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A Note on Transitivity of Ideals in Rings

KEYWORDS

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ABSTRACT *A group G is called a T-group if normality is a transitive relation in G. On the similar pattern, in this paper we discuss transitivity of ideals in rings. In general transitive property fails for ideals. Still there are some classes of rings which satisfy this property. We call such a ring a T-ring. This note carries out some comments on T-rings.*

Definition 1 : Let R be a ring and I be an ideal of R. Let J be an ideal of I. if J is also an ideal of R, then R is called a T-ring.

Examples :
If R is a commutative ring, I an ideal of R and J a prime ideal of I, then J is also an ideal of R ([2, 7]). Therefore R is a T-ring in this case.

Z is a T-ring.

Every simple ring is a T-ring.

But in general transitive property fails for ideals. i.e. If I is an ideal of a ring R, J an ideal of I, then it is not necessary than J is an ideal of R.

Examples :
Take R = Z [$\sqrt{2}$] = {a +b$\sqrt{2}$ | a, b $\in$ Z}

I = {3a +3b$\sqrt{2}$ | a, b, $\in$ Z}

J = {9a + 3b$\sqrt{2}$ | a, b $\in$ Z }

Here I is an ideal of R which is a commutative ring with unity, J is an ideal of I but J is not an ideal of R.

[1] consider the ring R of all reals and let R* be the ring of all maps from R into R where addition and multiplication are defined pointwise. Let S be the subring of R* consisting of all continuous maps form R to R. If

I = { fi | f $\in$ S, f(0) = 0|

J = {fi^2 + ni^2 | f $\in$ S, f(0) = 0, n $\in$ Z}

Where I is the identity function on R, then J is an ideal of I, I is an ideal of S but J is not an ideal of S.

If R is a ring and S a subring of R, then an ideal of S need not be an ideal of R. Z is a subring of the ring Q of all rationals but no ideal of Z, except for the trivial ideal (0) is an ideal of Q. Thus we make the following definitions.

Definition 2 : Suppose R is a ring such that every ideal of a subring of R is also an ideal of R, then R is called a TS-ring.

Remarks :
Every TS-ring is a T-ring but not conversely, Q is a T-ring which is not a TS-ring.

A homomorphic image of a T-ring (TS-ring) is also a T-ring (TS-ring).

We know that a subring of an Artinian ring need not be Artinian. But if R is an Artinian ring which in addition is also a TS-ring, then every subring of R is also an Artinian ring.

A group G is called a T-group if normality is a transitive relation in G. Abelian groups, simple groups, the symmetric groups S_n, n $\neq$ 4 are T-groups. (A survey of T-groups has been provided in [6].

Question 1 : If R is a T-ring and G is a T-group, what about group ring RG ? Is RG also a T-ring ?

The answer is no in general.

Example : Consider the ring R= {0,1}, a finite ring of characteristic 2, R is trivially a T-ring.

Take G = < x >, a finite cyclic group of order 4.

i.e. G= { 1, x, x^2, x^3 }. Then G is a T-group.

The group ring RG is not a T-ring.

J = {0,1 + x, 1 + x^2 , x^3 } is an ideal of RG.

J ' = {0,1 + x^2} is an ideal of j. But J' is not an ideal of RG.

Question 2 : Suppose R is a T-ring and G is a T-group under what conditions on G or R or on both, RG is a T.ring ?

We note that if R = {0,1} a finite ring of characteristic 2 and G = < x > a cyclic group of order 3, then the group ring RG is a T-ring.

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