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On The End Equitable Edge Domination of Graphs

KEYWORDS: Edge domination, equitable edge domination, end equitable edge domination.

A. C. Dinesh	Puttaswamy
Bangalore Institute of technology, K.R.Road V.V, Puram, Bangalore, India	PES College of engineering Mandya, India

ABSTRACT *In a graph ,a set is an equitable edge domination set of if every edge not in is adjacent to at least one an edge such that .The minimum cardinality of such an equitable edge domination set is denoted by and is called equitable edge domination number of .In this paper, we introduce the end equitable edge domination in graphs, exact values for some standard graphs are found. The relation with the others domination parameters is studied. Some bounds for the end equitable edge domination are obtained.*

1. Introduction

In this paper, by graph $G=(V,E)$, we mean undirected graph without loops or multiple edges and at least contains one edge. Let G be a graph, as usual $p=|V|$ and $q+|E|$ denote the number of vertices and edges of G, respectively. In general we use $\langle X\rangle$ to denote to the sub graph induced by the set X. $N(v)$ and $N[V]$ denote the open and closed neighborhood of the vertex v in G, respectively. For any notion not defined here we refer to Harary $[4]$.

A set $D\subseteq V(G)$ is said to be a dominating set of G, if every vertex in $v\in V-D$ is adjacent to some vertex in D. The minimum cardinality of vertices in such set is called the domination number of G and denoted by $\gamma(G)$.
The concept of edge domination was introduced by Michell and Hedetiemi $[6]$. A subset X of E is called an edge domination set of G if every edge not in X is adjacent some edge in X. The edge domination number $\gamma^{'}(G)$ of G is the minimum cardinality taken over all edge dominating sets of G. For more details about edge domination number in graph we refer the reader to $[6]$.

The equitable edge domination in graphs has introduced by Alwardi and N.D.Soner $[2]$. Let G be a graph for any edge $f\in E$ the degree of $f=uv$ in G is defined by $\deg(f)=\deg(u)+\deg(v)-2$. A set $S\subseteq E$ of edges is an equitable edge domination set of G if every edge f not in S is adjacent at least one edge $f^{'}\in S$ such that $\left|\deg(f)-\deg(f^{'})\right|\leq 1$. The minimum cardinality of such an equitable edge dominating set is called the equitable edge domination number and denoted by $\gamma_e^{'}(G)$. Graphs and degree equitably has studied in $[1]$. For more details and terminology about equitable domination and equitability in graphs we refer the reader to $[2]$.

The end domination was introduced for the first time by Hatting and Henning $[5]$. A dominating set $D\subseteq V(G)$ is said to be an end dominating set of G if D contains all the end vertices of $V(G)$. The minimum cardinality of such set is called the end domination number of G and denoted by $\gamma_e(G)$.

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End edge domination in graphs has introduced by Sedamkar and Muddebinal [5]. An edge dominating set $S \subseteq E(G)$ is said to be an end edge dominating set of G if S contains all the end edges of $E(G)$. The minimum cardinality of such set is called the end edge domination number of G and denoted by $\gamma'_e(G)$.

The end edge domination and equitable edge domination in graphs motivated us to introduce the end equitable domination in graph. We found exact values for some standard graphs, relations between the end equitable edge domination number and others domination parameters are obtained. Bound of the new parameter is established.

2. End Equitable Edge Domination of Graphs

Definition 2.1. *Let G be a graph with at least one edge. An equitable edge dominating set $S \subseteq E$ is said to be an end equitable edge dominating set of G if S contains all the end edges of G. The minimum cardinality of edges in such set is called the end equitable edge domination number of G and denoted by $\gamma'_e(G)$.*

Example 2.2. Let G be a graph as in Figure 1.

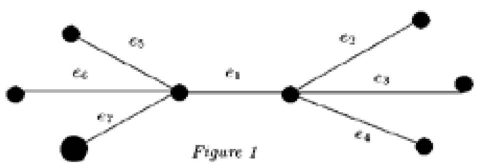

Figure 1

Clearly the edge e_1 adjacent to all the other edges of the graph. Therefore $\gamma'_{ee}(G)=1$. The edges e_2, e_3, e_4, e_5, e_6 and e_7 are all of degree 3 but e_1 is of degree 6. That means e_1 is equitable isolated edge. Therefore $\gamma_e\ (G)=3$. But the end equitable edge domination number $\gamma'_{ee}(G)=7$ that means all the edges of G. Hence, $\gamma(G)=1$ $\gamma_e\ (G)=3$ and $\gamma'_{ee}(G)=7$.

Proposition 2.3. *For any graph G, $\gamma'(G) \le \gamma'_e(G) \le \gamma'_{ee}(G)$.*

Proof. Let G be a graph with at least one edge and let $S \subseteq E(G)$ be any minimum end equitable edge dominating set in G. Then from the definition of the end equitable edge dominating set in graph, S is an edge equitable dominating set.

Therefore, $\gamma'(G) \le |S|$ i.e. $\gamma'(G) \le \gamma'_{ee}(G)$.

Similarly, if D any minimum equitable edge dominating set in G, then D is also dominating set from the definition of the equitable edge domination in graph. Therefore $\gamma'(G) \le |D| \le \gamma'_e(G)$.

Proposition 2.4. *Let G be a graph with no end edges and all the edges an equitable edges. Then*

$$\gamma'(G) = \gamma'_e(G) = \gamma'_{ee}(G).$$

Proof: From the definition of the end equitable edge domination if no end edge in G, then if $S \subseteq E$ is any minimum equitable edge dominating set will be an end equitable edge dominating set of G. i.e. $\gamma'_e(G) \le \gamma'_{ee}(G)$. Therefore, $\gamma'_{ee}(G) = \gamma'_e(G)$, and since for any edge in G is equitable edge, then any minimum edge dominating set in G will be end equitable edge dominating set. i.e., $\gamma'_e(G) \le \gamma'(G)$ and by Proposition 2.3 $\gamma'(G) \le \gamma'_e(G)$. Therefore, $\gamma'(G) = \gamma'_e(G)$.

Hence, $\gamma'(G) = \gamma'_e(G) = \gamma'_{ee}(G)$.

Remark 2.5. The converse of Theorem 2.3 not true in general. See the following example.

Example 2.6. Let G be a *graph* as in Figure 2.

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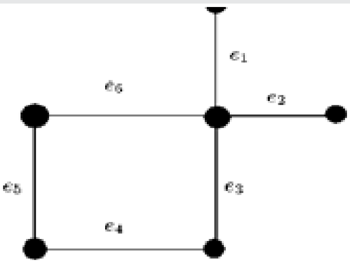

Figure 2

it is easy to see that $\gamma'(G) = \gamma'_e(G) = 2$. But, $e_1\, e_2$, e_3 and e_6 are not equitable edges.

Example 2.7. Let $G \cong P_5$. Then clearly $\gamma'_e(P_5) = 2$. Similarly, $\gamma'_{ee}(G) = 2$. But there are end edges in the graph

By Proposition 2.4 the proofs of the following results are straightforward.

Proposition 2.8. For any complete graph K_P with $p \geq 2$ vertices

$$\gamma'_{ee}(K_p) = \begin{cases} \frac{p}{2} & , if\ \ p \text{ is even}; \\ \frac{p-1}{2} & , if\ \ p \text{ is odd} \end{cases}$$

Proposition 2.9. *For any cycle graph C_P with $p \geq 3$ vertices*

$$\gamma'_{ee}(C_p) = \begin{cases} \frac{p}{3}, & if\ p \equiv 0 \bmod 3; \\ \frac{p+2}{3}, & if\ p \equiv 1 \bmod 3; \\ \frac{p+1}{3}, & if\ p \equiv 2 \bmod 3. \end{cases}$$

Proposition 2.10. For any Path graph P_p with $p \geq 6$ vertices

$$\gamma'_{ee}(P_p) = \begin{cases} \frac{p+2}{3}, & if\ p \equiv 1 \bmod 3; \\ \frac{p+1}{3}, & if\ p \equiv 2 \bmod 3; \\ \frac{p+3}{3}, & if\ p \equiv 0 \bmod 3. \end{cases}$$

Proof: Let G be path P_p on $p \geq 0$ vertices and labeling the edges of P_p as $e_1, e_2, \ldots\ldots, e_{p-1}$ there are two pe e_1 and e_{p-1}. For any minimum end equitable edge dominating set must be contains e_1 and e_{p-1} and clearly e_1 e_2 and e_{p-1} will adjacent e_{p-2}, so only we need to dominate $\langle E - \{e_1, e_{p-1}\}\rangle$. Therefore, $\gamma'_{ee}(G) = 2 + \gamma'_{ee}(P_{p-}$

.

Also by Proposition 2.4 $\gamma'_{ee}(P_{p-4}) = \gamma'_e(P_{p-4}) = \gamma'(P_{p-4}) = \left[\frac{p-5}{3}\right]$

Therefore, $\gamma'_{ee}(G) = 2 + \left[\frac{p-5}{3}\right] = \frac{p-5}{3}$, if $p \equiv 0 \bmod 3$

$\gamma'_{ee}(G) = 2 + \left[\frac{p-5}{3}\right] = 2 + \frac{p-4}{3} = \frac{p+2}{3}$, if $p \equiv 1 \bmod 3$

Similarly, if $p \equiv 2 \bmod 3$

We have $\left[\frac{p-5}{3}\right] = \frac{p-5}{3}$

. Hence, $\gamma'_{ee}(G) = \frac{p+1}{3}$, if $p \equiv 2 \bmod 3$

Therefore,

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$$\gamma'_{ee}(G)=\begin{cases} \frac{\ }{3}, & if\ p\equiv 1 \bmod 3; \\ \frac{p+1}{3}, & if\ p\equiv 2 \bmod 3; \\ \frac{p+3}{3}, & if\ p\equiv 0 \bmod 3. \end{cases}$$

Proposition 2.11. For any star *graph* $K_{1,n}$ *with* $n+1$ *vertices* $gamma'_{ee}\left(K_{1,n}\right)=n$.

Proof. Let G be the star graph with $n+1$ vertices. Clearly for any edge e in $K_{1,n}$, $\deg(e)=n-1$, that means all edges of $K_{1,n}$ are equitable adjacent. Therefore, one is enough to equitable but the end equitable edge dominating set must have all the end edges and since all the edges of $K_{1,n}$ are end edges and there are n edges in $K_{1,n}$.

Hence, $\gamma'_{ee}\left(K_{1,n}\right)=n$.

Proposition 2.12. Let $G\cong K_{m,n}$, *where* $2\le m\le n$. *Then* $\gamma'_{ee}(G)=m$.

Proof: For any complete bipartite graph $K_{m,n}$ where $2\le m\le n$ any edge $e\in E\left(K_{m,n}\right)$, we have $\deg(e)=m+n$ that mean any two adjacent edges are also equitable adjacent edges. Therefore $K_{m,n}$ has no end edge.

Then by Proposition 2.4 $\gamma'_{ee}\left(K_{m,n}\right)=\gamma'_{e}\left(K_{m,n}\right)=\gamma'\left(K_{m,n}\right)$ and as we know $\gamma'\left(K_{m,n}\right)=\min(m,n)$ $\gamma'_{ee}\left(K_{m,n}\right)=m$

Therefore, $\gamma'_{ee}\left(K_{m,n}\right)=m$, where $2\le m\le n$.

A spider graph is a tree with property that the removed of all end paths of length two result in an isolated vertex called head of spider. The spider of *n* end path denoted by SP_n.

Proposition 2.13. *Let* $G\cong SP_n$ *with* $n\ge 3$. Then, $\gamma'_{ee}(G)=n+1$.

Proof: Let $G\cong SP_n$ as in Figure 3.

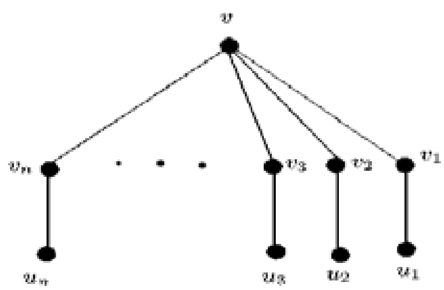

Figure 3

It is easy to see that all the edges vv_i, where $i=1,2,......,n-1$ are of degree $n-1$. But the edge v_iu_i, where $i=1,2,......,n$ are of degree one only. That means all the edges v_iu_i are equitable isolated and also end edges of the form v_iu_i must be included in any end equitable edge dominating set and to dominate the edges vv_i, $i=1,2,......,n$ we need only one edge. So let $S=\left\{v_iu_i : i=1,2,......,n\right\}\cup vv_i$. Clearly S is end equitable edge dominating set.

Therefore, $\gamma'_{ee}(G)\le n+1$.

Suppose $\gamma'_{ee}(G)<n+1$. What that is γ'_{ee} -set of size less than $n+1$ say's¿ There are n end edges must be belongs to the set S. Therefore all the edge belong to S are end edges and no edge will dominate in vv_i, $i=1,2,......,n$.

Hence,

$$\gamma'_{ee}(G)<n+1.$$

Theorem 2.14. Let G be a graph. Then $\gamma'_{ee}(G)=1$, if and only if $G\cong K_2$ or K_4-e, for some edge e.

Proof. Let G be a graph with $\gamma'_{ee}(G)=1$, we have two cases:

Case 1: If G has end edges. Then clearly $G\cong K_2$.

Case 2: If G has no end edges and $\gamma'_{ee}(G)=1$. Suppose there are q edges in G, then there is one edge say f such that

$\deg(f) = q-1$ and all the others edges e, we have $\deg_e = q-1$ or $q-2$. Since $\deg(f) = q-1$. Then clearly $\deg_e(f) = q-1$.

Subcase 2.1 If all the others edge of size $q-1$ then G is $K_{1,q}$ which is contradiction because G has no Pendant edge.

Subcase 2.2 Some edges have degree $q=2$ and other has degree $q-1$ there is only one case one edge of degree $q-1$ and the others will be of degree $q-2$ which is $K_4 - e$ for some edge e, then $\gamma'_{ee}(G) = 1$.

Theorem 2.15. Let a graph G is bi-star G, where $m, n \geq 2$ or star $K_{1,n}$. Then $\gamma_{ee}(G) = q$, where q is the number of edges in G.

Proof: If $G \cong K_{1,n}$, then by Proposition 2.9, $\gamma_{ee}(G) = q$.

Suppose $G \cong B(m,n)$, $m, n \geq 2$ as in Figure 4.

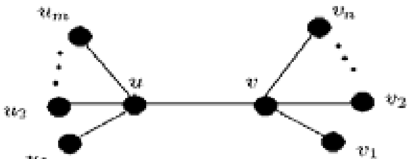

Figure 4

Clearly all the edges of $B(m,n)$ are end edges and must belong to any minimum end equitable edge dominating set and it is easy to see that uv is an isolated equitable edge $\deg_e(uv) = 0$. Hence, $\gamma'_{ee}(B(m,n)) = q$.

Remark 2.16. The converse of Theorem 2.13 is not true in general see example 2.15

Example 2.17. Let G be the graph as in Figure 5. From the figure we see that $\deg(e_1) = 1$, $\deg(e_2) = 3$, $\deg(e_3) = 5$, $\deg(e_4) = 7$,

$\deg(e_8) = \deg(e_9) = \deg(e_{10}) = \deg(e_{11}) = \deg(e_{12}) = 5$ and $\deg(e_5) = \deg(e_6) = \deg(e_7) = 3$

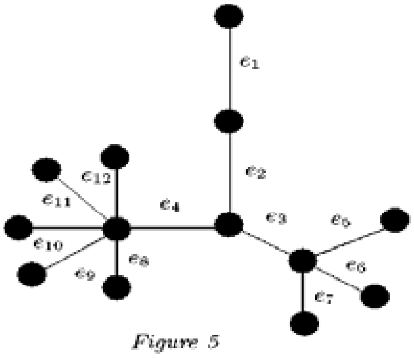

Figure 5

So it is easy to see that $\gamma'(G) = 3$, $\gamma'_e(G) = 6$, but $\gamma'_{ee}(G) = 12$. Therefore, but G is neither Bi-star nor star.

Theorem 2.18. For any graph G, $\gamma_{ee}(G) < \gamma_e(G) + t$ where t is the number of pendant edges in G.

Proof: Let G be a graph with minimum equitable edge dominating set $D \subset E(G)$ and let the set of pendant edges is F. That means $F = \{e : e \in E(G) \,\&\, e \text{ is pendant}\}$. Now, it is clear that the set $D \subset F$ is end equitable dominating set in G. Therefore, $\gamma_{ee}(G) < D \cup F$ and since sometimes $D \cap F = \phi$, implies to $|D \cup F| \leq |D| + |F| = \gamma'_e(G) + t$, where $t = |F|$.

Hence, $\gamma_{ee}(G) < \gamma_e(G) + t$.

Corollary 2.19. For any graph G without pendant edges we have $\gamma'_{ee}(G) = \gamma'_e(G)$.

Proof: By Theorem 2.16 if G has no pendant edges, then $\gamma'_{ee}(G) \leq \gamma'_e(G)$ and we $\gamma'_e(G) \leq \gamma'_{ee}(G)$ by Proposition 2.3.

Hence $\gamma'_{ee}(G)=\gamma'_e(G)$.

Remark 2.20. The converse of Corollary 2.17 is not true always see the following example

Example 2.21. Let $G \cong P_5$. Then it is easy to see that $\gamma'_{ee}(P_5)=\gamma'_e(P_5)=2$, but P_5 has two pendant edges.

Theorem 2.22. Let G be a graph such that for any edge $e\in E(G), \deg_e(e)=0$. Then $\gamma'_{ee}(G)=\gamma'_e(G)$.

Proof: Let G be a graph and for any edge $e\in E(G), \deg_e(e)=0$, that mean e is not equitable adjacent to any other edge (equitable isolated edge). Let $F=\{e\in E(G): e \text{ is pendant}\}$. By Proposition 2.3, we have $\gamma'_e(G)\le\gamma'_{ee}(G)$. Suppose that D is any minimum equitable edge dominating set in G, we have to prove that D is also an end equitable edge dominating set in G. Suppose, by contrary, that D is not end equitable edge dominating set of G, then there exists at least one end edge in G does not belong to D. Suppose that an edge $f\in F$ but $f\notin D$ and since $\deg_e(f)=0$ implies that D is not equitable edge dominating set which is a contradiction. Therefore, D is an end equitable edge dominating set of G.
Thus $\gamma'_{ee}(G)\le\gamma'_e(G)$. Hence, $\gamma'_{ee}(G)=\gamma'_e(G)$.

Remark 2.23. The converse of Theorem 2.20, is not true in general see example 2.19, $\gamma'_{ee}(G)=\gamma'_e(G)$ but the end edges are not equitable isolated.

Theorem 2.24. Let G be any connected graph with $p>2$ vertices and q edges. Then $\gamma'_{ee}(G)=q$ if and only if one of the following condition:
(i) All the edges of G are end edges. (ii) For any edge $f\in E(G)$ such that f not end edge $\deg_e(f)=0$.

Proof: If all the edge in G are end edges mean that $G\cong K_{1,2}$ and clearly by Proposition 2.9, $\gamma'_{ee}(G)=q$.
Similarly, let $F=\{e\in E(G): e \text{ is pendant}\}$. Let $H\cong(E-F)$. since all the edges which is not end edge has equitable degree zero it follows that H is totally equitable disconnected graph and $\gamma'_e(G)=q-|F|$ and since no edge from F is equitable adjacent any edge in $E-F$. Therefore, $\gamma'_{ee}(G)=\gamma'_e(G)+|F|=|E-F|+|F|=|E|$. Hence, $\gamma'_{ee}(G)=q$.
Conversely, suppose that G is connected graph with $\gamma'_{ee}(G)=q$, then we have two cases:

Case 1 If G has no end edges and $\gamma'_{ee}(G)=q$ implies that $\gamma'_e(G)=q$ then all the edges of G are equitable isolated that mean condition (ii) is holding.
Case 2 If G has pendant edges, then we have two subcases.

Subcase 2.1 All the edges are pendant edges that means condition (i) is holding.

Subcase 2.2 Some of the edges are not end edges, so if $F=\{e\in E(G): e \text{ is pendant}\}$ and $H=(E-F)$. Since $\gamma'_{ee}(G)=q=|E-F|+|F|$ it follows that $\langle E-F\rangle$ is totally equitable disconnected graph. Hence for any edge $f\in E-F$, we have $\deg_e(f)=0$ Hence, condition (ii) is holding.

Theorem 2.25. Let G be equitable connected graph with p vertices and q edges. Then $\gamma'_{ee}(G)+\gamma_e(G)\le\beta_e(G)+\alpha_e(G)$ where $\beta_e(G)$ and $\alpha_e(G)$ are the equitable independent number and equitable covering number. Further equality holds if $G\cong K_{1,2}$.
Proof: Let G be an equitable connected graph with p vertices and q edges. Then there are two cases:

Case 1: If G has end edge, then suppose that $F=\{e\in E(G): e \text{ is an end edge}\}$ and let I be the minimum equitable edge dominating set of the graph $\langle E-F\rangle$. Now, clearly $I\cup F$ is an end equitable edge dominating set of G ,that means

$$\gamma'_{ee}(G)\le|I\cup F| \quad \text{............. (1)}$$

Let $\overline{F}=\{f\in E/ f \text{ is equitablaadjacentforsameedgein } G\}$. Suppose H_1 is the minimum dominating set of $(F\cup\overline{F})$ and suppose H_2 is the minimum dominating set of $(E-(F\cup\overline{F}))$Now, $H_1\cup H_2$ is equitable dominating set of G. Therefore,

$$\gamma'_{ee}(G)\le|H_1\cup H_2| \quad \text{.......... (2)}$$

By (1) and (2) $\gamma'_{ee}(G)+|\,gamma_e(G)\le|FcupI|\;|H_1\cup H_2|$ and obviously $|F\cup I|+|H_1\cup H_2|\le p$ and we have $p=\alpha_e(G)+\beta_e(G)$ by Theorem in $[3]$. Hence, $\gamma'_{ee}(G)+\gamma_e(G)\le\beta_e(G)+\alpha_e(G)$. For equality, let $G\cong K_{1,2}$, then clearly $\gamma'_{ee}(K_{1,2})=2$, $\beta_e(K_{1,2})=2$ and $\alpha_e(G)=1$, then the equality is holding.

Case 2: If G has no end edge, then let D be a minimum equitable dominating set of G and let S be an end equitable dominating set of G, let D' be the covering set of set S and containing D. i.e., $D\subseteq D'$. Now, let $D'-D=s'$ and

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$\gamma'_{ee}(G)+\gamma_e(G)\le |S|+|D|\le |S'|+|D|$

and if $|S'|\ne V-D$, then $|S'|=p=\alpha_e(G)+\beta_e(G)$ Hence, $\gamma'_{ee}(G)+\gamma_e(G)\le \beta_e(G)+\alpha_e(G)$.

Remark 2.26. If G is not equitable connected, then $\gamma'_{ee}(G)+\gamma_e(G)\le \beta_e(G)+\alpha_e(G)$ is not true in general.

Example2.27. For any $K_{1,n}$, $n\ge 3$ we have $\gamma'_{ee}(K_{1,n})=n$, $\gamma'_e(K_{1,n})=n+1$, $\beta_e(K_{1,n})=n+1$ and $\alpha_e(G)=0$.

Hence $\gamma'_{ee}(G)+\gamma_e(G)\le \beta_e(G)+\alpha_e(G)$ is not true here because $K_{1,n}$, $n\ge 3$ is not equitable connected.

Theorem 2.28. For any graph G with at least two end edge have common vertex, $\gamma'_{ee}(G)\ge M_e$, where M_e is the equitable matching in G.

Proof: Let G be a graph which has at least two end edges have common vertex and let D be any minimum end equitable dominating set such that $D=F\cup F'$, where F is the set of all end edges in G and D' is the minimum equitable edge dominating set of graph $\langle E-(F\cup F')\rangle$, where F' is the set of edges which equitable adjacent to the end edges F, where $F\cup F'=\phi$. Now let J be the maximum equitable matching contains D''. That means $J=D'\cup S$, where $S\subseteq F$. Therefore,

$\gamma'_{ee}(G)=|D'|+|F|>|D'|+|S|\ge \beta_e(G)$.

Hence $\gamma'_{ee}(G)\ge \beta_e(G)$.

Remark 2.29. The equality of $\gamma'_{ee}(G)\ge M_e$ hold see example 2.26.

Example 2.30. Let G be a graph as in Figure 6.

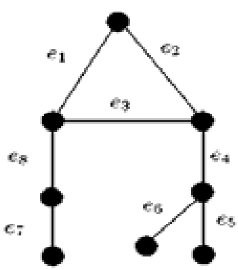

Figure 6

Obviously $\deg_e(e_1)=\deg_e(e_2)=\deg_e(e_8)=3$, $\deg_e(e_3)=\deg_e(e_4)=4$, $\deg_e(e_5)=\deg_e(e_6)=2$ and $\deg_e(e_7)=1$. The set of end edge is $\{e_5,e_6,e_7\}$. The set $\{e_3,e_5,e_6,e_7\}$ is a minimum end equitable edge dominating set in G. That means $\gamma'_{ee}(G)=4$ The set of $\{e_1,e_4,e_6,e_7\}$ is a maximum equitable matching in G. There are also the sets $\{e_4,e_6,e_7,e_8\}$, $\{e_4,e_5,e_7,e_8\}$ and $\{e_4,e_6,e_7,e_8\}$ all are maximum equitable matching in G. Thus $M_e(G)=4=\gamma'_{ee}(G)$.

Here we will state the following open problem for future work. What is the sufficient and necessary conditions for $\gamma'_{ee}(G)=M_e=?$

Theorem 2.31. For any equitable connected graph G, $\gamma'_{ee}(G)=\gamma'_e(G)\le p$, where p is the number of vertices. Further the equality holds if $G\cong K_{1,n}$ or C_4 or K_4.

Proof: Let G be an equitable connected graph, that means for any two vertices in G there is at least one equitable path connect between them. Let H be minimum equitable edge dominating set in G. Let $F=\{e\in E(G): e\ is\ an\ end\ edge\ in\ G\}$ and let I be the minimum equitable edge dominating set of the graph $\langle E-F\rangle$. Clearly $I\cup F$ is an end equitable edge dominating set of G. Since G is equitable connected, then G has at least $p-1$ edges and the minimum equitable edge dominating set H contains at least one edge and the set $I\cup F$ contains at most q edges. Therefore, $|H|+|F\cup I|\le q+1$ and since the number of edges here at least $p-1$, then $|H|+|F\cup I|\le p$. Finally, $\gamma'_{ee}(G)+\gamma'_e(G)\le |H|+|F\cup I|\le p$. Hence $\gamma'_{ee}(G)=\gamma'_e(G)\le p$.

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For equality, if $G \cong K_{1,n}$, then $\gamma'_{ee}(G) = n$ and $\gamma'_e(G) = 1$. Thus $\gamma'_{ee}(G) + \gamma'_e(G) = n+1$ which is the number of vertices in $K_{1,n}$.

Similarly, if $G \cong C_4$, we have $\gamma'_{ee}(G) = \gamma'_e(G) = 2$. Hence, $\gamma'_{ee}(G) + \gamma'_e(G) = 4$ the number of vertices in C_4.

For complete graph K_4, $\gamma'_{ee}(G) + \gamma'_e(G) = 4$ the number of vertices.

Here we will state the following open problem.
What is the sufficient and necessary conditions for $\gamma'_{ee}(G) + \gamma'_e(G) = p$?

Definition 2.32. The subdivision graph $S(G)$ of a graph G obtained by inserting vertex of degree two to every edge of G.

In the following results we get the exact values of the end equitable edge domination number of subdivision graph of some standard graphs.

Proposition 2.33. For any Path graph P_P with $p \geq 6$ vertices

$$\gamma'_{ee}\left(S\left(P_P\right)\right) = \begin{cases} \dfrac{2p+1}{3}, & if\ 2p \equiv 2 \bmod 3; \\ \dfrac{2p}{3}, & if\ 2p \equiv 0 \bmod 3; \\ \dfrac{2p+2}{3}, & if\ 2p \equiv 1 \bmod 3. \end{cases}$$

Proof: From the definition of subdivision graph of a graph $S\left(P_P\right) = P_{2p-1}$ and by apply Proposition 2.10 the result is holding.

Proposition 2.34. For any cycle graph C_P on p vertices.

$$\gamma'_{ee}\left(C_p\right) = \begin{cases} \dfrac{2p}{3}, & if\ 2p \equiv 0 \bmod 3; \\ \dfrac{2p+2}{3}, & if\ 2p \equiv 1 \bmod 3; \\ \dfrac{2p+1}{3}, & if\ 2p \equiv 2 \bmod 3. \end{cases}$$

Proof: The proof straightforward since $S(C_n) = C_{2p}$ and by apply Proposition 2.9.

Proposition 2.35. Let $B(m,n)$ be the bi-star graph of p vertices. Then $\gamma'_{ee}(S(B(m,n))) = p$.

.

Proof: Let G be the subdivision of $B(m,n)$ as in Figure 7.

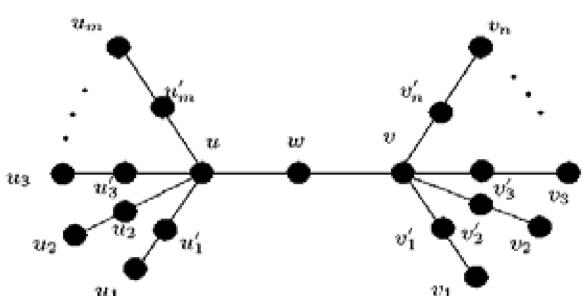

Figure 7

The end edge set is $F = \left\{ u_1u'_1, u_2u'_2, \ldots\ldots, u_mu'_m, v_1v'_1, v_2v'_2, \ldots\ldots v_2v'_2 \right\}$ which is of size $m+n$ all the edges in F has equitable degree zero, that means all the end edges of G are equitable isolated edges and any other two adjacent edges in $E-F$ are also equitable adjacent. So to equitable dominate the edges in $E-F$ we need only two edges without lose the generality vv'_1 and uu'_1 and clearly the set $\left\{ vv'_1, uu'_1 \right\}$ is minimum equitable edge for the set $E-F$. Hence, $\gamma'_{ee}(G) = m+n+2 = p$.

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References

1. Al-Kaenani, N. D. Soner and A. Alwardi Graphs and degree equitablity, Applied mathematics 4(8)(2013), 1199-1203.
2. A. Alwardi and N. D. Soner, Equitable edge domination in graphs, Bulleten of international mathematics, 3(2013), 7-13.
3. K. D. Dharmalingam, Studies in Graph Theorey-Equitable Domination and Bottleneck Domination, Ph.D Thesis, Madurai Kamaraj University, Madurai, 2006.
4. F. Harary, Graph Theory Adison Wesley Mass, 1972.
5. J. H. Hatting and M. A. Henning, Characrerization of trees with equal domintion parameter. Journal of graph theory, 34(2000), 142-153.
6. S. Michell and S. T. Hedetniemi, Edge domination in trees, Congr Number, 19(1977), 489-509.