



## Reliability Test Plans Based on Dagum Distribution Using Truncated Life Tests

### KEYWORDS

Reliability Test Plans, Economic sampling plan, Dagum distributions, Termination Ratio,

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### ABSTRACT

The acceptability of any product following a specified distribution is realized by reliability sampling plans. This paper deals with reliability sampling plans for truncated life test, when the lifetime of a product follows Dagum (D) distribution. The test termination ratio realising the final quality of acceptance is found by fixing the producer's risk at a fixed sample size and acceptance number. The operating characteristic curve for the sampling plan is presented. The results were given by suitable examples. The tables and graphs are also illustrated.

## Introduction

The Dagum (Burr III) comes under a four parameter generalized beta distribution of second kind (GB2) with density,

$$f(x) = \frac{ax^{ap-1}}{b^{ap} B(p,q) [1 + (\frac{x}{b})^a]^{p+q}} ; x, a, b, p, q > 0$$

The Dagum distribution is a GB2 distribution with two shape parameters  $p$  and ( $q = 1$ ) and scale parameter  $b$ . The density function is,

$$f(x) = \frac{apx^{ap-1}}{b^{ap} [1 + (\frac{x}{b})^a]^{p+1}} ; x, a, b, p > 0$$

It is worthy to note that the Dagum (D) and the Singh-Maddala distribution (SM) (Kleiber, Christian, 1996) are one to one related by the relation,

$$X \sim D(a, b, p) \Leftrightarrow \frac{1}{X} \sim SM\left(a, \frac{1}{b}, p\right)$$

This relationship permits to translate several results regarding to the Singh-Maddala family into corresponding results for the Dagum distribution; it may also be the reason for the name "inverse Burr distribution" often found in the actuarial literature for the Dagum distribution.

The CDF becomes

$$F(X) = [1 + (\frac{x}{b})^{-a}]^{-p} ; x > 0$$

Dagum refers a system known as "generalized logistic-Burr system". This is due to the fact that the Dagum distribution with  $p = 1$  is also known as the log-logistic distribution.

Dagum (1977, 1980) introduces two further variants of original distribution, hence the previously discussed (CDF) standard version will be referred to as the Dagum type I distribution. The Dagum type II distribution has the CDF,

$$F(x) = \alpha + (1 - \alpha) \left[ 1 + \left( \frac{x}{b} \right)^{-a} \right]^{-p} ; x, a, b, p > 0, \alpha \in (0, 1).$$

The type II distribution was proposed as a model for income distributions with null and negative incomes. The CDF of Dagum type III distribution is given by

$$F(x) = \alpha + (1 - \alpha) \left[ 1 + \left( \frac{x}{b} \right)^{-a} \right]^{-p} ; x, a, b, p > 0, \alpha > 0,$$

Both of the Dagum type II and the type III are the members of Dagum's generalized logistic-Burr distribution.

### Reliability Test Plans Based on Dagum Distribution from Truncated Life Tests

Assume that the lifetime of a product follows Dagum distribution which has the following probability density function (pdf) and cumulative distribution function (cdf) respectively;

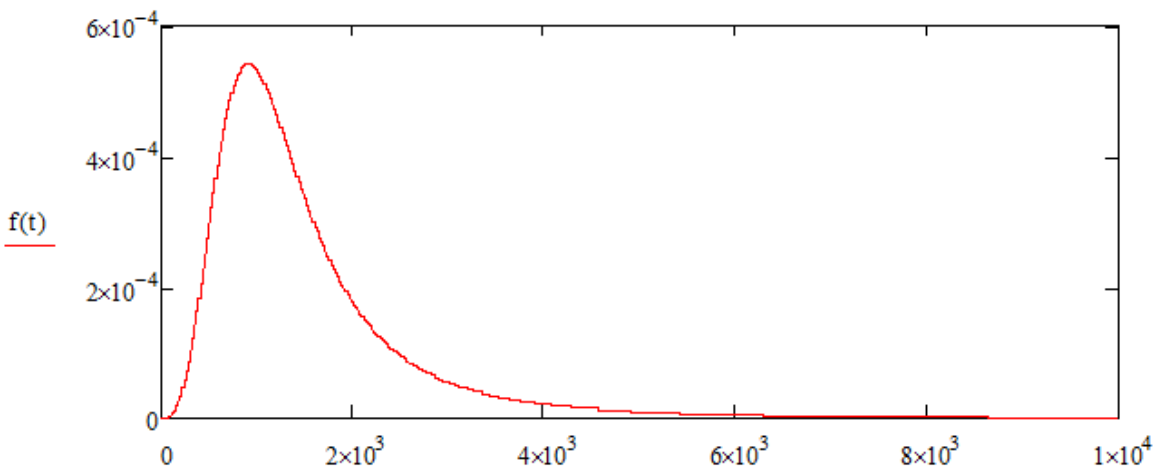
$$f(x) = \frac{apx^{ap-1}}{b^{ap} \left[ 1 + \left( \frac{x}{b} \right)^a \right]^{p+1}} ; x, a, b, p > 0 \quad (1)$$

And

$$F(x) = \left[ 1 + \left( \frac{x}{b} \right)^{-a} \right]^{-p} ; x, a, b, p > 0 \quad (2)$$

Where  $b$  is the scale parameter and  $a$  and  $p$  are the shape parameters.

The following figure shows the p.d.f for a random variable of Dagum distribution with parameters  $p=1.5, a=2.5, b=1000$



A common practice in life testing is to design a life test for a predetermined time  $t$ , with the probability of rejecting a bad lot is at least  $p^*$ , so that the maximum number of allowable bad items to be accepted in the lot is  $c$ . The acceptance sampling plan for a truncated life test is to set up the minimum sample size  $n$  for this given acceptance number  $c$  such that the consumer's risk- the probability of accepting a bad lot, does not exceed  $(1 - p^*)$ , i.e., the probability of accepting a bad lot (the one for which the true average life is below the specified average life  $\frac{t}{b_0}$ ) not to exceed  $1 - p^*$ , so that  $p^*$  is a minimum confidence level with which a lot of true average life below  $b_0$  is rejected, by the sampling plan. Therefore, for a given  $p^*$ , the proposed acceptance sampling plan can be characterized by the triplet  $(n, c, \frac{t}{b_0})$ .

In literature, R.R.L.Kantam, K.Rosaiah and G.Srinivasa Rao. (2001), K.Rosaiah and R.R.L.Kantam. (2005), Kantam, R.R.L and Srinivasa Rao, G.and Sriram B. (2006), Muhammad Aslam and Muhammad Qaisar shahbaz. (2007), Muhammad Aslam. (2008), Srinivasa Rao.G and M.E Ghitany, R.R.L Kantam. (2011), T. B Ramkumar and O.K Sajana (2011), G. Srinivasa Rao, R.R.L. Kantam , K.Rosaiah (2012), developed truncated life test plans based on different distributions .For a fixed  $p^*$  the sampling plan is determined for an average life ratio  $\frac{t}{b_0}$  along with minimum sample size  $n$  and acceptance number  $c$ . Considering sufficiently large sized lots the binomial distribution is suitable to distinguish type II error (A J Ducan. 1986 ). Thus the problem become determining the smallest positive integer  $n$  for given values of  $p^*$  ( $0 < p^* < 1$ ),  $b_0$  and  $c$  asserting that  $b > b_0$  must satisfy

$$\sum_{i=0}^c \binom{n}{i} p_0^i (1 - p_0)^{n-i} \leq 1 - p^* \quad (3)$$

Where  $p_0 = F_{\frac{t}{b_0}}(t)$  given by (2), which is the failure probability of the lot following Dagum distribution before time  $t$ . Hence it is evident that the product quality depends on the life time ratio  $\frac{t}{b_0}$  so that it is sufficient to design the life test experiment. The minimum values of  $n$  satisfying the inequality (1) are obtained and displayed in the Table 1 for the probabilities  $p^* = 0.90, 0.95, 0.99$  and life ratio  $\frac{t}{b_0} = 1, 1.25, 1.5, 1.75, 2, 2.25, 2.5, 3, 3.5, 4$ . The graph of minimum sample size is shown in Figure 3

## Economic test plan

Fixing 'n' and 'r' be a natural number less than sample size (n). The acceptance criteria can be restated as follows:

Put 'n' items on test.

Stop the process if  $r = (c+1)^{\text{th}}$  failure occur before  $t$  and the lot is rejected.

One may be interested that the probability of acceptance should be as large as possible for a given  $b=b_0$  and specify 'n' as a multiple of 'r' ( $r=1, 2, \dots$ ). Then

$$L(p_0) = \sum_{i=0}^c \binom{n}{i} p_0^i (1 - p_0)^{n-i} \geq 1 - p^* \quad (4)$$

Given the values of  $n$  (where  $n=r \cdot k$ ),  $r$  and  $k$  (integer) the above inequality can be solved for ' $p_0$ ' using cumulative probabilities of Binomial distribution. Then the values of ' $p_0$ ' can be used in the cumulative density function for  $a=2.5$  and  $p=1.5$  to find the values of  $\frac{t}{b_0}$ . With the choices of  $r$  ( $r=1, 2, \dots, k$ ) and  $p^*$ , the termination ratio could be found satisfying the above inequality. The termination ratios are given in Table 4 for various values of  $r$  and  $n$ .

### Operating Characteristic of the Sampling Plan

The OC function  $L(p)$  of the sampling plan  $(n, c, \frac{t}{b_0})$  is the probability of accepting a lot at various values of failure probability  $p$ . It is given as

$$L(p) = \sum_{i=0}^c \binom{n}{i} p_0^i (1 - p_0)^{n-i} \quad (4)$$

The average life time of the product increases with increase in values of  $b$  and, therefore, the failure probability  $p_0 = F_{\frac{t}{b_0}}(t)$  decreases for increases in  $b$ . It implies that the operating characteristic function is monotonic w.r.t  $b$ . The producer's risk is the probability of rejecting the lot although  $b > b_0$  holds. It is evidenced from the operating characteristic function that the type I error depends on  $\frac{b}{b_0}$ .

$$L(p(b)) = L(F_{\frac{t}{b}}(t))$$

$$p(b) = F_{\frac{t}{b_0} \frac{b_0}{b}}(t)$$

For specified sampling plan the operating characteristic function depending on  $\frac{b}{b_0}$  are displayed in Table 2. The plot for operating characteristic function is shown in Figure 4.

### Producer's Risk

The producer's risk is defined as the probability of rejecting the lot when  $b > b_0$ . For a given value of the producer's risk say  $\gamma$ , it is interested to know the value of  $d_q = \frac{b}{b_0}$  as to ensure the producer's

risk less than or equal to  $\gamma$ , provided the sampling plan  $(n, c, \frac{t}{b_0})$  is developed at a specified confidence level  $p^*$ . Thus one needs to find the smallest value  $d_q$  according to (4) as,

$$\sum_{i=0}^c \binom{n}{i} p_0^i (1 - p_0)^{n-i} \geq 1 - p^*$$

Where  $p = F_{\frac{t}{b_0}, \frac{b_0}{b}}(t)$ . The acceptability of a lot under  $a = 2.5, p = 1.5$  at the producer's risk of  $p^*=0.05$  are presented in Table 3.

### Illustration of Tables

Assume that the life time distribution is Dagum with  $a = 2.5$  and  $p = 1.5$  and the experimenter is interested in knowing that the true unknown average life is at least 800 hours. Let the consumers risk is set to be  $1 - p^* = 0.05$ , it is desired to stop the experiment at  $t = 1000$  hours. Then for an acceptance number  $c = 2$ , the required sample size  $n = 10$ , corresponding to the value  $p^* = 0.95, \frac{t}{b_0} = 1.25$  and  $c=2$  (From Table1). Thus 10 units have to be put on test. If during 1000 hours, not more than 2 failures out of 10 are observed then the experimenter can assert with confidence level  $p^* = 0.95$  that the average life is at least 800 hours using binomial approximation. That is, the sampling plan is  $(n = 10, c = 2, \frac{t}{b_0} = 1.25)$  with consumers risk 0.05. For this sampling plan  $(n = 10, c = 2, \frac{t}{b_0} = 1.25)$  with confidence level  $p^*=0.95$  under the Dagum distribution the operating characteristic values from Table 3

$\frac{b}{b_0}$	0.5	1	1.5	2	4	8
L(p)	0.0000	0.0500	0.5510	0.9020	1.0000	1.0000

From Table 3, the value of  $\frac{b}{b_0}$  for various values of  $c$  and  $\frac{t}{b_0}$ . Thus for the above discussion it is obtained that the value of  $\frac{b}{b_0} = 2.22$ , i.e. the product should have an average life time in order that under the above acceptance sampling plan  $(10, 2, 1.25)$ , the product is accepted with probability of at least 0.95. The actual average life time necessary to accept 95% of the lot is provided in Table 3.

## Example

Consider a simulated problem associated with failure times of bulbs whose lifetime follows four-parameter Burr distribution with  $\gamma=500$ ,  $k=2$ , and  $\alpha=2$ . This data can be regarded as an ordered sample of size  $n=10$  with observations 1580, 1045, 548, 4701, 1862, 5266, 1578, 1694, 1545 and 1331.

### Illustration of Example using Reliability Test Plan

Let the required average life time be 1000 hours and the testing time be  $t = 1050$ , leading to ratio  $\frac{t}{b_0} = 1.05$  with a corresponding sample size  $n=10$  and acceptance number  $c=2$  obtained from Table 1, for  $p^* = 0.95$ . Therefore the sampling plan for the above sample data is  $(n=10, c=2, \frac{t}{b_0}=1.05)$ . Based on the data it is to decide whether the lot is accepted or rejected. Accept the product only if the number of failures before 1050 hours is less than or equal to 2.

In the above sample of size 10, there are only two failures before the termination time  $t=1050$  hours and the acceptance number of the plan  $c=1$ . Therefore accept the product.

### Illustration of Example using Economic sampling Plan

From, the life test termination Table 4, corresponding to  $r=2$  ( $r=c+1$ ), for column  $5r$ , the value is 0.44. As the acceptable average lifetime is 1000 hours (given) for Four – parameter Burr distribution then the termination time is obtained as  $t=0.44*1000=440$ .

According to the above sampling plan, if the first failure is realized before 440<sup>th</sup> hour of the test, reject the lot otherwise accept the lot. In either case terminating the experiment as soon as the first failure is reached or 440<sup>th</sup> hour of the test time is realized whichever is earlier. In the case of acceptance, the assurance is that the average life of the submitted products is at least 1000 hours.

There are no failure before 440<sup>th</sup> hour, and accept the lot (in the example) by this approach. Thus for both approaches the sample size, acceptance number (termination number), the risk probability and the decision about the lot are same. The decision on the first approach can be reached only at the 1050<sup>th</sup> hour while in the second approach decision is realized at the 440<sup>th</sup> hours, showing that the economic sampling plan require less waiting time.

## Comparative Study and Summary

In order to compare the economic sampling plan with reliability test plan, the values are

presented for  $\alpha = 2, 1-p^*=0.05,0.01$  in Table 4. The entries given in the first row represents economic sampling plan and the values of second row shows reliability sampling plan. Termination values in the example is taken, for  $r=2, n=5r$  and  $1-p^*=0.05$  from Table 4. For any  $r, n$  and  $1-p^*$ , the present test plan gives minimum termination time than reliability test plan saving money and time of experiment

This paper presents the economic test plan for lifetime of products following Dagum Distribution. The proposed plan suggests a way for achieving optimum reliability test plan for Dagum Distribution. Comparative study affirms the conclusion that the present test plan works with minimum termination time (waiting time) ensuring minimum experimental cost at more or less same sampling design.

The continuous improvement and review of acceptance sampling plan is important to improve the quality of the products and to ensure customer satisfaction.

*Table1: Minimum Sample Size for the specified ratio  $\frac{t}{b_0}$ , confidence level  $p^*$ , acceptance number  $c, a=2.5$  and  $p = 1.5$  using binomial approximation.*

$p^*$	$c$	$\frac{t}{b_0}$									
		1	1.25	1.5	1.75	2	2.25	2.5	3	3.5	4
0.9	0	6	4	3	2	2	2	2	1	1	1
0.9	1	10	7	5	4	4	3	3	3	2	2
0.9	2	14	9	7	6	5	5	4	4	3	4
0.9	3	17	12	9	8	7	6	6	5	4	5
0.9	4	21	14	11	9	8	7	7	6	6	6
0.9	5	24	16	13	11	10	9	8	8	7	7
0.9	6	28	19	15	12	11	10	10	9	8	8
0.9	7	31	21	16	14	12	11	11	10	9	9
0.9	8	35	23	18	15	14	13	12	11	10	10
0.9	9	38	26	20	17	15	14	13	12	11	11
0.9	10	41	28	22	19	17	15	15	14	12	12
0.95	0	7	5	4	3	2	2	2	2	2	1
0.95	1	12	8	6	5	4	4	3	3	3	3
0.95	2	16	10	8	7	6	5	5	4	4	4
0.95	3	20	13	10	8	7	7	6	6	5	5
0.95	4	24	16	12	10	9	8	8	7	6	6
0.95	5	27	18	14	12	10	10	9	8	8	7
0.95	6	31	21	16	13	12	11	10	9	9	8
0.95	7	34	23	18	15	13	12	11	11	10	10
0.95	8	38	25	20	17	15	14	13	12	11	11
0.95	9	41	28	21	18	16	15	14	13	12	12
0.95	10	45	30	23	20	18	16	15	14	13	13
0.99	0	11	7	5	4	3	3	3	2	2	2
0.99	1	16	10	8	6	5	5	4	4	4	3
0.99	2	21	13	10	8	7	6	6	5	5	5
0.99	3	25	16	12	10	9	8	7	7	6	6

0.99	4	29	19	14	12	10	9	9	8	7	7
0.99	5	33	22	16	14	12	11	10	9	9	8
0.99	6	37	24	19	15	14	12	11	10	10	9
0.99	7	41	27	21	17	15	14	13	12	11	10
0.99	8	44	29	23	19	17	15	14	13	12	12
0.99	9	48	32	24	20	18	17	15	14	13	13
0.99	10	52	34	26	22	20	18	17	15	14	14

Table 2: Values of the operating characteristic function of the sampling plan  $(n, c, \frac{t}{b_0})$  for given confidence level  $p^*$  with  $a = 2.5$  and  $p = 1.5$

$P^*$	n	c	$\frac{t}{b_0}$	$\frac{b}{b_0}$					
				0.5	1	1.5	2	4	8
0.9	14	2	1	0.000	0.080	0.699	0.956	1.000	1.000
0.9	9	2	1.25	0.000	0.083	0.624	0.926	1.000	1.000
0.9	7	2	1.5	0.000	0.071	0.524	0.873	1.000	1.000
0.9	6	2	1.75	0.000	0.057	0.423	0.798	0.999	1.000
0.9	5	2	2	0.001	0.072	0.404	0.759	0.998	1.000
0.9	5	2	2.25	0.000	0.037	0.269	0.623	0.995	1.000
0.9	4	2	2.5	0.004	0.090	0.363	0.677	0.994	1.000
0.9	4	2	3	0.002	0.042	0.207	0.475	0.977	1.000
0.9	4	2	4	0.000	0.011	0.069	0.207	0.870	1.000
0.95	16	2	1	0.000	0.042	0.620	0.937	1.000	1.000
0.95	10	2	1.25	0.000	0.050	0.551	0.902	1.000	1.000
0.95	8	2	1.5	0.000	0.034	0.419	0.824	0.999	1.000
0.95	7	2	1.75	0.000	0.022	0.299	0.715	0.998	1.000
0.95	6	2	2	0.000	0.023	0.253	0.640	0.997	1.000
0.95	5	2	2.25	0.000	0.037	0.269	0.623	0.995	1.000
0.95	5	2	2.5	0.000	0.020	0.175	0.487	0.987	1.000
0.95	4	2	3	0.002	0.042	0.207	0.475	0.977	1.000
0.95	4	2	4	0.000	0.011	0.069	0.207	0.870	0.999
0.99	21	2	1	0.000	0.008	0.434	0.880	1.000	1.000
0.99	13	2	1.25	0.000	0.010	0.358	0.820	1.000	1.000
0.99	10	2	1.5	0.000	0.007	0.254	0.715	0.999	1.000
0.99	8	2	1.75	0.000	0.008	0.205	0.629	0.997	1.000
0.99	7	2	2	0.000	0.007	0.151	0.524	0.994	1.000
0.99	6	2	2.25	0.000	0.009	0.142	0.474	0.990	1.000
0.99	6	2	2.5	0.000	0.004	0.077	0.330	0.977	1.000
0.99	5	2	3	0.000	0.006	0.072	0.269	0.951	1.000
0.99	5	2	4	0.000	0.001	0.013	0.072	0.759	0.998



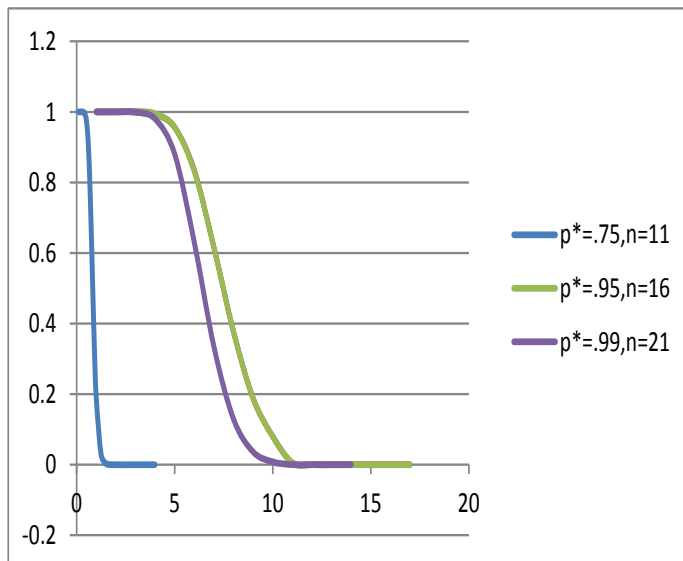
Table 3: Minimum Ratio of true  $b$  and required  $b_0$  for the acceptability of a lot with producers risk of 0.05 for  $a = 2.5$  and  $p = 1.5$

$p^*$	$c$	$\frac{t}{b_0}$									
		1	1.25	1.5	1.75	2	2.25	2.5	3	3.5	4
0.9	0	3.58	4	4.43	4.62	5.24	5.96	6.63	6.45	7.53	8.6
0.9	1	2.33	2.61	2.79	3.01	3.44	3.47	3.85	4.62	4.41	5.6
0.9	2	1.99	2.14	2.34	2.56	2.7	3.04	3	3.6	3.48	4.8
0.9	3	1.79	1.97	2.1	2.33	2.5	2.59	2.88	3.09	2.99	4.12
0.9	4	1.69	1.97	1.97	2.08	2.24	2.32	2.58	2.76	3.22	3.68
0.9	5	1.61	1.82	1.88	2.01	2.18	2.31	2.36	2.82	2.96	3.36
0.9	6	1.57	1.72	1.81	1.87	2.03	2.14	2.38	2.63	2.73	3.12
0.9	7	1.52	1.68	1.7	1.84	1.91	2.01	2.24	2.48	2.57	2.94
0.9	8	1.49	1.62	1.67	1.75	1.91	2.03	2.12	2.34	2.44	2.74
0.9	9	1.46	1.57	1.64	1.73	1.82	1.95	2.02	2.24	2.33	2.66
0.9	10	1.43	1.55	1.61	1.72	1.83	1.86	2.06	2.33	2.23	2.55
0.95	0	3.75	4.26	4.8	5.15	5.26	5.92	6.6	7.95	9.21	8.6
0.95	1	2.47	2.73	2.97	3.27	3.43	3.85	3.83	4.59	5.36	6.14
0.95	2	2.08	2.22	2.46	2.71	2.92	3.03	3.36	3.6	4.2	4.8
0.95	3	1.89	2.03	2.21	2.33	2.5	2.81	2.88	3.47	3.61	4.12
0.95	4	1.78	1.92	2.05	2.19	2.38	2.51	2.79	3.09	3.22	3.68
0.95	5	1.68	1.8	1.94	2.1	2.18	2.46	2.56	2.84	3.3	3.38
0.95	6	1.63	1.75	1.86	1.96	2.14	2.28	2.38	2.63	3.07	3.13
0.95	7	1.57	1.68	1.81	1.92	2.01	2.14	2.24	2.69	2.88	3.29
0.95	8	1.54	1.63	1.76	1.88	1.99	2.14	2.26	2.55	2.73	3.12
0.95	9	1.51	1.61	1.68	1.8	1.9	2.05	2.16	2.43	2.61	2.98
0.95	10	1.48	1.57	1.65	1.78	1.9	1.96	2.07	2.33	2.49	2.86
0.99	0	4.24	4.69	5.13	5.62	5.9	6.66	7.4	7.95	9.21	10.52
0.99	1	2.7	2.91	3.27	3.47	3.72	4.19	4.3	5.13	6	6.4
0.99	2	2.26	2.44	2.66	2.85	3.12	3.29	3.65	4.05	4.71	5.3
0.99	3	2.04	2.19	2.36	2.56	2.79	2.99	3.13	3.75	4.04	4.6
0.99	4	1.89	2.04	2.18	2.38	2.51	2.68	2.98	3.35	3.61	4.12
0.99	5	1.8	1.95	2.06	2.26	2.39	2.59	2.73	3.06	3.58	3.76
0.99	6	1.73	1.85	2.01	2.1	2.32	2.41	2.54	2.85	3.33	3.5
0.99	7	1.68	1.79	1.94	2.05	2.19	2.36	2.51	2.86	3.13	3.28
0.99	8	1.63	1.73	1.88	1.99	2.15	2.25	2.39	2.72	2.97	3.4
0.99	9	1.59	1.7	1.79	1.91	2.05	2.23	2.28	2.58	2.83	3.22
0.99	10	1.57	1.66	1.76	1.88	2.03	2.4	2.28	2.48	2.71	3.1

Table 4: Proportions of life test termination time  $(\frac{t}{b_0})$  for the present sampling plans and reliability test plan with producer's risk  $1-p^* = 0.05, 0.01$ .

n	2r	3r	4r	5r	6r	7r	8r	9r	10r	n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1-p*=0.05										1-p*=0.01									
1	0.39	0.35	0.32	0.3	0.29	0.27	0.26	0.26	0.25	1	0.25	0.22	0.2	0.19	0.18	0.18	0.17	0.16	0.16
2	1.92	1.5	1.27	1.15	1	0.98	0.93	0.9	0.86	2	2.78	2	1.61	1.43	1.25	1.18	1.11	1.05	1
3	0.59	0.51	0.47	0.44	0.41	0.39	0.38	0.36	0.35	3	0.46	0.39	0.36	0.34	0.32	0.31	0.3	0.29	0.28
4	1.85	1.39	1.18	1.05	0.97	0.91	0.86	0.83	0.79	4	2.38	1.67	1.39	1.22	1.12	1.05	0.98	0.94	0.89
5	0.69	0.59	0.54	0.5	0.47	0.45	0.43	0.41	0.4	5	0.57	0.49	0.44	0.41	0.39	0.37	0.36	0.34	0.33
6	1.75	1.32	1.12	1	0.94	0.87	0.83	0.79	0.76	6	2.22	1.56	1.3	1.15	1.05	0.98	0.93	0.89	0.85
7	0.76	0.64	0.58	0.53	0.5	0.48	0.46	0.44	0.43	7	0.64	0.54	0.49	0.46	0.43	0.41	0.39	0.38	0.37
8	1.7	1.25	1.09	0.98	0.9	0.85	0.81	0.77	0.74	8	2.08	1.47	1.24	1.1	1.01	0.94	0.89	0.84	0.81
9	0.81	0.67	0.6	0.56	0.53	0.5	0.48	0.46	0.45	9	0.69	0.59	0.53	0.49	0.46	0.44	0.42	0.41	0.4
10	1.67	1.25	1.06	0.96	0.89	0.83	0.79	0.75	0.73	10	1.96	1.41	1.19	1.06	0.97	0.91	0.86	0.82	0.79
11	0.84	0.7	0.63	0.58	0.54	0.52	0.49	0.48	0.46	11	0.73	0.61	0.55	0.51	0.48	0.46	0.44	0.43	0.41
12	1.64	1.23	1.05	0.94	0.87	0.82	0.78	0.75	0.71	12	1.91	1.38	1.16	1.04	0.95	0.89	0.84	0.81	0.78
13	0.87	0.72	0.64	0.59	0.56	0.53	0.51	0.49	0.47	13	0.76	0.64	0.58	0.53	0.5	0.48	0.46	0.44	0.43
14	1.61	1.21	1.04	0.94	0.86	0.81	0.77	0.74	0.71	14	1.85	1.34	1.14	1.02	0.94	0.88	0.83	0.79	0.76
15	0.89	0.74	0.66	0.6	0.57	0.54	0.52	0.5	0.48	15	0.79	0.66	0.59	0.55	0.51	0.49	0.47	0.45	0.44
16	1.59	1.2	1.03	0.92	0.86	0.8	0.76	0.73	0.7	16	1.82	1.33	1.12	1	0.92	0.86	0.82	0.78	0.75
17	0.91	0.75	0.67	0.61	0.58	0.55	0.52	0.5	0.49	17	0.81	0.68	0.61	0.56	0.52	0.5	0.48	0.46	0.44
18	1.58	1.18	1.02	0.91	0.85	0.79	0.76	0.72	0.69	18	1.79	1.3	1.11	0.99	0.91	0.86	0.81	0.77	0.74
19	0.92	0.76	0.68	0.62	0.58	0.55	0.53	0.51	0.49	19	0.83	0.69	0.62	0.57	0.54	0.51	0.49	0.47	0.45
20	1.55	1.17	1.01	0.91	0.84	0.79	0.75	0.72	0.69	20	1.75	1.28	1.09	0.98	0.9	0.84	0.8	0.76	0.74

Figure 3: The plot for the operating characteristic curve as a function of  $\frac{t}{b_0}$  and n for c=2.



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