RESEARCH PAPER | Mathematics | Volume : 5 | Issue : 6 | June 2015 | ISSN - 2249-555X

A Study of Inventory System with Ramp Type Demand Rate and Shortage in The Light of Inflation-II

KEYWORDS: Inventory, Deteriorating items, Inflation, Ramp type demand.

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ABSTRACT *An order-level inventory system consisting of deteriorating items under inflation with ramp type demand rate and partial backlogging and inventory starts with shortages. Three costs are considered under inflation as significant: deterioration, holding, shortage. The backlogging rate is an exponentially decreasing, time-dependent function specified by a parameter. In this paper we derive results, which ensure the existence of a unique optimal policy and provide the solution procedure for the problem. Numerical example is presented to illustrate the model.*

Introduction:

Most of the classical inventory models did not take into account the effects of inflation and time value of money. Perhaps, it was believed that inflation would not influence the cost and price components to any significant degree. However, in the last several years most of the countries have suffered from large-scale inflation and sharp decline in the purchasing power of money. As a result, while determining the optimal inventory policy , the effects of inflation and time value of money cannot be ignored, so several efforts have been made by researchers to reformulate the optimal inventory management polices taking into account inflation.

The first attempt in this direction was done by Buzacott [2], where he dealt with an economic order quantity model with inflation subject to different types of pricing policies. After Buzacott [2], several other researchers have extended his approach to various interesting situations taking into consideration the inflation rate. In this connection the works of Misra [6,7], Aggarwal [1], Jeya Chandra and Bahner [3] etc. are worth mentioning. But in all these studies, the marked demand rate has been assumed to be constant and unsatisfied demand is completely backlogged.

However, for fashionable commodities and high-tech products with short product life cycle, the willingness for a customer to wait for backlogging during a shortage period is diminishing with the length of the waiting time. Hence the longer the waiting time, smaller the backlogging rate would be. To reflect this phenomenon, Papachristos and Skouri [8] established a partially backlogged inventory model in which the backlogging rate decreases exponentially as the waiting time increases.

In a recent communication Gupta, Srivastava and singh [4] attempted to study of inventory system with ramp type demand rate and shortage in the light of inflation – I. We extended their work where inventory starts with shortages. This type of demand pattern is generally seen in the case of any new brand of consumer goods coming to the market. The demand rate for such items increases with time up to a certain time period and then ultimately stabilizes and becomes constant. It is believed that this type of demand rate is quite realistic.

The above investigation led us to develop an inflationary model for deteriorating items with ramp type demand rate and partial – exponential type backlogging started with shortages.

We attempt to provide the exact solution for the problem in the light of numerical example.

ASSUMPTIONS AND NOTATIONS:

- The mathematical model of the deterministic inventory replenishment problem with ramp type demand rate and starts with shortages is based on the following assumptions:
- The replenishment rate is infinite, thus replenishments are instantaneous.
- The lead-time is zero.
- The on hand inventory deteriorates at a constant rate θ (0< θ <1) per unit time. The deteriorated items are withdrawn immediately from the warehouse and there is no provision for repair or replacement.
- The rate of demand R (t) is ramp type demand function of t.

$$R(t) = D_0\left[t-(t-\mu)H(t-\mu)\right] \qquad D_0 > 0$$

- Where H(t–μ) is Heaviside's function defined as follows:

$$H(t-\mu) = \begin{cases} 1 & t \geq \mu \\ 0 & t < \mu \end{cases}$$

- Unsatisfied demand is backlogged at rate $e^{-\alpha x}$, where x is the time up to the next replenishment and 'α' a parameter

$$0 < \alpha < \frac{1}{T}$$

- The unit price is subject to the same inflation rate as other related costs.

The following notations are used throughout this investigation:

T – The fixed length of each ordering cycle.
S – The maximum inventory level for each ordering.
r – The inflation rate.
C_h – The inventory holding cost per unit per unit of time.
C_s – The shortage cost per unit per unit of time.
C_d – Deterioration cost per unit of deteriorated item.
I(t) – The on hand inventory at time t over [0, T).
CI – The amount of inventory carried during a cycle.
DI – The total number of items which deteriorate during a cycle.

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SI – The amount of shortage during a cycle.

MATHEMATICAL MODEL AND SOLUTION:

The objective of the inventory problem here is to determine the optimal order quantity so as to keep the total relevant cost as low as possible under inflation has been subjected to the inventory starts with shortages.

The work of further investigation and we shall discuss the inventory model for deteriorating items under inflation, where the inventory starts with shortages.

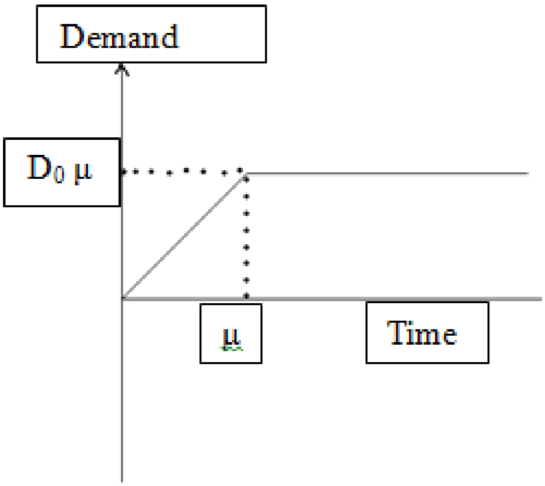


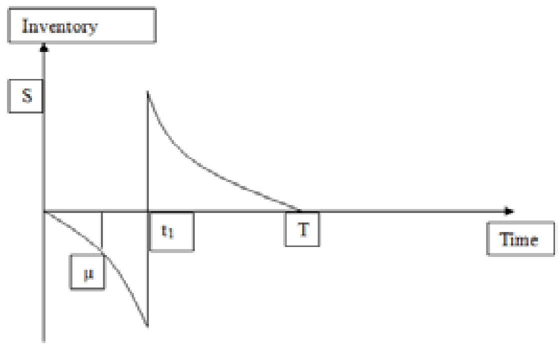
Figure 1. A ramp type function of the demand rate [Adapted from Mandal and Pal [5]

The behavior of the inventory system at any time during a given cycle is depicted in figure 2. There are two different situations may arise due to time t_1, (i) $\mu < t_1$ and (ii) $\mu > t_1$.

$$\frac{dI(t)}{dt} = -e^{-\alpha(t_1-t)}R(t) \qquad 0 \le t \le t_1 \qquad \text{.... (1)}$$

$$\frac{dI(t)}{dt} + \theta I(t) = R(t) \qquad t_1 \le t < T \qquad \text{.... (2)}$$

Situation I: When $\mu < t_1$



In this situation, the above two governing equation become

$$\frac{dI(t)}{dt} = -e^{-\alpha(t_1-t)}\,D_0\,t \qquad 0 \le t \le \mu \qquad \text{.... (3)}$$

$$\frac{dI(t)}{dt} = -e^{-\alpha(t_1-t)}\,D_0\,\mu \qquad \mu \le t \le t_1 \qquad \text{.... (4)}$$

and $$\frac{dI(t)}{dt} + \theta I(t) = -D_0\,\mu \qquad t_1 \le t < T \qquad \text{.... (5)}$$

The solutions of the differential equation (3)-(5) with the boundary conditions I(0) = 0 and I(T) = 0 are

$$I(t) = -\frac{D_0}{\alpha^2}\left[e^{-\alpha(t_1-t)}(\alpha t-1) + e^{-\alpha t_1}\right] \qquad 0 \le t \le \mu \qquad \text{.... (6)}$$

$$I(t) = -\frac{D_0}{\alpha^2}\left[e^{-\alpha(t_1-t)}\mu\alpha - e^{-\alpha(t_1-\mu)} + e^{-\alpha t_1}\right] \qquad \mu \le t \le t_1 \qquad \text{.... (7)}$$

$$I(t) = \frac{D_0\,\mu}{\theta}\left[e^{\theta(T-t)} - 1\right] \qquad t_1 \le t < T \qquad \text{.... (8)}$$

Following the approach of Misra [5,6], we obtain the present value of the holding cost duri
[t1, T) as

$$= C_h \left[\frac{D_0 \mu}{r(\theta+r)} e^{-rT} + \frac{e^{\theta T-(\theta+r)t_1}}{\theta(\theta+r)} D_0 \mu - \frac{D_0 \mu \, e^{-rt_1}}{r\theta} \right] \qquad \text{.... (9)}$$

The present value of the deterioration cost during the period [t1, T) as

$$= C_d \, \theta \left[\frac{D_0 \mu}{r(\theta+r)} e^{-rT} + \frac{D_0 \mu}{r(\theta+r)} e^{\theta T-(\theta+r)t_1} - \frac{D_0 \mu \, e^{-rt_1}}{r\theta} \right] \qquad \text{.... (10)}$$

The present value of the shortage cost during the period [0, t1] as

$$= C_S \frac{D_0}{\alpha^2} \left[\frac{\alpha^2}{r(\alpha-r)^2} e^{-\alpha t_1} \left(1-e^{(\alpha-r)\mu}\right) + \frac{e^{-(\alpha+r)t_1}}{r} \left(e^{\alpha\mu}-1\right) + \frac{\mu\alpha \, e^{-rt_1}}{(\alpha-r)} \right] \qquad \text{.... (11)}$$

The order quantity during the period [0, T) is

$$Q = \frac{D_0 \mu}{\theta} \left[e^{\theta(T-t_1)} - 1 \right] + \left[\frac{D_0 \mu}{\alpha} + \frac{D_0 \, e^{-\alpha t_1}}{\alpha^2} - \frac{D_0}{\alpha^2} e^{-\alpha(t_1-\mu)} \right] \qquad \text{.... (12)}$$

Thus, the total relevant cost of the system during the time interval [0, T), which is

$$X_2 = \left(C_h + C_d \, \theta\right) \left[\frac{D_0 \mu}{r(\theta+r)} e^{-rT} + \frac{D_0 \mu}{\theta(\theta+r)} e^{\theta T-(\theta+r)t_1} - \frac{D_0 \mu e^{-rt_1}}{r\theta} \right]$$
$$+ C_S \frac{D_0}{\alpha^2} \left[\frac{\alpha^2 \, e^{-\alpha t_1}}{r(\alpha-r)^2} \left(1-e^{(\alpha-r)\mu}\right) + \frac{e^{-(\alpha+r)t_1}}{r} \left(e^{\alpha\mu}-1\right) + \frac{\mu\alpha \, e^{-rt_1}}{(\alpha-r)} \right] \qquad \text{.... (13)}$$

Thus, the average total cost per unit time is

$$T C_2(t_1) = \frac{X_2}{T} \qquad \text{.... (14)}$$

The optimal value of t_1 for the minimum average total cost per unit of time is the solution

$$\frac{d \, T C_2(t_1)}{d \, t_1} = 0 \qquad \text{.... (15)}$$

Provided that the value of t_1 satisfies the condition

$$\left. \frac{d^2 \, TC_2(t_1)}{d \, t_1^2} \right|_{t=t_1^*} > 0 \qquad \text{.... (16)}$$

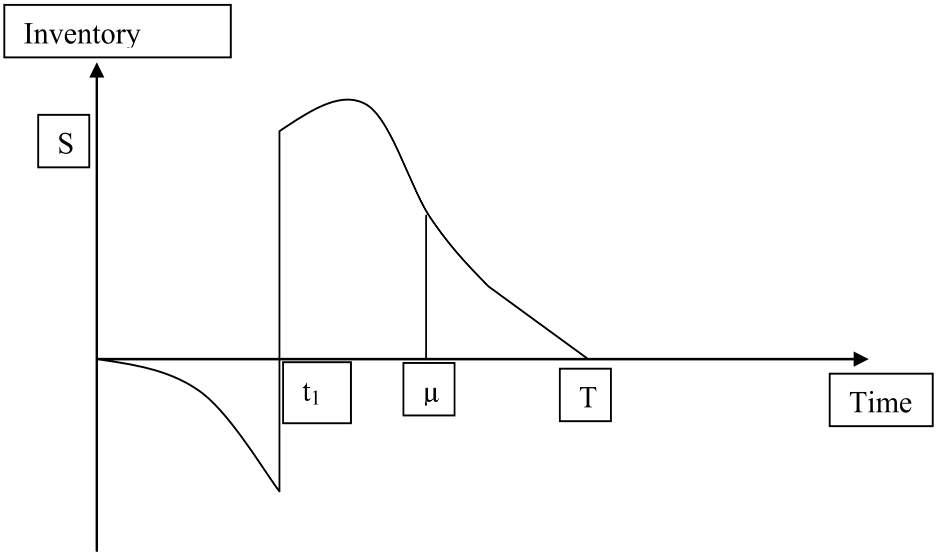
Equation (15) is equivalent to

$$\left(C_h + C_d\,\theta\right)\frac{D_0\,\mu}{\theta}\left[e^{-rt_1} - e^{\theta T-(\theta+r)t_1}\right] + C_s\left[\frac{D_0\,\alpha}{r(\alpha-r)^2}\left(e^{(\alpha-r)\mu}-1\right)e^{-\alpha t_1} - \frac{D_0(\alpha+r)}{\alpha^2 r}e^{-(\alpha+r)t_1}\left(e^{\alpha\mu}-1\right) - \frac{D_0\,\mu\, e^{-rt_1}\,r}{\alpha(\alpha-r)}\right] = 0 \quad \text{.... (17)}$$

For different values of the various parameters, equation (17) can be solved and find the optima value t_1*.

By using the optimal value of t_1*, the minimum average total cost per unit of time can be obtained from equation (14).

Situation II: ($\mu > t_1$)



In this situation, the above two governing equations, equation (1) and equation (2), becomes

$$\frac{d\,I(t)}{d\,t} = -\,e^{-\alpha(t_1-t)}\,D_0 t \qquad 0 \le t \le t_1 \qquad \text{.... (18)}$$

$$\frac{d\,I(t)}{d\,t} + \theta\, I(t) = -D_0 t \qquad t_1 \le t \le \mu \qquad \text{.... (19)}$$

and $$\frac{d\,I(t)}{d\,t} + \theta\, I(t) = -D_0\,\mu \qquad \mu \le t < T \qquad \text{.... (20)}$$

The solutions of the differential equations (37)-(39) with the boundary conditions I(0) = 0 and I(T) = 0 are

$$I(t) = -\frac{D_0}{\alpha^2}\left[e^{-\alpha(t_1-t)}(\alpha t-1)+e^{-\alpha t_1}\right] \qquad 0 \le t \le t_1 \qquad \text{.... (21)}$$

$$I(t) = Se^{-\theta(t-t_1)} - \frac{D_0}{\theta^2}\left\{(\theta t-1)-(\theta t_1-1)\,e^{-\theta(t-t_1)}\right\} \qquad t_1 \le t \le \mu \qquad \text{.... (22)}$$

$$I(t) = \frac{D_0\,\mu}{\theta}\left[e^{\theta(T-t)}-1\right] \qquad \mu \le t < T \qquad \text{.... (23)}$$

Now, in equations (22)-(23), the values of I(t) at t = μ should coincide, which implies that

$$S = \frac{D_0\,\mu}{\theta}\,e^{\theta(T-t_1)} - \frac{D_0}{\theta^2}\left[e^{\theta(\mu-t_1)}+\theta t_1-1\right] \qquad \text{.... (24)}$$

As discussed in situation I, the average total cost per unit of time is

$$TC_3(t_1) = \frac{1}{T}\left[(C_h+C_d\,\theta)\left\{\frac{D_0\,e^{-\mu r}}{r^2(\theta+r)}+\frac{D_0\,e^{-(\theta+r)t_1}}{\theta^2(\theta+r)}\left(\mu\theta e^{\theta T}-e^{\mu\theta}\right) + \frac{D_0\,\mu e^{-rT}}{r(\theta+r)}-\frac{D_0}{\theta^2 r^2}(\theta r t_1-r+\theta)\,e^{-rt_1}\right\} + C_S\frac{D_0}{\alpha^2}\left\{\frac{(\alpha t_1-1)}{(\alpha-r)}e^{-rt_1}-\frac{\alpha}{(\alpha-r)^2}e^{-rt_1}-\frac{e^{-(\alpha+r)t_1}}{r}+\frac{\alpha^2}{r(\alpha-r)^2}e^{-\alpha t_1}\right\}\right] \qquad \text{.... (25)}$$

The optimal value of t_1 for the minimum average total cost per unit of time is the solution

$$\frac{d\,TC_3(t_1)}{d\,t_1} = 0 \qquad \text{.... (26)}$$

Provided that the value of t_1 satisfies the condition

$$\left.\frac{d^2TC_3(t_1)}{d\,t_1^2}\right|_{t=t_1^*} > 0$$

Equation (26) is equivalent t

$$(C_h+C_d\,\theta)\left\{\frac{D_0}{\theta}(\theta t_1-1)\,e^{-rt_1}-\frac{D_0}{\theta^2}\,e^{-(\theta+r)t_1}\left(\mu\theta e^{\theta T}-e^{\mu\theta}\right)\right\} + C_S\frac{D_0}{\alpha^2}\left\{\frac{(\alpha+r-\alpha r t_1)}{(\alpha-r)}e^{-rt_1}+\frac{\alpha r e^{-rt_1}}{(\alpha-r)^2}+\frac{(\alpha+r)}{r}e^{-(\alpha+r)t_1}-\frac{\alpha^3 e^{-\alpha t_1}}{r(\alpha-r)^2}\right\} = 0 \qquad \text{.... (27)}$$

Equation (27) can be solved and find the optimal value of t_1^*. Using the optimal value t_1^*, the optimal value S* and the minimum average total cost per unit of time can be obtained from equation (25) and (26) respectively.

The optimal order quantity Q* is

$$Q^* = S^* + \frac{D_0\,t_1^*}{\alpha} - \frac{D_0}{\alpha^2}\left(1-e^{-\alpha t_1^*}\right)$$

Volume : 5 | Issue : 6 | June 2015 | ISSN - 2249-555X

Numerical Example:
The theory presented above can be illustrated using an numerical example adopted from Kun-Shan WU and Liang-Yuh-Ouyang [10]. The input parameters are as follows:

C_h = $ 3 per unit per year, C_d = $ 5 per unit, C_s = $ 15 per unit per year, D_0 = 100 units, μ = .12 year, q = .001, T = 1 year, a = .08, r = 0.05.

The optimal solutions for this model are given in Table 1. Notice that the exact solution for this Model are better than to the result of Kun-Shan Wu and Liang-Yuh-Ouyang [10] because the former has a smaller minimum average total cost per unit of time (13.0835 vs. 13.3968).

Optimal Solution of the Proposed Inventory System

Optimal Solution	Model	
	Situation-I: $\mu < t_1^*$	Situation-II: $\mu > t_1^*$
t_1^*	0.219	---
S^*	9.3757	---
Q^*	11.2710	---
TC^*	13.0835	---

--- denotes the infeasible solution.

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