



Importance of Statistical Measures in Image Processing

KEYWORDS

Statistical measures, noise, filtering.

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ABSTRACT

This paper presents the comprehensive study of the various statistical measures and their application in digital image processing at root level. We have simulated the majority of statistical measures and reviewed their existing applications. Also we have explored and proposed their importance in some more research area of digital image processing. We have done their comparative analysis with the help of MATLAB simulation to ease the selection of statistical parameter for a specific image processing technique like image enhancement, denoising, restoration, edge detection etc.

I. INTRODUCTION

Statistics is the study of the collection, organization, analysis, and interpretation of data. It deals with all aspects of this, including the planning of data collection in terms of the design of surveys and experiments.

The various statistical measures [1, 7] are mean, mode, median, variance, standard deviations, covariance, skewness and kurtosis. All of these measures are used in a wide range of scientific and social research, including: biostatistics, computational biology, computational sociology, network biology, social science, sociology and social research etc.

In this paper we have studied various type of statistical measure in respect to image processing and simulated all of these. Main purpose is to highlight the application of these measures in the various fields of digital image processing like, image enhancement, image restoration, image denoising, and edge detection etc. at the basic level and ease the selection of statistical parameter for a specific image processing technique.

The organization of the paper is as follows. Following the brief introduction proposed statistical model is discussed in section II. From section III to section XIII details of various statistical parameters is discussed. Section XIV discusses about analysis and simulation results followed by concluding remarks in section XV.

II. PROPOSED STATISTICAL MODEL

Although research has already been done on few of these measures at quite advance level, we have proposed a simple statistical model in Fig.1 for image processing to optimize it features.

The proposed statistical model consists of the following steps:

1. *Statistical analysis of input image, $li(x, y)$* : In this step statistical analysis of input image is done using various measures like mean, mode, median, variance, standard deviation, covariance, skewness, kurtosis etc.
2. *Selection of statistical measure*: Depending upon the requirements in output optimized image, the statistical parameter is chosen.
3. *Image Filtering*: Image filtering is done using the filter based on the statistical parameter selected in previous step.

Depending upon requirements, for the image filtering we can choose from a very basic filter to any multiparameter complex filtering. In the next few sections we have discussed the statistical analysis of an image using various statistical measures.

III. MEAN

Mean [1, 2] is most basic of all statistical measure. Means are often used in geometry and analysis; a wide range of means have been developed for these purposes. In context of image processing filtering using mean is classified as spatial filtering and used for noise reduction. In this section we have discussed about various type of mean and analysed their use for removing various type of noise in image processing.

A. Arithmetic Mean

The arithmetic mean filter [2], also known as averaging filter, operates on an sliding ' $m \times n$ ' window by calculating the average of all pixel values within the window and replacing the center pixel value in the destination image with the result. Its mathematical formulation is given as follows

$$f(x, y) = \frac{1}{mn} \sum_{(r,c) \in W} g(r, c)$$

Where 'g' is the noisy image, $f(x,y)$ is the restored image, and 'r' and 'c' are the row and column coordinates respectively, within a window 'W' of size ' $m \times n$ ' where the operation takes place.



Figure 1 Proposed Statistical Model for Image Optimization

The arithmetic mean filter causes a certain amount of blurring (proportional to the window size) to the image, thereby reducing the effects of noise and local variations. It can be used to reduce noise of different types, but works best for Gaussian, uniform, or Erlang noise. Fig. 2 (C) shows the image after arithmetic mean filtering of Gaussian noise

added image.

B. Geometric Mean

The geometric mean [2] filter is a variation of the arithmetic mean filter and is primarily used on images with Gaussian noise. This filter is known to retain image detail better than the arithmetic mean filter. Its mathematical formulation is as follows:

$$f(x, y) = \left[\prod_{(r,c) \in W} g(r, c) \right]^{1/mn}$$

In this case each restored pixel is given by the product of the pixel in the sub image window raised to the power '1/mn'. Fig. 2 (D) shows the image after geometric mean filtering of Gaussian noise added image.

C. Harmonic Mean

The harmonic mean filter [2] is yet another variation of the arithmetic mean filter and is useful for images with Gaussian or salt noise. Black pixels (pepper noise) are not filtered. The filter's mathematical formulation is as follows:

$$f(x, y) = \frac{mn}{\sum_{(r,c) \in W} \left(\frac{1}{g(r, c)} \right)}$$

D. Contraharmonic Mean

The contra-harmonic mean filter [2] is another variation of the arithmetic mean filter and is primarily used for filtering salt or pepper noise (but not both). Images with salt noise can be filtered using negative values of R, whereas those with pepper noise can be filtered using positive values of R.

$$f(x, y) = \frac{\sum_{(r,c) \in W} (g(r, c))^{R+1}}{\sum_{(r,c) \in W} (g(r, c))^R}$$

Where R is the order of filter. shows image after Contraharmonic mean filtering of image with Salt & Pepper noise for R=1 Fig. 2 (I) R=-1.

IV. MEDIAN

Median [2] is measure of intensity level of pixel which is separating the high intensity value pixels from lower intensity value pixels. It is a type of order-statistic filter. The most popular and useful of the rank filters is the median filter. It works by selecting the middle pixel value from the ordered set of values within the 'm×n' neighborhood 'W' around the reference pixel. If 'mn' is an even number (which is not common), the arithmetic average of the two values closest to the middle of the ordered set is used instead. Mathematically,

$$\tilde{f}(x, y) = \text{median} \{ g(r, c) | (r, c) \in W \}$$

There have been many variants, extensions, and optimized implementations of the median filter proposed in the literature. This filter simply sorts all values within a window, finds the median value, and replaces the original pixel val-

ue with the median value. It is commonly used for salt and pepper noise. Fig. 2 (J) shows image after median filtering of image with Gaussian noise. Fig. 2 (K) shows image after median filtering of image with Salt & Pepper noise.

V. MAX AND MIN FILTERS

These are another type of rank-statistics filter [2]. The min and max filters also work on a ranked set of pixel values. Contrary to the median filter, which replaces the reference pixel with the median of the ordered set, the min filter, also known as the zeroth percentile filter, replaces it with the lowest value instead.

A quick way to get rid of salt noise in an image is to use the min filter, which simply takes the minimum value of a window when the values are ordered. Mathematically

$$\tilde{f}(x, y) = \min \{ g(r, c) | (r, c) \in W \}$$

Similarly, the max filter, also known as the 100th percentile filter, replaces the reference pixel within the window with the highest value that is

$$\tilde{f}(x, y) = \max \{ g(r, c) | (r, c) \in W \}$$

The max filter is used for filtering pepper noise, similar to the technique of the min filter. Fig. 2 (L) shows image after min filtering of image with Salt & Pepper noise. Fig. 2 (M) shows image after max filtering of image with Salt & Pepper noise

VI. STANDARD DEVIATION

It is a most widely used measure of variability or diversity used in statistics. In terms of image processing it shows how much variation or "dispersion" exists from the average (mean, or expected value). A low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data points are spread out over a large range of values. Mathematically standard deviation is given by

$$\tilde{f}(x, y) = \sqrt{\frac{1}{mn-1} \sum_{(r,c) \in W} \left(g(r, c) - \frac{1}{mn-1} \sum_{(r,c) \in W} g(r, c) \right)^2}$$

A standard deviation filter calculates the standard deviation and assigns this value to the center pixel in the output map. As it has capability in measuring the variability, it can be used in edge sharpening, as intensity level get changes at the edge of image by large value.

Standard deviation filters [10] can be useful for radar images. The interpretation of radar images is often difficult: you cannot rely on spectral values because of back scatter (return of the pulse sent by the radar). This often causes a lot of 'noise'. By using a standard deviation filter, you may be able to recognize some patterns

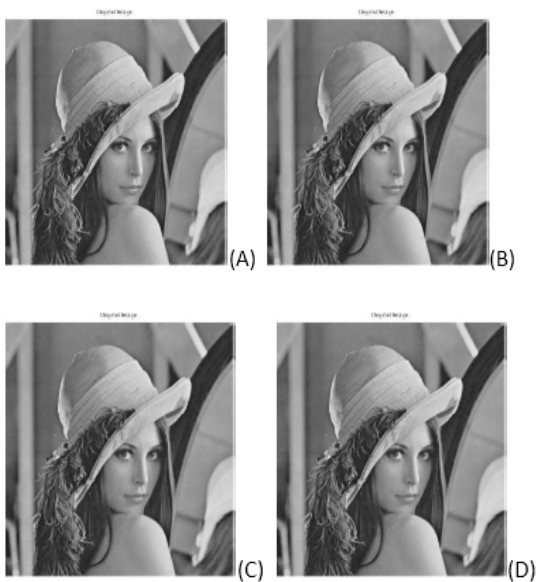


Figure 1 (A)Original Image (B) Image with Gaussian noise of mean 0 & variance 0.005 (C) Image after arithmetic mean filtering of (B) with 3*3 kernel size (D) Image after Geometric mean filtering of (B) with 3*3 kernel size

CONCLUSION

In this paper we have discussed the details of various statistical measures in reference to digital image processing. We have presented a statistical model for selecting the statistical measure properly before going for a complex image processing technique. In the proposed model criteria of the output requirement is taken into account while selecting the statistical measure. MATLAB simulations are done based on various statistical measures filtering technique to prove the use of each at root level. Hence we can conclude that proposed statistical model is can be used as pre-processing model for various digital image processing technique to improve the effectiveness of complex image processing technique in the next levels.

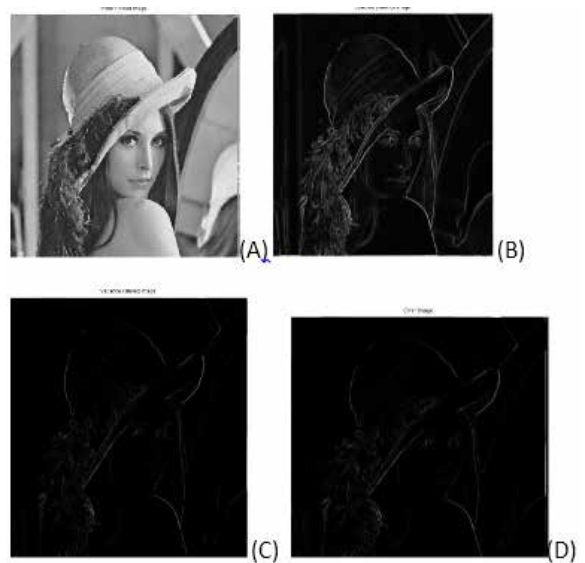


Fig. 2 (A) Image after Mode filtering of Fig. 2 (E) with 3*3 kernel size. (B) Image after standard deviation filtering of Fig. 2 (A) with 3*3 kernel size. (C) Image after Covariance filtering of Fig. 2 (A) with 3*3 kernel size. (D) Image after variance filtering of Fig. 2

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