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Variance of Time to Recruitment in a Single Graded Manpower System with Two Thresholds and Inter-Policy Decisions as an Order Statistics

KEYWORDS: Single grade manpower system, two sources of depletion, two thresholds, order statistics and univariate policy of recruitment.

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ABSTRACT *In this paper the problem of time to recruitment is studied for a single graded manpower system with two sources of depletion and two thresholds using a univariate policy of recruitment based on shock model approach. The variance of the time to recruitment is estimated when the inter-policy decision times form an order statistics and the number of transfer decisions is governed by a Poisson process.*

Introduction

Early studies related to manpower planning were reported by a number of researchers namely Bartholomew[1],[2], Grinold and Marshal[5], Mehlmann[8], Mukherjee and Chattopadhyay[10] in the context of providing elementary theory of wastages and their measures for a manpower system. For a single grade manpower system which is subjected to loss of manpower due to the policy decisions taken in the system and having a mandatory breakdown threshold for the cumulative loss of manpower, Sathiyamoorthi and Elangovan [12] who have initiated the study on finding the time to recruitment in this system, obtained the variance of the time to recruitment using a univariate policy of recruitment based on shock model approach when the loss of manpower are independent and identically distributed Poisson random variables, inter-policy decisions are independent and identically distributed exponential random variables and the mandatory breakdown threshold is a geometric random variable. Kasturi and Srinivasan[7] have studied this problem when the inter-policy decision times are exchangeable and constantly correlated exponential random variables. Venkat Lakshmi[14] studied this problem using a bivariate policy of recruitment. In[11], Muthaiyan et.al have studied the work in [12] for exponential loss of manpower and exponential breakdown threshold when the inter-policy decisions form an order statistics In [4], Esther clara has studied this problem for the first time by considering optional and mandatory thresholds for the cumulative loss of manpower when the inter-policy decision times are either independent and identically distributed exponential random variables or exchangeable and constantly correlated exponential random variables using univariate and bivariate policies of recruitment. Like policy decisions which form one source of depletion, transfer decisions also produces loss of manpower independently and hence, a second source of depletion can be considered to make manpower planning models more realistic. In this context, Elangovan et.al[3] have studied the problem of time to recruitment for a single grade manpower system with two sources of depletion when the inter-policy decisions and inter-transfer decisions form two ordinary renewal processes by

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considering only a mandatory exponential threshold. Usha et.al[13] have extended this work for correlated inter-policy decision times. Recently, Mercy Alice and Srinivasan[9] have studied the work of Elangovan et.al [3] for a manpower system with optional and mandatory exponential breakdown thresholds. The present paper extends the work of Mercy Alice and Srinivasan[9] when the inter- policy decision times follow an order statistics. For a single grade manpower system, the present work also extends the work of Elangovan et.al[3] for a system with two thresholds and that of Esther Clara[4] for a system with two sources of depletion.

Model Description

Consider an organization taking decisions at random epochs in $(0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. Let $X_{iP}, i = 1,2, \ldots.$ be the continuous random variables representing the amount of depletion of manpower due to the ith policy decision in the organization. It is assumed that X_{iP} form a sequence of independent and identically distributed exponential random variables with density function $f_P(.)$. Let $\bar{X}_{mP}$ be the cumulative loss of manpower in the first m policy decisions. For $j = 1,2, \ldots.$, let X_{jT} be continuous random variables representing the amount of depletion of manpower in the organization caused due to the jth transfer decision. It is assumed that X_{jT} form a sequence of independent and identically distributed exponential random variables with probability density function $f_T(.)$. Let $\bar{X}_{nT}$ be the cumulative loss of manpower in the first n transfer decisions. For each i and j, let X_{iP} and X_{jT} be statistically independent. Let Y (Z) be a continuous random variable denoting the optional (mandatory) threshold level such that Z > Y. It is assumed that Y and Z are independent. Let U_{iP} be the time between $(i-1)^{th}$ and $(i)^{th}$ inter policy decision times with distribution function $G_P(.)$ and mean $\frac{1}{\lambda_P}$ $(\lambda_P > 0)$. Let $U_{(1P)}\left(U_{(kP)}\right)$ be the smallest (largest) order statistic with probability density function $g_{U_{(1P)}}(.)\left(g_{U_{(kP)}}(.)\right)$. It is assumed that the number of transfer decisions follow a Poisson process with rate λ_T. Let $G_{mP}(.)[G_{nT}(.)]$ be the distribution of the waiting time upto m[n] policy decisions. Let $g^*_{mP}(.)$ be the Laplace transform of $g_{mP}(.)$and R be the correlation between U_{iP} and $U_{jP}, i \neq j$, and b = $\frac{1}{\lambda_P}$ (1 – R). Let W be the time to recruitment in the organization with distribution L(.), mean E(W) and variance V(W). Let $N_P(t)$ be the number of policy decisions and $N_T(t)$ be the number of

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$$+q\left(\frac{A_2}{s+A_2}+B_2\sum_{m=1}^{\infty}(1-B_2)^{m-1}\,g_{mP}^{*}(s+A_2)\left(\frac{s}{s+A_2}\right)\right)$$

$$-q\left(\frac{A_3}{s+A_3}+B_3\sum_{m=1}^{\infty}(1-B_3)^{m-1}\,g_{mP}^{*}(s+A_3)\left(\frac{s}{s+A_3}\right)\right) \quad (5)$$

where,

$$A_1=\lambda_T[1-f_T{}^{*}(\alpha_1)]=\frac{\lambda_T\alpha_1}{\mu_T+\alpha_1}$$

$$A_2=\lambda_T[1-f_T{}^{*}(\beta_1)]=\frac{\lambda_T\beta_1}{\mu_T+\beta_1}$$

$$A_3=\lambda_T[1-f_T{}^{*}(\alpha_1)f_T{}^{*}(\beta_1)]=\frac{\lambda_T[\mu_T(\alpha_1+\beta_1)+\alpha_1\beta_1]}{(\mu_T+\alpha_1)(\mu_T+\beta_1)}$$

$$B_1=1-f_P{}^{*}(\alpha_1)=\frac{\alpha_1}{\mu_P+\alpha_1}$$

$$B_2=1-f_P{}^{*}(\beta_1)=\frac{\beta_1}{\mu_P+\beta_1}$$

and $B_3=1-f_P{}^{*}(\alpha_1)f_P{}^{*}(\beta_1)=\frac{\mu_P(\alpha_1+\beta_1)+\alpha_1\beta_1}{(\mu_P+\alpha_1)(\mu_P+\beta_1)}$ (6)

The probability density function of r^{th} order statistic is given by

$$g_{U_{(rP)}}(t)=r\binom{k}{r}[G_P(t)]^{r-1}g_P(t)[1-G_P(t)]^{k-r}, r=1,2,3,\dots\dots,k \quad (7)$$

CASE (i): $g_P(t)=g_{U_{(1P)}}(t)$

In this case $\mathrm{g}_{\mathrm{P}}^{*}(\mathrm{s})=\mathrm{g}_{\mathrm{U}_{(1\mathrm{P})}}^{*}(\mathrm{s})$ (8)

From (7) it is found that, $g_{U_{(1P)}}(t)=kg_P(t)[1-G_P(t)]^{k-1}$ (9)

Since $g_P(.)$ is exponential with parameter λ_P ,

$$\mathrm{g}_{\mathrm{U}_{(1\mathrm{P})}}^{*}(\mathrm{s})=\frac{k\lambda_P}{k\lambda_P+s} \quad (10)$$

$$+q\left(\frac{A_2}{s+A_2}+B_2\sum_{m=1}^{\infty}(1-B_2)^{m-1}\, g_{mP}^*(s+A_2)\left(\frac{s}{s+A_2}\right)\right)$$

$$-q\left(\frac{A_3}{s+A_3}+B_3\sum_{m=1}^{\infty}(1-B_3)^{m-1}\, g_{mP}^*(s+A_3)\left(\frac{s}{s+A_3}\right)\right) \tag{5}$$

where,

$$A_1=\lambda_T[1-f_T{}^*(\alpha_1)]=\frac{\lambda_T\alpha_1}{\mu_T+\alpha_1}$$

$$A_2=\lambda_T[1-f_T{}^*(\beta_1)]=\frac{\lambda_T\beta_1}{\mu_T+\beta_1}$$

$$A_3=\lambda_T[1-f_T{}^*(\alpha_1)f_T{}^*(\beta_1)]=\frac{\lambda_T[\mu_T(\alpha_1+\beta_1)+\alpha_1\beta_1]}{(\mu_T+\alpha_1)(\mu_T+\beta_1)}$$

$$B_1=1-f_P{}^*(\alpha_1)=\frac{\alpha_1}{\mu_P+\alpha_1}$$

$$B_2=1-f_P{}^*(\beta_1)=\frac{\beta_1}{\mu_P+\beta_1}$$

and$B_3=1-f_P{}^*(\alpha_1)f_P{}^*(\beta_1)=\frac{\mu_P(\alpha_1+\beta_1)+\alpha_1\beta_1}{(\mu_P+\alpha_1)(\mu_P+\beta_1)}$ (6)

The probability density function of r^{th} order statistic is given by

$$g_{U_{(rP)}}(t)=r\binom{k}{r}[G_P(t)]^{r-1}g_P(t)[1-G_P(t)]^{k-r}, r=1,2,3,\dots\dots,k \tag{7}$$

CASE (i): $g_P(t)=g_{U_{(1P)}}(t)$

In this case $g_P^*(s)=g_{U_{(1P)}}^*(s)$ (8)

From (7) it is found that, $g_{U_{(1P)}}(t)=kg_P(t)[1-G_P(t)]^{k-1}$ (9)

Since $g_P(.)$ is exponential with parameter λ_P ,

$$g_{U_{(1P)}}^*(s)=\frac{k\lambda_P}{k\lambda_P+s} \tag{10}$$

It is known that

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$$E[T] = -\frac{d}{ds}\left(l^*(s)\right)\Big|_{s=0}\ ,\ E[T^2] = \frac{d^2}{ds^2}\left(l^*(s)\right)\Big|_{s=0}\ \text{and}\ V[T] = E[T^2] - (E[T])^2 \tag{11}$$

From (5), (10) and (11) we get

$$E[T] = \frac{1}{A_1+B_1k\lambda_P} + \frac{q}{A_2+B_2k\lambda_P} - \frac{q}{A_3+B_3k\lambda_P} \tag{12}$$

$$E[T^2] = 2\left[\frac{1}{(A_1+B_1k\lambda_P)^2} + \frac{q}{(A_2+B_2k\lambda_P)^2} - \frac{q}{(A_3+B_3k\lambda_P)^2}\right] \tag{13}$$

Equations(11), (12) and (13) give the mean and variance of the time to recruitment for case(i), where A_i and $B_i, i = 1,2,3$ are given by equation (6).

CASE (ii) $g_P(t) = g_{U_{(kP)}}(t)$

In this case $g_P^*(s) = g_{U_{(kP)}}^*(s)$ (14)

From (7) it is found that

$$g_{U_{(kP)}}(t) = kg_P(t)[G_P(t)]^{k-1} \tag{15}$$

From (10) and (15) it can be shown that

$$g_{U_{(kP)}}^*(s) = \frac{k!\lambda_P^k}{(s+\lambda_P)(s+2\lambda_P)......(s+k\lambda_P)} \tag{16}$$

From (5), (11), (16) and on simplification

$$E[T] = \frac{1}{A_1}\left[\frac{\prod_{m=1}^k(A_1+m\lambda_P)-k!\lambda_P^k}{\prod_{m=1}^k(A_1+m\lambda_P)-(1-B_1)k!\lambda_P^k}\right] + \frac{q}{A_2}\left[\frac{\prod_{m=1}^k(A_2+m\lambda_P)-k!\lambda_P^k}{\prod_{m=1}^k(A_2+m\lambda_P)-(1-B_2)k!\lambda_P^k}\right]$$
$$-\frac{q}{A_3}\left[\frac{\prod_{m=1}^k(A_3+m\lambda_P)-k!\lambda_P^k}{\prod_{m=1}^k(A_3+m\lambda_P)-(1-B_3)k!\lambda_P^k}\right] \tag{17}$$

$$E[T^2] = \frac{2}{{A_1}^2}\left[1 - \frac{B_1k!\lambda_P^k\left[\prod_{m=1}^k(A_1+m\lambda_P)\left(1+\sum_{n=1}^k\frac{A_1}{A_1+n\lambda_P}\right)-(1-B_1)k!\lambda_P^k\right]}{\left[\prod_{m=1}^k(A_1+m\lambda_P)-(1-B_1)k!\lambda_P^k\right]^2}\right]$$
$$+\frac{2q}{{A_2}^2}\left[1 - \frac{B_2k!\lambda_P^k\left[\prod_{m=0}^k(A_2+m\lambda_P)\left(1+\sum_{n=1}^k\frac{A_2}{A_2+n\lambda_P}\right)-(1-B_2)k!\lambda_P^k\right]}{\left[\prod_{m=1}^k(A_2+m\lambda_P)-(1-B_2)k!\lambda_P^k\right]^2}\right]$$

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$$-\frac{2q}{{A_3}^2}\left[1-\frac{B_3 k!\lambda_P^k\left[\prod_{m=0}^{k}(A_3+m\lambda_P)\left(1+\sum_{n=1}^{k}\frac{A_3}{A_3+n\lambda_P}\right)-(1-B_3)k!\lambda_P^k\right]}{\left[\prod_{m=1}^{k}(A_3+m\lambda_P)-(1-B_3)k!\lambda_P^k\right]^2}\right] \quad (18)$$

Equations (11), (17) and (18) give the mean and variance of the time to recruitment for case (ii), where A_i and $B_i, i = 1,2,3$ are given by equation (6).

CONCLUSION:

The manpower planning model developed in this paper is new in the context of considering order statistics for the inter policy decision times. Further, this model can be used to plan for the adequate provision of manpower for the organization at graduate, professional and management levels. There is a scope of studying the applicability of designed model using simulation. The goodness of fit for the distributions assigned in this paper can be tested by collecting relevant data. The findings given in this paper enable one to estimate manpower gap in future, thereby facilitating the assessment of manpower profile in predicting future manpower development not only on industry but also in a wider domain.

REFERENCE

1) Bartholomew, D. J. (1971). The statistical approach to manpower planning, Statistician, 20, 3-26. | 2) Bartholomew, D. J. (1973). Stochastic Model for Social Processes(2nd ed.). John Wiley, New York. | 3) Elangovan, R., Sathiyamoorthi, R. and Susiganeshkumar, E. (2011) Estimation of expected time to recruitment under two sources of depletion of manpower, Proceedings of the International Conference on Stochastic Modelling and Simulation, Allied Publishres Pvt.Ltd., Chennai, 99-104 | 4) Esther Clara, J. B., (2012). Contributions to the study on some stochastic models in manpower planning, Ph.D. Thesis, Bharathidasan University, Tiruchirappalli. | 5) Grinold, R. C. and Marshall, K. T. (1977). Manpower Planning Models, North Holland, New York. | 6) Gurland, J. (1955). Disribution of the maximum of the arithmetic mean of correlated random variables Ann. Math. Statist., 26, 24 – 300. | 7) Kasturi, K. and Srinivasan, A. (2005). Expected time to recruitment for correlated inter- decision time of exits when threshold distribution has SCBZ property, Acta Ciencia Indica, 31 M(1), 277-283. | 8) Mehlmann, A. (1977). Markovian manpower models in continuous time, J. Appl. Probability, 14, 249-259. | 9) Mercy Alice, B. and Srinivasan, A.(2015) Estimation of Variance of the time to recruitment in a single graded manpower system with two sources of depletion and two thresholds, International Research Journal of Human Resources and Social Sciences, 2(6), 29-36. | 10) Mukherjee, S. P. and Chattopadhyay, A. K. (1985). An optimal recruitment policy, Indian Association for Productivity Quality and Reliability Transactions, 11(1-2), 87-96. | 11) Muthaiyan, A., Sulaiman, A. and Sathiyamoorthy, R.(2009). A stochastic model based on order statistics for estimation of expected time to recruitment, Acta Ciencia Indica, 5(2), 501-508. | 12) Sathiyamoorthi, R. and Elangovan, R. (1998). Shock model approach to determine the expected time for recruitment, Journal of Decision and Mathematika Sciences, India, 3(1-3), 67-78. | 13) Usha, K., Leelathilagam, P. and Ramanarayanan,R. (2013). A stchastic model for estimation of mean time to recruitment with two sources of depletion under correlated inter decision times, International Journal of Pure and Applied Mathematics, 82(5), 683-693. | 14) Venkat Lakshmi, K. (2007). Mean and variance of the time to recruitment for a single graded manpower system associated with a bivariate policy of recruitment, M.Phil Dissertation, Bharathidasan University, Tiruchirappalli.