(\theta_1+\theta_2)\alpha-3\alpha^2}{(\theta_1-3\alpha)(\theta_2-3\alpha)}\right\} \quad (14)$$

and

$$Var(W)=\left[\frac{(\theta_1\theta_2)^2}{(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)}\right] X$$

$$\left\{[E(U^2)]+(E(U))^2\frac{2(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)[(\theta_1+\theta_2)\alpha-(3\alpha)^2]-(\theta_1\theta_2)^2(\theta_1-3\alpha)(\theta_2-3\alpha)}{(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)(\theta_1-3\alpha)(\theta_2-3\alpha)}\right\} \quad (15)$$

(15) represents variance of time to recruitment for this model.

4. Model Description and Analysis for Model– II

Description and expectation of time of recruitment for Model II are same as in Model I except the condition on inter –decision times. In Model II , It is assumed that the inter-policy decisions are exchangeable and constantly correlated exponential random variables with mean u (u>0) . Let ρ be the correlation between U_i and U_j, i≠j and v=u(1-ρ).

5.Main Results

In the present model,

$$E(R_{i+1}^{\,2})=u^2[\rho^2 i(i+1)+(i+1)(i+2)] \quad (16)$$

Using (16) in (10) we get,

$$E(W^2)=[(E(U)]^2\left[\frac{-\theta_1\theta_2}{(\alpha+\theta_1)(\alpha+\theta_2)}\right]\left\{2(\rho^2+1)\left[\frac{\theta_1\theta_2}{(\theta_1-2\alpha)(\theta_2-2\alpha)}\right]-2\rho^2\left[-\frac{\theta_1\theta_2}{(\theta_1-2\alpha)(\theta_2-2\alpha)}\right]\right\}$$

and

$$Var(W)=[(E(U)]^2\left[\frac{-\theta_1\theta_2}{(\alpha+\theta_1)(\alpha+\theta_2)}\right]\left\{2(\rho^2+1)\left[\frac{-\theta_1\theta_2}{(\theta_1-2\alpha)(\theta_2-2\alpha)}\right]-2\rho^2\left[-\frac{\theta_1\theta_2}{(\theta_1-2\alpha)(\theta_2-2\alpha)}\right]\right\}-$$

$$\left[E(U)\left[\frac{(\theta_1\theta_2)^2}{(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)}\right]\right]^2 \quad (17)$$

(17) represents variance of time to recruitment for this model.

Conclusion:

The models discussed in this paper are found to be more realistic in the context of considering components for breakdown threshold, since loss in manpower can occur by people working in the system as well as by people who go out of the system once for all. The probabilistic analysis carried out in this paper does require only the first two moments of inter –decision times unlike the Laplace transform technique which requires the knowledge about distribution of these random variables.

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