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Research on an Estimation Method for the Coefficients of Discrete Models

KEYWORDS: Kalman filter, models, estimation of coefficients

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ABSTRACT *One of the major tasks in designing automation systems is the modelling of separate elements and/or the entire structure. The wide use of high-tech software in design has lead to the more intense application of discrete models of processes under complex environmental influences. Taking into account the conditions defining the Kalman filter recursion and its basic features, the challenge here is in examining a method based on this recursion and developed for the purpose of parameter estimation in discrete linear models.*

INTRODUCTION

The identification of the automation elements and systems is made on the basis of experimental results, which in terms of time are most often the familiar transient characteristics. As a rule, the digital identification data obtained experimentally contain subjective and/or objective inaccuracies (errors). In cases when an inaccuracy is ignorable in relation to a given process value, it is omitted in the synthesis and analysis of analytical models. When the inaccuracies are significant, two approaches are applied: the impact is reduced through pre-processing of the experimental data (smoothing, filtration, modelling, etc.) and usage of determinate methods and models; the second approach consists in using stochastic models of the processes and values, and application of appropriate methods of synthesis and analysis [1, 4-6, 8].

One of the popular methods of synthesis defined for noisy (stochastic) data which leads to optimal results (evaluations) in linear models is the Kalman filter. There are, however, enunciations of this method for non-linear models that yield quasi-optimal results. It is known that there are discrete enunciations of the Kalman filter for the estimation of states, of states and parameters (extended filter), and of parameters only (reverse filter) [2, 3, 7].

The aim of this paper was to study the method of parameter estimation through Kalman recursions in [2, 7] with regard to various linear discrete models and the absence or presence of random environmental influences (noises) on the object. With noises present, it aimed to examine the cases when these were measurable or immeasurable with known or unknown statistical distribution and characteristics; to study the impact of different initial conditions on the algorithm convergence; to compare the method results to those of other MATLAB® accessible methods; to specify the algorithm and calculation predicaments arising for the software implementation of the method.

COEFFICIENT ESTIMATION OF LINEAR DISCRETE MODELS USING THE KALMAN FILTER

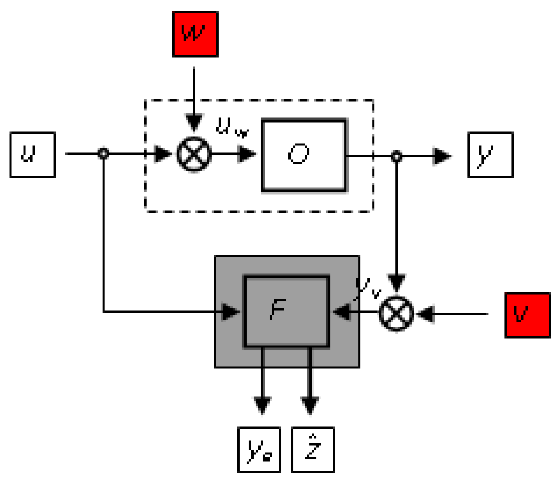

Figure 1: system object (O), Kalman filter (F), and input and output signals

The method of estimating the coefficients of linear discrete models using the Kalman filter has been given in [7]. Fig.1 represents a chart of the object system (O), the Kalman filter (F), and the input-output signals. *u* is the determinate part of the input signal to be measured without error, and *y* is object response – a determinate, measurable signal. This figure also shows: random signal *w*, measurable or not (there was assumed normal distribution, mathematical expectation m_w = 0, and variance D_w = Q); measuring noise *v*, measurable or not random signal (normal distribution, mathematical expectation m_v = 0 and variance D_v = *R*); random signal at object input u_w; noise signal from the object output to the filter input y_v; filter output y_e (estimation of object output); coefficients in the model studied Ž. It was assumed that the object model was a difference equation of the type:

$y_v(k)+a_1*y_v(k-1)+a_2*y_v(k-2)+\ldots+a_{na}*y_v(k-n_a)=b_1*u_w(k-1)+b_2*u_w(k-2)+\ldots+b_{nb}*u_w(k-n_b)+e(k)$, (1)

where:

$u_w(k) = u(k) + w(k)$; (2)

$y_v(k) = y(k) + v(k)$. (3)

The discrete transfer function of the type (z^{-1}) corresponding to (1) was

$W(z) = (b_1 z^{-1}+b_2 z^{-2}+\ldots+b_{nb} z^{-nb})/(1+ a_1 z^{-1}+a_2 z^{-2}+\ldots+a_{na} z^{-na})$ (4)

Without losing totality, to make description easier, it was assumed $n_a = n_b = n$. Then from equation (1), $y(k)$ was expressed, and the result was

$y_v(k) = -a_1*y_v(k-1)-a_2*y_v(k-2)-\ldots-a_n*y_v(k-n)+b_1*u_w(k-1)+b_2*u_w(k-2)+\ldots+b_{nb}*u_w(k-n)+e(k)$, (5)

where e(κ) was the generalised error in the model. By reading (2) and (3), equation (4) in vector form was

$y_v(k) = [H(k)+hB(k)]*Z(k)+e(k)$, (6)

where:

- $Z(k) = [a_1, a_2, \ldots a_n, b_1, b_2, \ldots b_n]$ is a vector column of the model's coefficients;

- $H(k) = [-y(k-1), \ldots -y(k-n), u(k-1), \ldots u(k-n)]$ is a vector series, of the measured input-output values (regressors);

- $hB(k) = [v(k-1), \ldots v(k-n), w(k-1), \ldots w(k-n)]$ is a vector series, of the random parts of input-output values.

The equations of the model within the state space were written down as follows:

- equation of state was $Z(k+1) = Z(k)$;
- equation of output was $y_v(k)=[H(k)+hB(k)]*Z(k)+v(k)$.

The equations of the Kalman filter were searched under the minimum error condition.

$\varepsilon(k) = Z(k)-\check{Z}(k)$, for $k = n, n+1, n+2, \ldots$ (7)

Coefficients $\check{Z}(k)$ were determined by the filter equations [2, 4, 7]

$\check{Z}(k) = \check{Z}(k-1)+G(k)*[y_v(k)-H(k)*\check{Z}(k-1)]$, (8)

where:

$G(k)=P(k-1)*H'(k)[H(k)*P(k-1)*H'(k)+M\{hB(k)*S(k)*hB(k)'\}+D_v]^{-1}$; (9)

$P(k) = [1-G(k)*H(k)]*P(k-1)$ and $S(k)=M\{\check{Z}(k)*\check{Z}(k)'\}$. (10)

The initial conditions were:

$S(0)=M\{\check{Z}(0)*\check{Z}(0)'\}=c^2*I; \quad c^2\to\infty; \quad P(0)=S(0)$ (11)

Equations (8), (9) and (10) are also known as Kalman recursion, and the algorithm for estimation of output y_e and the model's coefficients $Z(k)$ are referred to as Reverse Kalman filter [2, 4, 7].

DESCRIPTION OF THE ALGORITHM AND PROGRAM FOR EXPLORING THE METHOD, ALGORITHM CALCULATIONS AND PREDICAMENTS

The general algorithm pattern for testing the method of model parameter estimation is given in fig. 2. The algorithm is linear, including the basic stages successively, as illustrated in the figure.

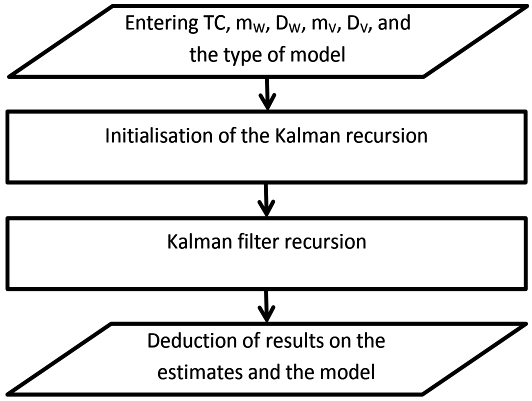

Figure 2: general algorithm of the method

Stage 1 Assignment of input data:

- Entering Transient Characteristic y_v, assuming that the values are read within a continuous interval of time T_d = *const*; further, entering the discretisation period and input signal *u*;

- Entering the random signal values *w* and *v* or their statistical estimates D_w and D_v;

- Entering the net delay (if any) as number of discretisation periods.

Stage 2 Initialisation of the Kalman recursion that includes:

- Assigning initial values to the vector of the searched coefficients $\check{Z}(0)$;

- Assigning initial values to the variance matrix *P(0)*;

- Structuring vectors *H* and *hB* according to the model order n_a (in the example below $n_a=2$);

- Initialisation of the cycle of counting in which the equations of the two variants are executed.

Stage 3 Executing the Kalman filter recursion. Two variants (V1 and V2) for second order models by means of the recursion equations (12-19) are given below. V1 is for measurable and V2 for immeasurable noise. In the first variant, equation (13) is complementary, and (14) and (18) are executed instead of (15) and (19).

```
for k=k0:1:length(t),
    H=[-yv(k-1),-yv(k-2),u(k-1-tau),u(k-2-tau)];        (12)
    hB=[v(k-1),v(k-2),w(k-1),w(k-2)];                   (13)
    G=P*H'/(H*P*H'+hB*Z*Z'*hB'+var(v));                 (14)
    Z=Z+G*(yv(k)-H*Z);                                  (16)
    P=(I-G*H)*P;                                        (17)
    ye(k)=(H+hB)*Z+v(k);                                (18)
    ZE(:,k)=Z;
end
```

```
for k=k0:1:length(t),
    H=[-yv(k-1),-yv(k-2),u(k-1-tau),u(k-2-tau)];        (12)
    G=P*H'/(H*P*H'+0.001*Z'*Z+var(v));                  (15)
    Z=Z+G*(yv(k)-H*Z);                                  (16)
    P=(I-G*H)*P;                                        (17)
    ye(k)=H*Z;                                          (19)
    ZE(:,k)=Z;
end
```

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Figure 4: stage 3 - variant V2

Stage 4 Deduction of results on the estimates and the model relevance:

- Displaying the analytical expression of the model obtained as a discrete transfer function;

- Displaying the experimental transient characteristic and the model transient characteristic onto a single coordinate system;

- Displaying the mean square differences (errors): **MSE1** is between the experimental transient characteristic and the transient characteristic of the model, and **MSE2** is between the experimental transient characteristic and the transient characteristic of the model obtained with the **arx** function in MATLAB®.

When implementing the algorithm and the program the following traits were observed:

- Variant V1: read measuring noise *w*, i.e. the values were assumed, and the statistical characteristics were calculated. Various random signals were studied;

- Variant V2: did not read random signals, and the respective members in (13) were omitted, (14) transformed into type (15), (18) transformed into type (19). No information on the random signals was available, random signal *w* of normal distribution, centred, with minimum dispersion was assumed;

- Selection and assignment of initial values to the searched coefficients Ž(0) and the variance matrix of error *P(0)*;

- The sum $M\{hB(k)*S(k)*hB(k)'\}+D_v$ in equation (9) affected the rate of the recursion convergence. In [4] it is recommended that this sum be altered in the course of recursion in a specific way (reduction until saturation of the algorithm is reached). In the present experiment, for the bulk of cases, the best results were obtained for

$$M\{hB(k)*S(k)*hB(k)'\}=0.001*Ž'(k)*Ž(k) . \qquad (20)$$

EXPERIMENTAL DATA AND SIMULATION RESULTS

The method was examined through software implementation of an algorithm containing the Kalman recursion and simulation data identification. The MATLAB® computing environment was used for the simulation and the method comparison. In identification, no methods exist for an unambiguous definition of the model order if only the transfer characteristic is known, therefore the model order in the experiments was selected by intuition, hence an error resulting from an inappropriate order assumption was likely to occur. The model order and the program delay were set by means of properly structuring vectors *H* and *hB*. The presence of a net delay in the data was a priori read and set in *hB*. The method only accounted for the transient delay by the model order.

The adequacy of the models obtained in relation to the set transient characteristic was estimated by visually comparing the set transient characteristic to the theoretical transient characteristic (TC), and the models were compared using the mean square difference (**MSE1**). In all simulations the method examined was compared to the **arx** method of the MATLAB® **ident** extension, and the **MSE1** and **MSE2** differences were compared to the set TC. The **arx** method is a variation of the Least Square Method with QR factorisation. Being on the input of the identified object, the impact of the process noise *w* on the object output depended on the static and dynamic properties of the object. Therefore in this paper the impact of the process noise was neglected, i.e. only the effect of the additive measuring noise on the method results was explored.

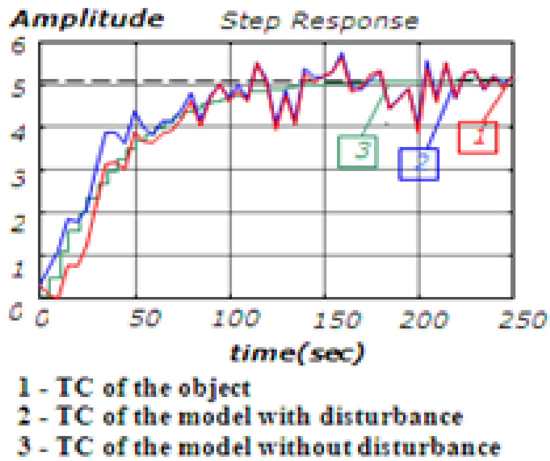

Figure 5: models TF1*TF1

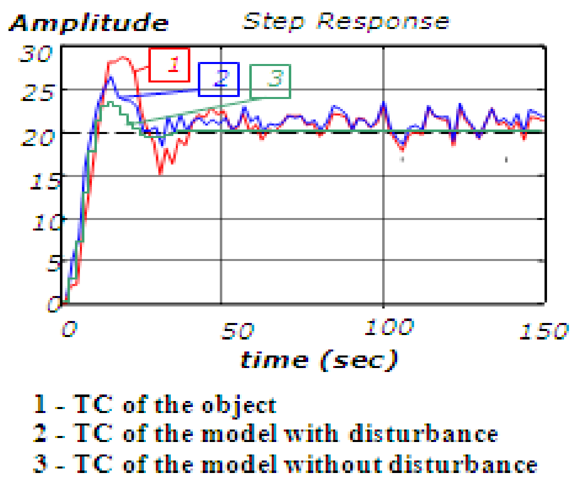

Figure 6: models TF1*K2

The program and the algorithm were explored regarding the two cases most often occurring in practice: measurable interferences (variant 1 – V1), and immeasurable interferences (variant 2 – V2). **With the measurable interferences** (V1) the method allowed for reading their observed values (the vector) and the variance var(v). Fig. 5, 6 illustrate the results for the approximation of a monotonic and oscillatory transient characteristic. In fig. 5, TC of the object is approximated to an aperiodic second order model with two consecutive aperiodic third order members (**TF1*TF1**). In fig. 6, TC of the object is approximated to a third order oscillatory model, i.e. aperiodic first order member consecutively to a second order fluctuating member (**TF1*K2**). Fig.5 and fig.6 (1) show TC disturbed by random signals of normal distribution, centred, and of dispersion, 10% and 8% respectively of the value found. Graphs (1) in these figures represent the noisy TC that could also be experimental but are in this case simulated random values. Graphs (2) represent the noisy response of the model y_e. For the purpose of compar-

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ison, graphs (3) represent the response of the model in the determinate case (without noise). Table 1 represents data and results of the two identified models (fig.5 and fig. 6).

TABLE - 1 DATA AND RESULTS OF THE MODELS (TF1*TF1, and TF1*K2)

	TF1*K2	TF1*TF1
Td(sec.)	2	2
u	1	1
w(%*u)	0	0
v(%*$y_{уст}$)	8	10
MSE1	0.4886	0.1834
MSE2	0.4982	0.1797

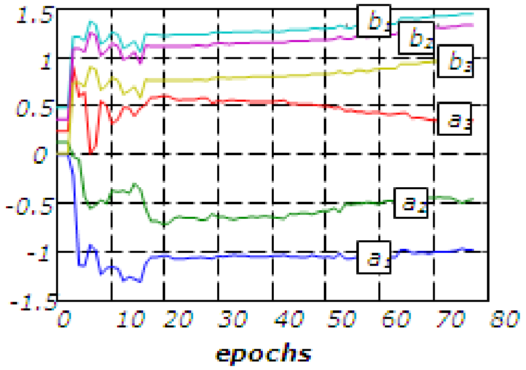

Figure 7: variation of the coefficients during Kalman recursion

Fig. 7 shows the variation of the coefficients in 80 cycles (epochs) of the method recursion for the oscillatory model. **MSE1** is the mean square difference between TC (1) and TC (2) with the Kalman filter, **MSE2** is the same difference obtained using the **arx** method, and **StE** is the static error of the obtained model TC.

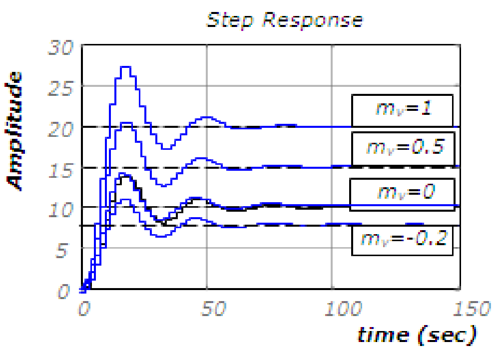

Figure 8a: effect of the mean of distribution m_v (D_v=0)

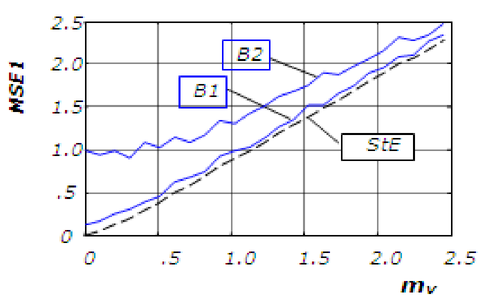

Figure 8b: effect of the mean of distribution mV (DV=0)

Figures 8a and 8b illustrate how the variation in the mean of distribution (m_v = 0 ÷ 2.5) affected the type of the model TC. The TCs refer to models derived by the V2 variant. Fig. 8b shows the **MSE1** difference for the two variants of the method and the **StE** error in a steady-state mode.

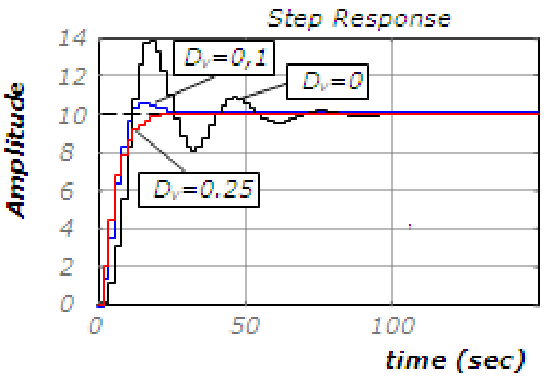

Figure 9a: effect of the dispersion D_v (m_v=0)

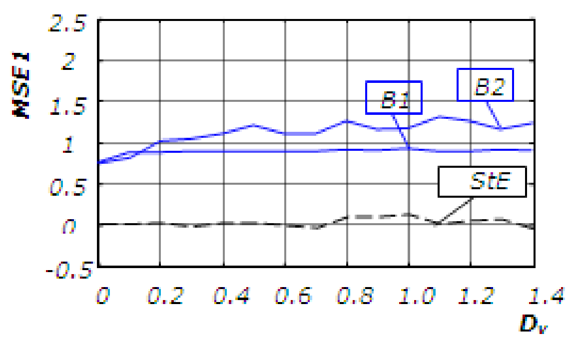

Figure 9b: effect of the dispersion D_v (m_v=0)

Figures 9a, and 9b illustrate how the variation in the dispersion (D_v = 0 ÷ 1.4) affected the type of the model TC. The TCs refer to models derived by the V2 variant. Fig 9b shows the **MSE1** difference for the two variants of the method and the **StE** error in a steady-state mode.

Fig. 10 demonstrates how the mean m_v, and the dispersion D_v together affected the model TC (the values displayed for m_v and D_v were part of the output value found).

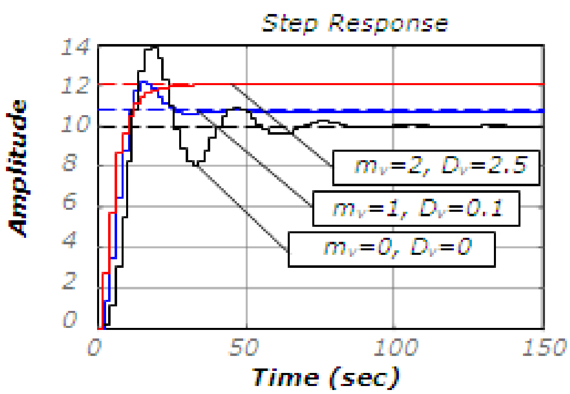

Figure 10: joint effect of mV and DV

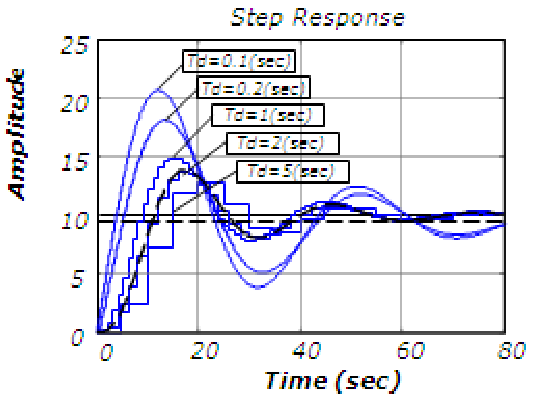

Figure 11: effect of discretisation step T_d

Figure 11 illustrates how the model TC varied with the variation in the discretisation step $T_d = (0.1 \div 5)$ s.

In the course of the study, the method was tested: for other types of linear models; for noises of distribution different from the normal ones (Weibull, or other); for presence of random noise at the object input; for various initial conditions, etc. The results are summarized in the next section.

ANALYSIS AND COMPARATIVE EVALUATION OF THE METHOD

The test results, some of which presented above in the figures and tables, allowed the following conclusions:

- The method was converging and applicable to the estimation of the parameters of linear (according to the superposition principle) discrete models (fig. 5 and 6). The method was applicable to damping/non-damping transient characteristics.

- The method accounted for the noises in the measuring part and was applicable for two variants (V1 and V2), for measurable and immeasurable noise respectively.

- The results obtained by using the method were comparable to those of the arx method in the MATLAB® ident extension, which is a variant of the Least Square Method with QR factorisation (fig. 12). In this figure, **MSE1** is the mean square difference between TC (1) and TC (2) with the Kalman filter, and **MSE2** is the same difference obtained by the **arx** function in MATLAB®.

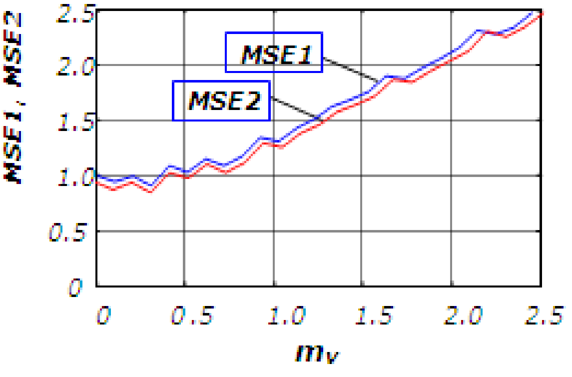

Figure 12: The method in comparison with the arx function (m_V=0-2.5, D_V=0, T_d=2)

- The results obtained depended on the statistical parameters of the interferences: noises had less effect on the estimations if it was possible to apply variant V1 (fig. 8 and 9); the method offered allowed to fully compensate centred interferences of normal distribution and intensity of 8-10% of the established value; the mean of distribution of a random noise of normal distribution had an analogical effect to that of a permanent interference of zero dispersion (fig. 8), and the increase in the mean of distribution proportionally changed only the static amplification factor of the model obtained; the dispersion only slightly influenced the static amplification factor but strongly affected the rest of the parameters of the model obtained (fig. 9); the symmetrical distributions (+ and – random quantity value) affected the results like a normal centred distribution, and the non-symmetrical distributions affected the parameters in a way analogical to the variation in the mean of distribution.

- The method results depended on the discretisation period T_d (fig. 11), and good results were obtained with discretisation step T_d 8-12 times as small as the time constant in the model (if the step was smaller, the error was considerable).

- The method and the implementation algorithm were applicable without any significant changes to both practical cases, i.e. with an priory available set of data or when the data were obtained in real time.

- The calculation specificities were of the type typical of the discrete Kalman filter, namely: fast convergence within 20 recursion cycles (fig. 7); initial values Ž(0) preferably assigned close to the actual ones (values significantly departing from the actual ones were likely to result in non-convergence of the algorithm), and it was not recommended to attribute zero values, as suggested by some authors, since these values were in the denominator of equation (15); the rate of convergence depended on the initial values of the variance matrix of error *P(0)* of the coefficients (it is suggested that [1, 5] be set as a diagonal matrix of infinite large values), but in the cases studied with $c^2 > 1000$ no better results were obtained; the algorithm for multi output models was sensitive to the input data due to the inversion in (14); in the present experiment, for equation (11) the best results were obtained for the type, shows in (20).

*M{hB(k)*S(k)*hB(k)'}=0.001*Ž'(k)*Ž(k) .I"*

- Due to the requirement for linearity, the method examined could not define the net delay in the difference equation (the net delay, if any, needed to be determined a priori).

- The impact of the process noise *w(t)* depended primarily on the static and dynamic parameters of the object, therefore its effect could not be studied.

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