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Connectedness in Biclosure Space

KEYWORDS: Closure space, connectedness in closure space, biclosure space, connectedness in biclosure space.

U.D.TAPI
Department of Applied Mathematics and Computational Science Shri G. S. Institute of Technology and Science (M.P.), India

BHAGYASHRI A. DEOLE
Department of Applied Mathematics and Computational Science Shri G. S. Institute of Technology and Science (M.P.), India

ABSTRACT *Our aim, in the present paper, is to introduce connectedness in biclosure space and study some of its fundamental properties.*

INTRODUCTION:

Čech Closure space was introduced by E. Čech [3] in 1966. The notions of closure system and closure operator are very useful tools in several areas of classical mathematics. They play an important role in topological spaces, Boolean algebra, convex sets etc.

Biclosure space was introduced by Chandrasekhar Rao, Gowri and Swaminathan [4]. Such spaces are equipped with two arbitrary closure operators. They extended some of the standard results of separation axioms in closure space to biclosure space. Thereafter a large number of papers have been written to generalize the concept of closure space to biclosure space. **In this paper we introduce connectedness in Biclosure space and study some of its fundamental properties.**

1. PRELIMINARIES:

Definition 2.1 [1]:- An operator $u: P(X) \to P(X)$ defined on the power set P(X) of a set X satisfying the axioms:

1. $u\phi=\phi$,
2. $A \subseteq uA$, for every $A \subseteq X$,
3. $u(A \cup B) = uA \cup uB$, for all $A, B \subseteq X$.

is called a Čech closure operator and the pair (X, u) is a Čech closure space.

Definition 2.2[2]:-Two maps u_1 and u_2 from power set of X to itself are called biclosure operators for X if they satisfies the following properties:

(1) $u_1\phi = \phi$, $u_2\phi = \phi$;
(2) $A \subseteq u_1A$, $A \subseteq u_2A$, for all $A \subseteq X$;
(3) $u_1(A \cup B) = u_1 A \cup u_1B$, $u_2(A \cup B) = u_2 A \cup u_2B$, for all $A \subseteq X$.

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(3) $u_1(A \cup B) = u_1 A \cup u_1B$, $u_2(A \cup B) = u_2 A \cup u_2B$, for all $A \subseteq X$.

A structure (X, u_1, u_2) is called a biclosure space.

Definition 2.3[6]:- A closure space (X, u) is connected if there exists any continuous mapping f from X to the discrete space {0, 1} is constant .A subset A in a closure space (X, u) is said to be connected if A with the subspace topology is a connected space.

Definition 2.4[5]:- Let (X, u_1, u_2) and (Y, v_1, v_2) be biclosure spaces and let $i \in \{1, 2\}$. Then a mapping f: $(X, u_1, u_2) \rightarrow (Y, v_1, v_2)$ is called:

(i) i -open (respectively, i -closed) if the mapping $f: (X, u_i) \rightarrow (Y, v_i)$ is open (respectively, closed).

(ii) Open (respectively, closed) if f is i -open (respectively, i -closed) for all $i \in \{1, 2\}$.

(iii) i -continuous if the mapping $f: (X, u_i) \rightarrow (Y, v_i)$ is continuous.

(iv) Continuous if f is i -continuous for all $i \in \{1, 2\}$.

CONNECTEDNESS IN BICLOSURE SPACE:

Definition 3.1:- A biclosure space (X, u_1, u_2) is connected if there exists any continuous mapping f from X to the discrete space {0, 1} is constant. A subset A in biclosure space (X, u_1, u_2) is said to be connected if A with the subspace topology is a connected space.

Example 3.2:- Let X = {a, b, c} bea non empty set.

Consider a closure operator u_1: P(X) $\rightarrow$ P(X) such that
$u_1\{a\} = \{a, b\}$,$u_1\{b\} = u_1\{c\}= u_1\{b, c\}=\{b, c\}$,
$u_1\{a, b\}= u_1\{a, c\}= u_1\{X\}=X$, $u_1\{\phi\}= \phi$,
Hence (X,u_1) is closure space.
Open sets of closure space $(X, u_1) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, X, \phi\}$.

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Consider another closure operator u_2: P(X) $\rightarrow$ P(X) such that

$u_2\{a\}=\{a\}$, $u_2\{b\}=\{b\}$, $u_2\{c\}=\{c\}$,$u_2\{a, b\} = u_2\{a, c\} = u_2\{b, c\}=X$.

$u_2\{\phi\}=\phi$, $u_2\{X\}=X$,

Hence (X, u_2) is a closure space.

Open sets of closure space (X, u_2) = {{a, b}, {a, c}, {b, c}, X, ϕ}.

Then (X, u_1, u_2) is a biclosure space.

Open sets of (X, u_1, u_2) = {{a, b}, {a, c}, X, ϕ}

We define a continuous mapping f: X $\rightarrow$ {0, 1} such that

$f^{-1}\{1\}=\{a\}=\{b\}=\{c\}=\{a, b\}=\{c, a\}=\{b, c\}=\{X\}$,$f^{-1}\{0\}=\phi$ is constant.

Hence (X, u_1, u_2) is a connected biclosure space.

Example 3.3:- Let X = {a, b, c, d} be a non empty set.

Consider a closure operator u_1: P(X) $\rightarrow$ P(X) such that

$u_1\{a\}=\{a, b\}$, $u_1\{b\}=\{a, b\}$, $u_1\{c\}=\{b, c\}$, $u_1\{d\}=\{c, d\}$,

$u_1\{X\} =X$, $u_1\{\phi\} =\phi$.

For all A contained in X, let

$$u_1\{A\} = \begin{cases} \phi;\ \text{if } A=\phi, \\ \cup\{u_1(a) : a \in A\}; \text{otherwise.} \end{cases}$$

Hence (X, u_1) is a closure space.

Open sets of (X, u_1) =

{{a}, {b}, {c}, {d}, {b, c}, {c, d}, {d, a}, {b, c, d}, {c, d, a}, {d, a, b}, X, ϕ}.

Consider another closure operator u_2:P(X) $\rightarrow$ P(X)

such that $u_2\{a\} = \{a, b, c\}, u_2\{b\} = \{b, c, d\}, u_2\{c\} = \{c, d, a\}$,

$u_2\{d\} = \{d, a, b\}, u_2\{X\} = \{X\}, u_2\phi = \phi$.

$$u_2\{A\} = \begin{cases} \phi;\ \text{if } A=\phi, \\ \cup\{u_2(a) : a \in A\}; \text{otherwise.} \end{cases}$$

Hence (X, u_2) is a closure space.

Open sets of (X, u_2) =

{{a}, {b}, {c}, {d}, {a, b},{b, c}, {c, d}, {d, a}, {a,b,c},{b, c, d}, {c, d, a}, {d, a, b}, X, ϕ}

Open sets of biclosure space (X, u_1, u_2)={{a}, {b}, {c}, {d}, {b, c}, {c, d}, {d, a},
{b, c, d}, {c, d, a}, {d, a, b}, X, ϕ}.

Let a continuous mapping f:X $\rightarrow$ {0, 1} such that

$f^{-1}\{1\} = \{a\}=\{b\}=\{c\}=\{d\}=\{b, c\}=\{c, d\}=\{d, a\}=\{b, c, d\}=\{c, d, a\}=\{d, a, b\}=X$.

$f^{-1}\{0\} = \phi$.

Hence (X, u_1, u_2) is a connected biclosure space.

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4. PROPERTIES OF CONNECTED BICLOSURE SPACE:

Proposition 4.1:- A biclosure space (X, u_1, u_2) is connected if and only if it has no separation.

Proof: - If (X, u_1, u_2) is connected biclosure space and it has a separation U| V. Define $f: X \rightarrow \{0, 1\}$ by $f(x) = 0$ if $x \in U$, $f(x) = 1$ if $x \in V$. Then f is continuous but not constant. It contradicts the connectedness of biclosure space (X, u_1, u_2). Hence biclosure space (X, u_1, u_2) has no separation.

Conversely, consider biclosure space (X, u_1, u_2) has no separation, if X is not connected. Then there exists a continuous map $f: X \rightarrow \{0, 1\}$ is not constant. $f^{-1}(0)$ and $f^{-1}(1)$ is non-empty and

$$f^{-1}(0) \cup f^{-1}(1) = X$$
$$f^{-1}(0) \cap f^{-1}(1) = \phi.$$

Thus biclosure space X has a separation $f^{-1}(0) \left| f^{-1}(1) \right.$, which is a contradiction. Hence biclosure space (X, u_1, u_2) is connected.

Proposition 4.2:- A continuous image of a connected biclosure space is connected.

Proof: - Let $f: X \rightarrow Y$ be a continuous surjective map where X, Y are biclosure spaces and X is connected biclosure space. Suppose biclosure space Y is not connected, then by proposition 4.1, Y has a separation, say $U|V$. Let $M = f^{-1}(U), N = f^{-1}(V)$, then U, V are disjoint open sets of X and form a separation of biclosure space X. This contradicts the fact that X is connected biclosure space. Therefore Y must be connected biclosure space.

Proposition 4.3:- The union of any family of connected sets of biclosure space (X, u_1, u_2) with a common point is connected.

Proof:-Let $\{X_\alpha\}$ be a family of connected sets and $p \in X_\alpha$ for all α. Let $f: \cup X_\alpha \rightarrow \{0, 1\}$ be any continuous map and $f_\alpha: X_\alpha \rightarrow \{0,1\}$ be the restriction of f to X_α. Since f is continuous, each f_α is

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continuous. X_α is connected so f_α is constant. Now let $p \in X_\alpha$ for all α and $f_\alpha(x_\alpha) = f(p)$, for all α and $f(\cup X_\alpha) = f(p)$ i.e. f is constant. Therefore $\cup X_\alpha$ is connected set of biclosure space (X, u_1, u_2).

Proposition 4.4:-Let A and B are subsets of a connected biclosure space (X, u_1, u_2) such that $A \subseteq B \subseteq u_1 A$ and $A \subseteq B \subseteq u_2 A$. If A is connected, then B is connected.

Proof:-If B is not connected; it has a separation $U|V$. Since A is connected so $A \subseteq U$ or $A \subseteq V$. Without loss of generality, let $A \subseteq U$.As $A \subseteq U \subset B$, we take closure of A and U in B,

$u_{1B}A \subseteq u_{1B}U$, $u_{2B}A \subseteq u_{2B}U$. Also, $u_{1B}\ A = u_{1B}\ A \cap B = B$, $u_{1B}U \subseteq B$, $u_{2B}A = u_{2B}A \cap B = B$, $u_{2B}U \subseteq B$ we have $B = U$. Thus $U|V$ is not a separation, and B is connected.

For each point p in a connected biclosure space (X, u_1, u_2), the component $C(p)$ of X is the largest connected set in X which contains the point p.

Proposition 4.5:-For each point p in a connected biclosure space (X, u_1, u_2), the component C(p) of X is a closed set of X.

Conclusion: - In this paper the idea of connectedness in biclosure space was introduced.

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