RESEARCH PAPER | Engineering | Volume : 5 | Issue : 3 | March 2015 | ISSN - 2249-555X

Sustainable Buildings with Brick Masonry Slabs

KEYWORDS: Brick masonry; heterogeneous; meridional stress; hoop stress; dome; sustainable buildings.

V. K. Dogra	Dr. S. N. Sinha
Assistant Professor, Shri Mata Vaishno Devi University Katra, Jammu and Kashmir, INDIA	Former Professor, Civil Engineering Department, I. I. T. Delhi, INDIA

ABSTRACT *Following paper explores the use of brick masonry for square and rectangular slabs. Brick masonry is strong in compression but weak in tension. Brick masonry shallow domes can cover circular areas successfully as the axial meridional and hoop stresses are only compressive in shallow domes (2014). Further in shallow domes and thin shells, bending moments are insignificant. Moreover, bending moments and shear forces are not necessary for the stability of domes. In case of conventional slabs, vertical load is transmitted to edge beams / walls through bending moments and shear forces, whereas, the same vertical load is transmitted to edge beams through axial stresses in shallow domes. This possibility of transmitting the vertical loads by generating axial compressive stresses by way of changing the geometrical profile of the square / rectangular slab has been discussed in this paper. Different potential geometrical profiles for the slabs are presented which can be conveniently built of brick masonry, projecting brick masonry as a potential material for slabs in low-rise sustainable buildings in low intensity earthquake zones.*

1.0Introduction

Brick masonry is a heterogeneous material composed of two different homogeneous constituents, viz., clay brick units and mortar. The logic for using the brick masonry as a roofing material has been worked out from the study of the behavior of arches, thin shells, domes, vaults and pyramids.

Brick masonry is being used in the construction of buildings since very ancient times (Stewart, 1971). It is efficient and effective in compression but is weak in tension and shear. Thus brick masonry is used mostly in the construction of walls and pilasters. It is also used in the construction of arches, vaults and domes as spanning members as can be seen in many ancient buildings. Care is required to be taken that the stresses in the brick masonry are all compressive, and tensile stresses, if any, are of small magnitude within the permissible limit.

Francis, Horman, and Jerems, (1970); Hilsdorf (1969); Khoo and Hendry (1973); Mojsilovic (2005); Tikalsky, Atkinson and Hammons (1995); Frunzio, Gesualdo and Monaco (1997); Gumaste, Nanjunda, Venkataraman and Jagadish (2007); Hemant, Durgesh and Sudhir (2007); Sarangapani, Venkatarama and Jagadish (2005); have tried to establish compressive strength of brick masonry based on the contribution of the influencing factors. Lopez-Almansa, Surrablo, Lourenco, Barros, Roca, da Porto and Modena (2010); Da Porto, Casarin, Garbin, Grendene, Modena and Valluzzi (2005); Roca, Lopez-Almansa, Miquel and Hanganu (2007) have done extensive work on the design, construction and testing of brick masonry vaults.

In a spherical dome, the surface elements of the dome are subjected to axial stresses in hoop and meridional directions (Vazirani, & Ratvani, 1992). The stress in meridional direction is all compressive from crown to the base, when the dome is subjected to uniformly distributed load on its surface. However, hoop stress is compressive near the crown with a maximum value at the crown. The compressive hoop stress reduces downwards and it is zero at a point which makes an angle of 51° 48′ with the vertical axis passing through the crown. Thereafter, hoop stress is tensile and increases downwards (Vazirani & Ratvani, 1992). In case the dome is subjected to a point load at the crown, hoop stress is all tensile. Combined effect of uniformly distributed load, which is always present, and the point load at the crown, if any, will shift the point of zero tensile hoop stress upwards and the angle shall get reduced. Thus circular shallow domes are subjected to axial compressive stresses only (Sinha S. N. and Dogra V. K., 2014). The focus is to cover square and rectangular areas so that it is more useful in the construction industry.

The slab being in brick masonry has to be curved in both directions, so that the bending moments / bending tensile stresses are well within the limits. The rise at centre has to be optimum, so that the compressive stresses are within the permissible limits and the quantity of filling is minimised.

Possible geometrical profiles for the square / rectangular slabs are derived as discussed below.

2.0Possible Geometrical shapes

Basic geometrical shapes such as circle, parabola, catenary, ellipse and pyramid are the main shapes proposed for the brick masonry roofs. The basic shapes are also modified and more geometrical profiles are derived.

2.1Spherical

The simplest shape is spherical. It is double curved and many domes have been constructed with this geometry successfully. Fitting of this spherical shape in square / rectangular plan and other shapes derived from the basic spherical shape are as follows:

2.1.1 Simple Spherical

In this case, spherical geometry is used to cover square / rectangular areas in plan. The profile along the radial lines is taken as circular and the y-coordinates are worked out. Each arc of a circle springs from the same horizontal level at the edge of the slab and rises to maximum at the centre of the slab. The radius of each of these circular lines, along different radial directions, is different and is worked out on the basis of the length of the radial line in plan and

Volume : 5 | Issue : 3 | March 2015 | ISSN - 2249-555X

the maximum rise at the centre. Each radial line springs from the same level along the edges and rises up to the same maximum level at the crown, located at the centre of the slab panel. The value of the maximum rise at the crown is fixed, the level of the edge beam is same, but the length of the arc is different along each radial line. This results in a different value of radii along different radial lines. The shape is shown in Fig. 1.

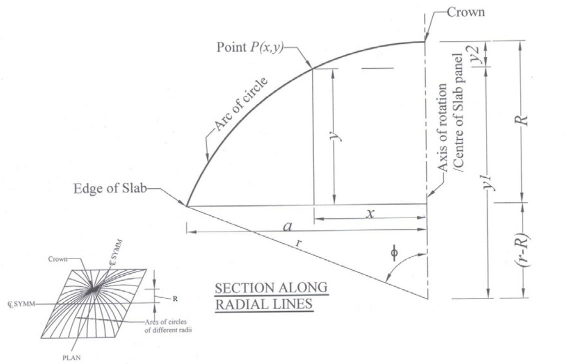

Fig. 1 Geometry of Simple Spherical Slab

The geometry is worked out as follows:
Radius r of the arc of a circle along a particular radial line is given by

$$r^2 = a^2 + (r-R)^2$$

or $$r^2 = a^2 + r^2 + R^2 - 2rR$$

or $$2rR = R^2 + a^2 \qquad \text{......(1)}$$

or $$r = \frac{R^2 + a^2}{2R}$$

And the vertical coordinate y, with respect to the vertical coordinate at the edge of the beam as zero, is given by

$$y_1 = \sqrt{\left(r^2 - x^2\right)}$$

$$y_2 = r - y_1 = r - \sqrt{\left(r^2 - x^2\right)}$$

$$y = R - y_2$$

or $$y = R - \left\{r - \sqrt{r^2 - x^2}\right\} \qquad \text{......(2)}$$

Where R is the maximum rise at the centre of slab with respect to the edge of beam, a is the maximum length of the radial line in plan from the centre of the slab to the edge of the beam and x is the distance in plan, along the radial line, from centre of slab to the point on the arc along the radial line.

2.1.2 Spherical with top flat
In this case, a circular portion of suitable dimension, say 500 mm diameter, at the centre of the slab is considered as flat. The radial lines drawn from the centre of the square slab are flat up to a distance of 250 mm, i. e., the edge of this flat portion. Thereafter, theses lines are curved as arcs of circle meeting the edge of the beam at lower level. Here also the radii of each arc is different as in case of 2.1.1, with the only difference that the radii are smaller than the corresponding radii in case of simple spherical. It gives two benefits; (i) it reduces the weight of the fill required to level the surface and (ii) the bending moments shall get further reduced. The flat area of 500 mm diameter may vary with span and thickness of slab.

The y coordinates along the radial lines are worked out from the geometry of the circle with the reduced chord length in plan, and the same value of rise. The shape is shown in Fig. 2.

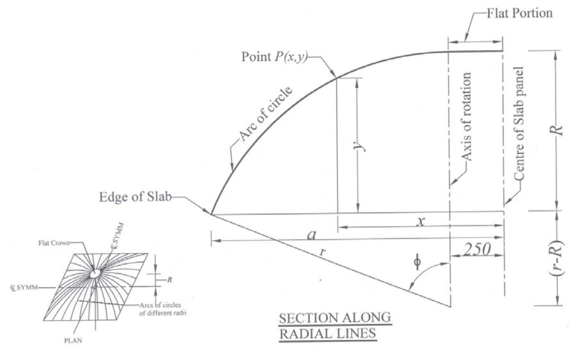

Fig. 2 Geometry of Spherical Flat Slab

The geometry is worked out as follows:

Radius r of the arc of a circle along the radial line is given by

$$r = \left(\frac{R^2 + (a-250)^2}{2R}\right) \qquad \text{......(3)}$$

And the vertical coordinate y, with respect to the vertical coordinate at the edge of the beam as zero, between the flat area and the edge of the beam, is given by

$$y = R - \left\{r - \sqrt{r^2 - (x-250)^2}\right\} \qquad \text{......(4)}$$

2.1.3 Spherical – 1
In this case, two circular arcs are drawn along the two diagonals. Each circle passes through the corner at the intersection of beams at the beam level, the centre of the slab panel at the crown and the opposite corner at the intersection of beams at the same beam level. The points at the same level, on these diagonal arcs are then joined by straight lines, to form the surface of the four sides of the slab panel. The arcs along the diagonal lines form curved ridges of the slab. The radius of the arc r and the y coordinate, along the diagonal, are worked out by using the equations 1 and 2 respectively. The geometry is shown in Fig 3.

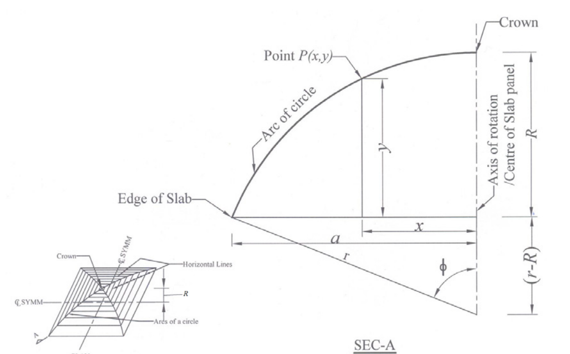

Fig. 3 Geometry of Spherical – 1 Slab

Volume : 5 | Issue : 3 | March 2015 | ISSN - 2249-555X

2.1.4 Spherical – 2

This geometry is achieved by drawing two circular arcs, from the midpoint of each beam, intersecting at the crown at the centre of the slab panel. Starting from the beam level, when we draw the arcs of the same radius and parallel to the arcs already drawn from the points on each beam, these arcs, from the adjacent sides, shall intersect at points along ridges. The points of intersection, along the diagonals, can be joined to form the ridges. In this case a discrete ridge, curved in elevation, is formed along the diagonal line. Except for the curved ridges, the radius of each arc springing from the edge beam is the same. The geometry is shown in Fig. 4.

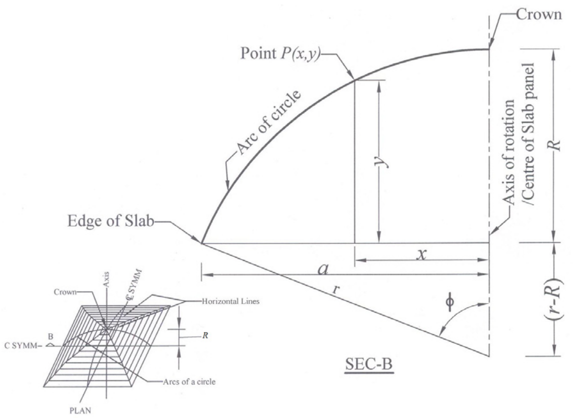

Fig. 4 Geometry of Spherical – 2 Slab

2.2Parabolic

Another type of double curvature roof can be parabolic. The parabola can be a simple parabola or cubic parabola. Some more shapes can also be derived from the parabola. These are as under:

2.2.1 Simple parabolic

In this case, the slab profile along the radial lines can be taken as parabolic. The equation for such simple parabola is,

$$y = R - k_1 x^2 \qquad \text{......(5)}$$

Where x is the distance in plan from the centre of the slab, along the radial line, y is the rise with respect to the leveled edge of the wall/beam, R is the maximum rise at the centre of slab as shown in Fig. 5. k_1 is a constant based on the maximum value of x and y for a particular radial line. The value of k_1 can be worked out from the boundary conditions, i.e., at $x = a$, y=0;

We get $0 = R - k_1 a^2$

Or $k_1 = \dfrac{R}{a^2}$

Substituting the value of a in equation 3, we get

$$y = R - \left(\frac{R}{a^2}\right) x^2 \qquad \text{......(6)}$$

Where a is the length of the radial line in plan, as shown in Fig. 5.

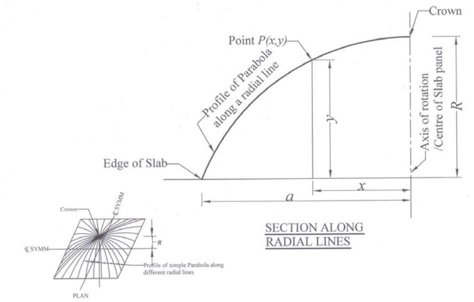

Fig. 5 Geometry of Simple Parabolic Slab

As a varies and its value is different along each radial line, the value of the constant k_1 also changes for calculation of y coordinate along each radial line.

2.2.2 Cubic parabolic

In this case, the slab profile is similar as for the parabola as above except that the equation for the parabola along the radial lines is taken as cubic as follows:

$$y = R - kx^3 \qquad \text{......(7)}$$

Where x is the distance, in plan, from the centre of the slab, along the radial line, y is the rise with respect to the leveled edge of the wall/beam, R is the maximum rise at the centre of slab. k is a constant based on the maximum value of x and y for a particular radial line whose value can be worked out from the boundary conditions, i.e., at $x = a$, $y = 0$,

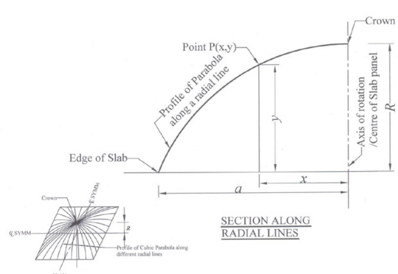

Fig. 6 Geometry of Cubic Parabola Slab

Therefore, $0 = R - ka^3$

Or $k = \dfrac{R}{a^3}$

Substituting the value of a in equation 7, we get

$$y = R - \left(\frac{R}{a^3}\right) x^3 \qquad \text{......(8)}$$

The value of the constant is different for each radial line which can be worked out. The maximum rise at the centre may be varied.

2.2.3 Parabolic with top flat

In this case, a circular portion of some reasonable diameter, say 500 mm, at the centre of the slab is considered flat at the top and the parabolic profile is considered along the radial lines, from the edge of the flat to the edge beam. This case is similar to the case of simple parabola, with the difference that the length of the radial lines get reduced by 250 mm. The Eq 8 gets modified as under:

$$y = R - \left(\frac{R}{(a-250)^2}\right)(x-250)^2 \qquad \text{......(9)}$$

Volume : 5 | Issue : 3 | March 2015 | ISSN - 2249-555X

2.2.4 Parabolic – 1

In this case, two parabolic curves are drawn along the two diagonals. Each curve passes through the corner, at the intersection of beams, the centre of the slab panel at the crown and the opposite corner at the intersection of beams. The points at the same level, on these diagonal curves are then joined by straight lines, to form the surface of the four sides of the slab panel. The parabolic lines along the diagonal lines form curved ridges of the slab. The profile of the parabola and the y coordinates, along the curved line in diagonal direction, are worked out by using the Eqs 5 and 6 respectively.

2.2.5 Parabolic – 2

This geometry is achieved by drawing two parabolic curved lines, from the midpoint of each beam, intersecting at the crown at the centre of the slab panel. These curved lines are now divided into an equal parts and the horizontal lines parallel to the beams are drawn from these points. These horizontal lines from adjacent sides, which are perpendicular to each other, and are at the same level, shall intersect on the diagonal line. The points of intersection, along the diagonals, can be joined to form the ridges. In this case a discrete ridge, curved in elevation, is formed along the diagonal line. Except for the curved ridges, the curved profile of other lines, springing from the edge beam, and which are not radial, is represented by same equation. Each triangular side of the slab panel is curved in one direction only. The profile and the y coordinates along the curved profile is worked out by using Eq. 6.

2.3Pyramidal

In this case, the basic shape of the roof is a pyramid. In simple pyramid, development of bending moments in the triangular plates is necessary for stability. Hence simple pyramid is not considered as the potential profile of the slab. Pyramid with varying slopes as described below has been considered:

2.3.1 Pyramid with varying slopes

Here the slope of the sides of pyramid are changed by providing folds on each triangular side of a regular pyramid. Thereby each triangular side is divided into three parts/plates. The Lowest and the middle plates are trapezoidal and the top plate is triangular. The lower two plates are bent at two different angles and the top plate is made horizontal. The four triangular horizontal plates from all the four sides to form a flat square plate at the centre as shown in Fig. 7.

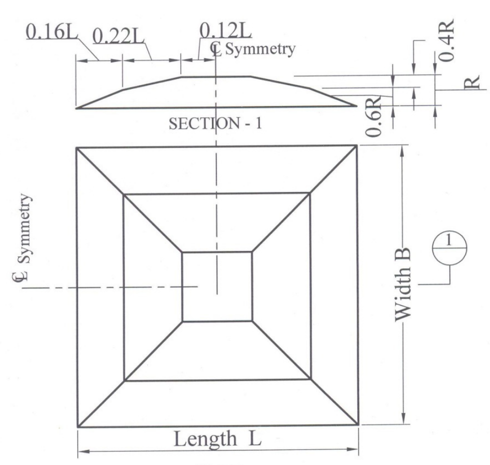

Fig. 7 Geometry of Pyramid with Varying Slopes

2.4Catenary

In this case, the geometry along the radial lines is worked out using the equation of the catenary as follows:

$$y = aCosh\left(\frac{x}{a}\right) \qquad \text{……(10)}$$

As the value of a is different, it is worked out separately for each radial line with the same rise at the centre. Geometry is shown in Fig. 8.

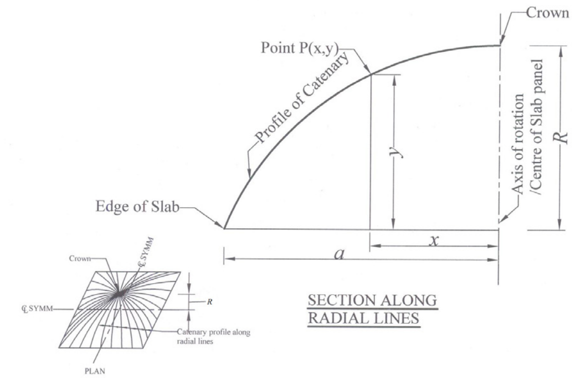

Fig. 8 Geometry of Slab with Catenary Profile along Radial Lines.

2.5Elliptical

In this case, the curved profile along the radial lines is considered elliptical given by,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{……(11)}$$

Where x and y are coordinates of points lying on the curved line and a, b are constants. The value of constant b is the maximum value of rise at the centre of slab and a is the horizontal distance from the centre of slab to edge beam along the radial line. The value of a is different along each radial line being maximum along the diagonal and minimum along the radial line passing through the middle of edge beam. The geometry is shown in Fig. 9.

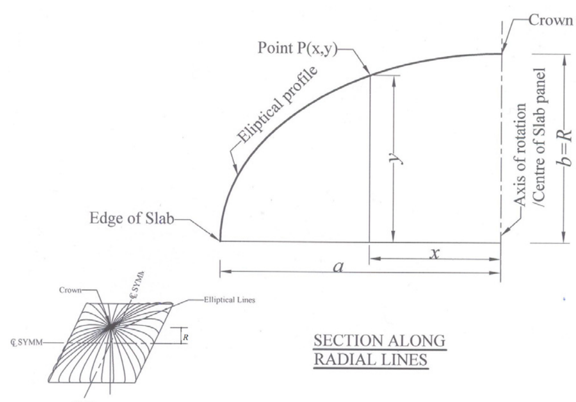

Fig. 9 Geometry of Slab with Elliptical Profile along Radial Lines.

2.7Conclusions

1. Twelve potential geometrical profiles for square / rectangular slabs have been presented as above for brick masonry slabs.

Volume : 5 | Issue : 3 | March 2015 | ISSN - 2249-555X

2. Eleven profiles are generated from circle, parabola, catenary and ellipse. These are curved in both directions. Moreover, change of curvature along adjacent radial lines is so gradual that a smooth curved profile in direction perpendicular to the radial lines is obtained.

3. One profile is based on pyramid / folded plates. The folds can be so proportioned that the bending moments in the trapezoidal plates are insignificant.

4. Any of the above twelve profiles can be analyzed and innovatively utilized for the construction of low cost houses in low intensity earthquake zones.

5. Construction of these brick masonry slabs is possible without the use of formwork and reinforcements.

REFERENCE

da Porto F., Casarin F., Garbin E., Grendene M., Modena C. & Valluzzi M. R., (2005), “Design assisted by testing of semi-prefabricated reinforced brick masonry vaults”, 10th Canadian Masonry Symposium, Banff, AB, Canada, pp. 261-270. | Francis, A. J., Horman, C. B. & Jerems, L. E. (1970), “The effect of joint thickness and other factors on the compressive strength of brickwork”, Proceedings 2nd International Brick Masonry Conference, Stroke-on Trent, England, pp. 31-37. | Frunzio, G., Gesualdo, A. & Monaco, M. (1997), “An anisotropic failure criterion for masonry”, Proceedings 4th International Symposium on Computer Methods in Structural Masonry, Florence, Italy, E & FN Spon Publ., pp. 1-7. | Gumaste K. S., Nanjunda Rao K. S., Venkatarama Reddy B. V. & Jagadish K. S., (2007), “Strength and elasticity of brick masonry prisms and wallets under compression”, Materials and Structures, 40(2), pp. 241-253. | Hemant B. Kaushik, Durgesh C. Rai & Sudhir K. Jain (2007), “Stress-strain characteristics of clay brick masonry under uniaxial compression”, Journal of Materials in Civil Engineering (ASCE), Vol. 19, No. 9, pp. 728-739. | Hemant B. Kaushik, Durgesh C. Rai & Sudhir K. Jain (2007), “Uniaxial compressive stress-strain model for clay brick masonry”, Current Science, Vol. 92, No. 4, pp. 497-501. | Hilsdorf, H. K. (1969), “Investigation into the failure mechanism of brick masonry loaded in axial compression”, Designing Engineering and Construction with Masonry Products, Gulf Publishing Co., Texas, pp. 34-41. | Khoo, C. L. & Hendry, A. W. (1973), “Strength tests on brick and mortar under complex stresses for the development of a failure criterion for brick masonry in compression”, Proceedings of British Ceramic Society, Load Bearing Brickwork (4), No. 21, pp. 51-66. | Lopez-Almansa, F., Roca P., Sarrablo, V., Cahis, X. & Canet, J. M. (2010), “Experiments on reinforced brick masonry vaulted light roofs”, ACI Structural Journal, Vol. 107, issue 3, pp. 355-363. | Lopez-Almansa, F., Surrablo V., Lourenco P. B., Barros J. A. O, Roca P., da Porto F. & Modena C., (2010), “Reinforced brick masonry light vaults: semi-prefabrication, construction, testing and numerical modeling”, Construction and Building Material, Volume 24, issue 10, pp. 1799-1814. | Mojsilovic, N. (2005), “A discussion of masonry characteristics derived from compression tests”, 10th Canadian Masonry Symposium, Banff, Alberta, June 2005. | Roca P., Lopez-Almansa F., Miquel J. & Hanganu A. (2007), “Limit analysis of reinforced masonry vaults”, Engineering Structures, Vol. 29, No.3, pp. 431-439. | Sarangapani G., Venkatarama Reddy B. V., & Jagadish K. S., (2005), “Brick-mortar bond and masonry compressive strength”, Journal of Material in Civil Engineering (ASCE), 17(2), pp. 229-237. | Dogra V. K. and Sinha S. N., (2014), “Brick masonry shallow dome as an alternative to conventional floor slab”, Indian Journal of Applied Research, vol. 4, Issue 9, pp. 192-203. | Stewart (1971), “The Pyramids and Sphinx”, Newsweek Book Division, New York, N.Y. | Tikalsky, P. J., Atkinson, R. H. & Hammons, M. I. (1995), “Compressive strength of reinforced masonry under lateral tension”, Journal of Structural Engineering, ASCE, Vol. 121, No. 2, pp. 283-289. | Vazirani V. N. & Ratvani M. M., (1992), “Domes”, Concrete Structures, 15th edition, Khanna Publishers, pp. 497-498. |