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1 -Near Mean Cordial Labeling of Wheel , Globe, Binary Trees, Odd Cycle with a Chord Graphs

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ABSTRACT Let G=(V,E) be a simple graph. A surjective function $f:V\rightarrow\{0,1,2\}$ is said to be a 1-Near Mean Cordial labeling if for each edge uv, the induced map

$$f(uv)=0 \text{ if } \frac{f(u)+f(v)}{2} \text{ is an integer}$$
$$1 \quad \text{Otherwise}$$

Satisfies the condition $|e_f(0)-e_f(1)| \le$ where $e_f(0)$ is the number of edges with 0 label and $e_f(1)$ is the number of edges with 1 label. G is said to be 1-Near Mean Cordial Graph if it has a 1-Near Mean Cordial Labeling. In this paper we proved that Wheel Graph $W=C_n^+$, Globe Graph $Gl(n)$, Binary Trees , Odd cycle C_n with a chord are 1-Near Mean Cordial Graphs.

1. INTRODUCTION

Let us consider the graphs to be finite, undirected and simple. The vertex set and edge set of a graph G are denoted by V(G) and E(G) respectively. The cardinality of V(G) and E(G) are respectively called order and size of G. Labeling of graphs has enormous application in many practical problems involved in circuit designing, communication network, astronomy etc. [1]. The concept of mean cordial labeling was introduced by Raja Ponraj, Muthirulan Sivakumar and Murugesan Sundaram in the year 2012 in [5]. Some results on Mean Cordial Labeling was discussed in [3,4,5,6]. Let f be a function

V (G) to $\{0,1,2\}$. For each edge uv of G assign the label $\frac{f(u)+f(v)}{2}$. f is called a mean cordial labeling of G if $|v_f(i)-v_f(j)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$, $i,j\in\{0,1,2\}$ where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with x $(x=0,1,2)$ respectively. A graph with a mean cordial labeling is called mean cordial graph. K.Palani, J.Rejila Jeya Surya [2] introduced a new concept called 1-Near Mean Cordial labeling and investigated the 1-Near Mean Cordial Labeling behavior of Paths, Combs, Fans and Crowns. Terms defined here are used as in F. Harary [7].

2. PRELIMINARIES

K.Palani,J.Rejila Jeya Surya [2] define the concept of 1-Near Mean Cordial labeling as follows

Let $G=(V,E)$ be a simple graph. A surjective function $f:V\rightarrow\{0,1,2\}$ is said to be 1-Near Mean Cordial Labeling if for each edge uv, the induced map

$$f^*(uv)=0 \text{ if } \frac{f(u)+f(v)}{2} \text{ is an integer}$$
$$=1 \text{ otherwise}$$

Satisfies the condition $|e_f(0)-e_f(1)|\le 1$ where $e_f(0)$ is the number of edges with zero label and $e_f(1)$ is the number of edges with one label.

G is said to be a 1-Near Mean Cordial Graph if it has a 1-Near Mean cordial labeling. G is said to be 1-Near Mean Cordial Graph if it has a 1-Near Mean Cordial Labeling. In this paper we proved that Wheel Graph $W=C_n^+$, Globe Graph $Gl(n)$, Binary Trees , Odd cycle C_n with a chord are 1-Near Mean Cordial Graphs.

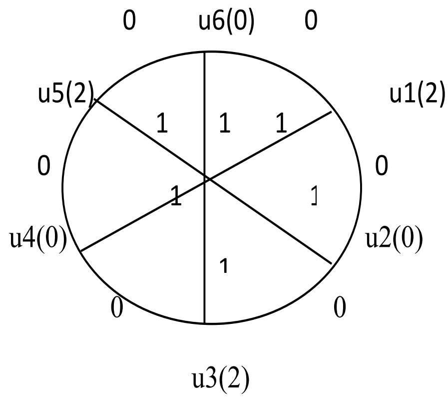

Fig.3.1.2 $W = c_6^+$

Theorem.3.2: Globe $Gl(n)$ is a 1-Near Mean Cordial Graph

Proof : Let $G = (V, E)$ be a simple graph and let G be $Gl(n)$. Let $V(G) = \{(u, v, w_i) : 1 \le i \le n\}$

and let $E(G) = [\{(uw_i) : 1 \le i \le n\} \cup \{(vw_i) : 1 \le i \le n\}]$

Define $f : V \to \{0,1,2\}$ by

Case .1 : When n is Odd

$f(u) = 0$

$f(v) = 1$

$f\left(w_{\frac{(n+1)}{2}}\right) = 2$ when $i = \frac{n+1}{2}$

$$\left.\begin{array}{l} f(w_i) = 1 \text{ if } i \equiv 1 \bmod 2 \\ \quad = 2 \text{ if } i = 0 \bmod 2 \end{array}\right\} \text{when } i \prec \frac{n+1}{2}$$

$$\left.\begin{array}{l} f(w_i) = 1 \text{ if } i = 0 \bmod 2 \\ \quad = 2 \text{ if } i \equiv 1 \bmod 2 \end{array}\right\} \text{when } i \succ \frac{n+1}{2}$$

Case .2 : When n is Even

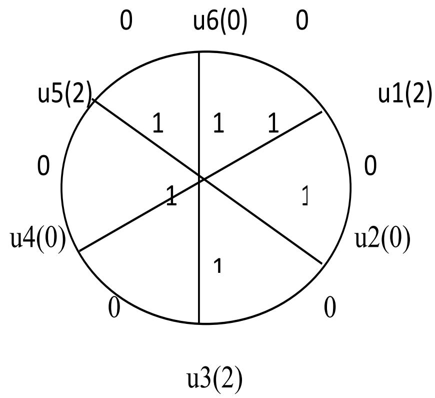

Fig.3.1.2 $W = c_6^+$

Theorem.3.2: Globe $Gl(n)$ is a 1-Near Mean Cordial Graph

Proof : Let $G=(V,E)$ be a simple graph and let G be $Gl(n)$. Let $V(G)=\{(u,v,w_i):1\le i\le n\}$

and let $E(G)=\left[\{(uw_i):1\le i\le n\}\cup\{(vw_i):1\le i\le n\}\right]$

Define $f:V\to\{0,1,2\}$ by

Case .1 : When n is Odd

$f(u)=0$

$f(v)=1$

$f\left(w_{\frac{(n+1)}{2}}\right)=2$ when $i=\frac{n+1}{2}$

$f(w_i)=1$ if $i\equiv 1 \bmod 2$
= 2 if $i=0 \bmod 2$ $\Big\}$ when $i\prec\frac{n+1}{2}$

$f(w_i)=1$ if $i=0 \bmod 2$
= 2 if $i\equiv 1 \bmod 2$ $\Big\}$ when $i\succ\frac{n+1}{2}$

Case .2 : When n is Even

$f(u)=0$

$f(v)=1$

$$f(w_i)=\begin{cases}1 \text{ if } i\equiv 1 \bmod 2 \\ 2 \text{ if } i=0 \bmod 2\end{cases} \quad \text{for } 1\le i\le n$$

Hence the induced edge labeling are

Case .1 : When n is Odd

$$f^*(uw_i)=\begin{cases}1 \text{ if } i\equiv 1 \bmod 2 \\ 0 \text{ if } i\equiv 0 \bmod 2\end{cases} \quad \text{when } i \prec \frac{n+1}{2}$$

$$f^*(uw_i)=\begin{cases}1 \text{ if } i\equiv 0 \bmod 2 \\ 0 \text{ if } i\equiv 1 \bmod 2\end{cases} \quad \text{when } i \succ \frac{n+1}{2}$$

$$f^*\left(uw_{\frac{(n+1)}{2}}\right)=0 \quad \text{when } i=\frac{n+1}{2}$$

$$f^*(vw_i)=\begin{cases}0 \text{ if } i\equiv 1 \bmod 2 \\ 1 \text{ if } i=0 \bmod 2\end{cases} \quad \text{when } i \prec \frac{n+1}{2}$$

$$f^*(vw_i)=\begin{cases}0 \text{ if } i=0 \bmod 2 \\ 1 \text{ if } i\equiv 1 \bmod 2\end{cases} \quad \text{when } i \succ \frac{n+1}{2}$$

$$f^*\left(vw_{\frac{(n+1)}{2}}\right)=0 \text{ when } i=\frac{n+1}{2}$$

Case .2 : When n is Even

$$f^*(uw_i)=\begin{cases}1 \text{ if } i\equiv 1 \bmod 2 \\ 0 \text{ if } i=0 \bmod 2\end{cases} \quad \text{for } 1\le i\le n$$

$$f^*(vw_i) = 0 \text{ if } i \equiv 1 \bmod 2 \quad \text{for } 1 \le i \le n$$
$$= 1 \text{ if } i \equiv 0 \bmod 2$$

Hence it satisfies the condition $|e_f(0) - e_f(1)| \le 1$

Therefore the Globe $Gl(n)$ is a 1-Near Mean Cordial graph.

Illustration:

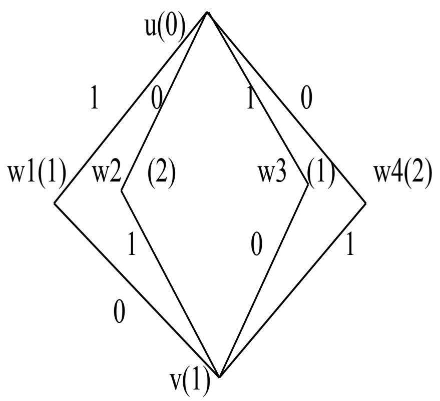

Fig.3.2.1 **:** Gl(4) graph

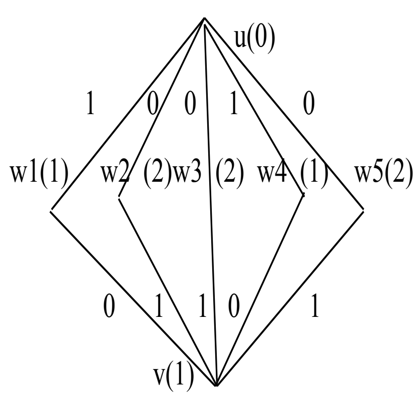

Fig.3.2.2: Gl(5) graph

Theorem .3.3: Binary Trees is 1-Near Mean Cordial Graph

Proof: Let $G = (V, E)$ is a simple graph be a Binary Tree. Let $v_1, v_2 ... v_n$ be the vertices and $E(G) = \{(v_1v_2) \cup (v_1v_3) \cup \{(v_iv_{2i}) : 1 \le i \le n\} \cup \{(v_iv_{(2i+1)}) : 1 \le i \le n\}\}$ be the edges of binary tree. Let us consider the root vertex to be v_1 at the top of the binary tree and other vertices are labeled from left to right side of the binary tree by level wise. Let the level of v_1 be 0 and level of $\{v_2, v_3\}$ be one and level of $\{v_3, v_4, v_5, v_6\}$ be two and so on.

For labeling the vertices

$f(v_1)=0$

$f(v_i)=1$ if $i=1 \bmod 2$ for $1 \le i \le n$

= 2 if $i=0 \bmod 2$

Then the induced labeling for edges

$f^*(v_1 v_2)=1$

$f^*(v_1 v_3)=0$

$f^*(v_i v_{2i})=0$ for i is even

=1 for i is odd

$f^*(v_i v_{2i+1})=1$ for i is even

= 0 for i is odd

Clearly it satisfies the condition $|e_f(0)-e_f(1)| \le 1$ where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1.

Hence Binary Trees 1-Near Mean Cordial graph.

Illustration:

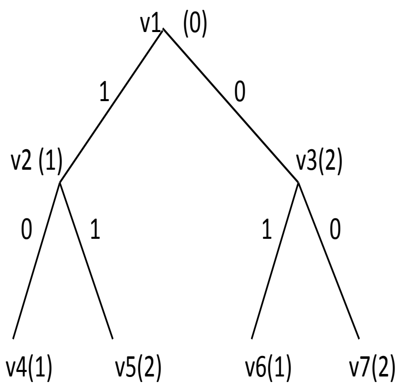

Fig. Binary Tree

Theorem.3.4: Odd Cycle c_n with a Chord is 1-Near Mean Graph

Proof: Let $G=(V,E)$ is a simple graph be a Odd Cycle graph with a Chord c_n. Let $V(G)=\{u_1,u_{21},u_{22},u_{31},u_{32}...\}$ be the vertices and

$E(G)=\left\{(u_1u_{21})\cup(u_1u_{22})\cup\left\{\left(u_{i1}u_{(i+1)1}\right):1\le i\le n-1\right\}\cup\left\{\left(u_{i2}u_{(i+1)2}\right):1\le i\le n-1\right\}\cup(u_{n1}u_{n2})\cup\left(u_{n2}u_{(n-1)1}\right)\right\}$ be the edges of Odd cycle c_n. Define the vertex labeling $f:V\to\{0,1,2\}$ as

$f(u_1)=1$

For $f(u_{ij})$ when i is even

$$f(u_{ij})=1 \text{ for } j=1\bmod 2, \quad = 2 \text{ for } j=0\bmod 2 \qquad 2\le i\le n, 1\le j\le 2$$

For $f(u_{ij})$ when i is odd

$$f(u_{ij})=0 \text{ for } j=1\bmod 2, \quad =2 \text{ for } j=0\bmod 2 \qquad 2\le i\le n, 1\le j\le 2$$

Then the induced labeling for edges

$f^*(u_1u_{2j})=0$ for $j=1\bmod 2$

= 1 for $j=0\bmod 2$

$f^*\left(u_{i1}u_{(i+1)1}\right)=1$ for $2\le i\le n-1$

$f^*\left(u_{i2}u_{(i+1)2}\right)=0 \; 2\le i\le n-1$

$f^*(u_{n1},u_{n2})=0$ when n is odd

$f^*(u_{n1},u_{n2})=1$ when n is even

The induced edge for the chord representing in the odd cycle c_n is given by

$f^*\left(u_{(n-1)1},u_{n2}\right)=1$ when n is odd

$f^*\left(u_{(n-1)1},u_{n2}\right)=0$ when n is even

Clearly it satisfies the condition $|e_f(0)-e_f(1)|\le 1$ where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1.

Hence Odd Cycle c_n is 1-Near Mean Cordial graph.

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Illustration :

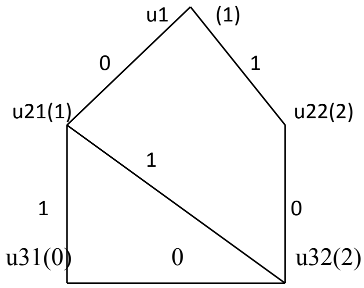

Fig . 3.4.1 : 5-Cycle c_5 with a chord

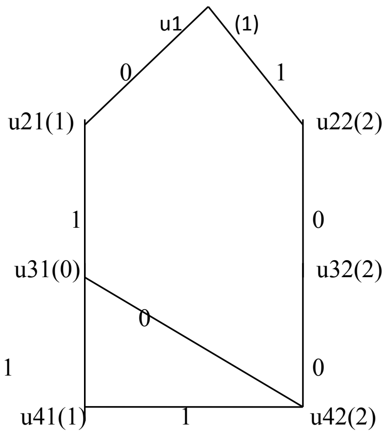

Fig .3.4.2: 7-Cycle c_7 with a Chord

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