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Probabilistic Seismic Hazard Analysis on the Base of the Stochastic Models of Seismicity

KEYWORDS: PSHA, clustering, Markov model, attenuation

Rashid Burtiev	Vasile Alcaz	Vladlen Cardanets
Institute of Geology and Seismology, Moldavian Academy of Sciences, 3 Academiei Str., Chisinau, Moldova	Institute of Geology and Seismology, Moldavian Academy of Sciences, 3 Academiei Str., Chisinau, Moldova	Institute of Geology and Seismology, Moldavian Academy of Sciences, 3 Academiei Str., Chisinau, Moldova

ABSTRACT *A universal method based on the stochastic models of seismicity to seismic hazards assessment is proposed. It means that in a certain location of the Earth's surface during the time of t will occurs a certain number n of shakes and m of them with an intensity of Ik (more Ik). To identify the area of earthquakes on the seismic zones cluster analysis is used. The Markov model for seismic events was elaborated. Intensity attenuation law for Moldova and Romania, was obtained on base of macroseismic data set of the intermediate earthquakes occurred in Vrancea seismic zone The equation coefficients of the attenuation for intermediate earthquakes of the Vrancea seismic zone were determined. The angle between the major axis of the attenuation ellipse and the positive direction of abscissa is γ0=510. Probabilistic seismic hazard analysis was performed and the map of seismic zoning of Moldova and adjacent territory elaborated.*

INTRODUCTION

The seismic hazard for the territory of the Republic of Moldova and mostly part of Romanian territory is determined by the seismic source "Vrancea". The first seismic hazard map of Moldova, based on seismological and geological data was compiled by the Institute of Earth Physics of the Academy of Sciences of USSR in 1957. Southwestern part of Moldova, near the board with Romania, referred to seismic intensity VIII, the rest of the territory – to VII-degree zone. According to map CP–69 (seismic zoning of USSR territory, 1969), was done corrections into the previous map of seismic zoning – for the Northern and North-Eastern territory of Moldova the level of seismic hazard was reduced from 7 to 6 MSK intensity. In 2007, the new map of seismic hazard Republic of Moldova territory was elaborated (Alkaz, 2007). The new map includes zones of 6, 7 and 8 MSK intensities, the 8 -intensity zone being smaller comparing with previous.

Probabilistic seismic hazard assessment (PSHA) consists an important element of realization the strategy of earthquake-proof construction (Alkaz, 2005; Borman, 2011; Petrova, 2010). Various studies have been performed to evaluate the seismic hazard (Baker, 2008; Rotaru and Kolev, 2010). However, the existing measure of seismic hazard – the 0.01 probability of exeedance some level of PGA, - (Cornell, 1968) is not sufficiently informative for optimal planning and earthquakes resistance construction in seismic active areas (Borman, 2011; Gavrilescu and Manta, 2011). For example, let us suppose that the observer (civil engineer) at the point $V(\varphi,\psi)$ is wondering about possible earthquake and its seismic effect in that point: what seismic situation will we have for the next 475 years? The probability of exceedance some level of PGA in 50 years is not enough informative for describing the full seismic situation.

The main idea of the proposed approach is as follows: suppose the site is located in the area of influence of several seismic zones $\{Z^s, s=1, S\}$. As the measure of seismic hazard, the following probability is proposed:

$$\mathrm{P}_{(\varphi,\psi)}(t,n,m,I^k) \qquad (1)$$

Eq. (1) means that in the location of the Earth's surface $V(\varphi,\psi)$, with geographical coordinates (φ,ψ), during the time t, ground shaking will occur n times and n_k of them will have intensity I^k (Burtiev, 1986; 2012, 2015). $P_{(\varphi,\psi)}(t,n,n_k,I^k)$ is a universal measure for seismic hazard, because from it is possible to derive all parameters of seismic hazards: the average number of shakes of intensity, the economic losses , the probability of exceeding of maximal value of shakes in European Unions' standards EUROCODE 8 (EC-8) (Solomos et al., 2008), the recurrence period of maximal events T_R, etc.

Seismic intensity is an integrated measure of ground shaking, directly related to the effects on people, objects and buildings (Radulian et al., 2002). If the intensity of ground shaking at $V(\varphi,\psi)$ follows the normal distribution (Pasolini, 2008; Burtiev, 2012, 2015), the necessary earthquake parameters for predicting seismic intensity are specifically: magnitude; location of epicenter φ, ψ; focal depth h; the frequency of earthquake occurrence.

PSHA analysis is a way to consider the probabilistic nature of seismicity and ground shaking. Hence, firstly we must identify the seismic zone in order to elaborate the stochastic model of seismicity. In turn, this will make possible to define the seismic parameters, to determine their distribution and eventually will lead to

The present approach consists of the following four steps: identification of seismic zones; elaboration of a probabilistic model of seismicity zones; determination of attenuation relation of ground shaking; implementation of PSHA. The FORTRAN program, created by author for the realization the proposed PSHA method allows for the statistical analysis of the macroseismic data set for evaluation of the attenuation equation. It also allows the identification of seismic zones and elaboration of the stochastic model of seismic zones seismicity and for calculation of seismic hazard.

1. IDENTIFICATION OF SEISMIC ZONES

Catalog of earthquakes is one of the basic PSHA elements. : Catalogue ROMPLUS (http://www1.infp.ro/arhiva-in-timp-real) contains data about the Romanian earthquakes but no information about seismic zones. Cluster analysis has been applied to identify the area of Romania seismic zone. (BURTIEV 2012). For this, two clustering procedures were used: agglomerative hierarchical clustering methods and nonhierarchical k-means clustering.

1.1. Agglomerative hierarchical clustering method

The Euclidean distance between earthquake epicenters $V_i(\varphi_i,\psi_i)$ and $V_j(\varphi_j,\psi_j)$ (Mingjin, 2005) is:

$$D_{ij}=\sqrt{(\varphi_i-\varphi_j)^2+(\psi_i-\psi_j)^2} \qquad (2)$$

where φ - latitude and ψ - longitude of epicenters. Simple mathematical derivation shows that the total sum of the squared errors $Q(X,G)$ is monotonically decreasing with g (Mingjin, 2005):

$$Q(X,G)=\sum_{g=1}^{G}\sum_{q\in C_g}\left\|v-c_g\right\|^2,\ c_g=\frac{1}{n_g}\sum_{q\in C_g}v \qquad (3)$$

where G –is the number of clusters, C_g – the g-th cluster.

The catalogue ROMPLUS contains gathered data from about N=11589 seismic events for the 1984-2014 period. Clustering was accomplished by the procedure of "Cluster analysis" of the statistical package SPSS (IBM SPSS Statistics v20). To determine how many clusters we needed to represent the data, need to identify the step J=11584 where the "distance coefficients" Eq. (3) makes a bigger jump from 2.753 to 3.472 and use the rule $G=N-J=5$. The best cluster decision refers to 5 clusters, which minimized the functionality of cluster quality Eq. (3) $Q(X,G)=0.347E+05$ is used as the initial guess for k-means clustering analysis described in next section.

1.2. k-means clustering

The *k-means* method an iterative procedure was taken from the initial guess $k=5$ to define optimal cluster solution. Numerous methods for estimating the optimal number of clusters were proposed and the optimal decision is based on the values of methods' index. In this case, the optimal number of seismic zones on the Romanian territory is (Mingjin, 2005):

Calinski and Harabasz's method:

$$Ch(k)=\frac{trace(B_k)(n-k)}{trace(W_k)(k-1)} \qquad (4)$$

The best cluster decision is $K=13$, that correspond to maximal value of Calinski-Harabasz index $Ch_k=0.120E+07$.

Krzanowski-Lai's method (Tibshirani R., et al. 2001):

$$KL(k)=\left|\frac{DIFF(k)}{DIFF(k+1)}\right|,\ DIFF(k)=(k-1)^{2/p}W(k-1)-k^{2/p}W(k) \qquad (5)$$

For number of clusters $K=13$, the value $KL(K)=11.2$ is maximal.

Sugar and James method (Tibshirani R., et al. 2001):

$$J_k=d_k^{-\lambda}-d_{k-1}^{-\lambda},\ d_{ij}^2=\sum_{h=1}^{m}\left(x_{ih}-x_{jh}\right)^2 \qquad (6)$$

The best decision is $k=13$, which correspond to the maximum of $J_k=0.311$.

Silhouette statistic (Milligan 1985):

$$s(i)=\begin{cases}0, & j-th\\ \dfrac{b(i)-a(i)}{\max(a(i),b(i))}, & another\end{cases}$$

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object is in a single cluster:

(7)

$$\overline{S}_k = \frac{1}{n}\sum_{i=1}^{n} s(i),\ \ k \in (2,...,K) \tag{8}$$

where K– the number of all clusters. The optimal number of the clusters defined from maximum functional:

$$\hat{G} = \arg\max_{2 \le k < M} \overline{S_k} \tag{9}$$

is $k=2$ – which corresponds to the maximal values $\hat{G}=0.897$. However, all cluster solutions are acceptable, because the values of the silhouette coefficient are within the tolerance range $(0,1)$.

The Hartigan's criterion (Mingjin 2005):

$$H(k) = (n - K - 1)\left[\frac{W(K)}{W(K+1)} - 1\right] \tag{10}$$

The optimal cluster decision corresponds to the number $K=13$, on which sample values $H(13)=-0.00267$ is less than 10. Therefore, the dominating decision for earthquakes from ROMPLUS catalogue is 13 clusters.

Optimization methods of cluster decision identified the dominant solution as 13 clusters (13 seismic zones, Fig.1) 1 wich is consistent with the existing 13 seismic zones earlier identified in Romania (Marmureanu, 2009).

The statistical capacity of listed above methods is not inferior to other methods and allows developing simpler FORTRAN program for their realization.

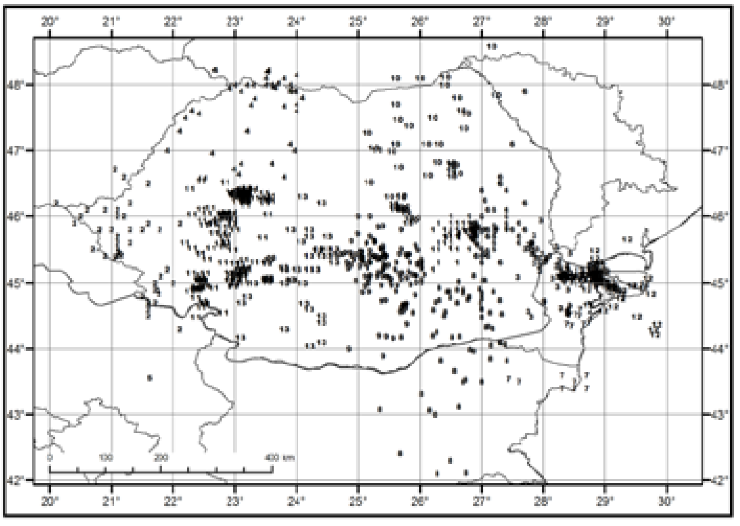

Fig. 1. The map of clusters. Symbols: 1, 2, 3,…, 13 – the epicenters of earthquakes in clusters with the same number.

1.3. Unimodality of clustering

Unimodality detection is a natural way to identify the presence of clusters, understanding each of them as a mode surrounded by a density and separated enough from other modes, if they exist (Alvarez, Daniel, 2013). Figure 2 shows how frequently certain numbers appear in the set of data of distance between the center of cluster and epicenters in first cluster. In the presented histogram, for seismic zone Vrancea - first cluster in Figure 1 has only one peak; this is a visual cue to know if a data set has unimodal distribution. The number of distance bins k is established by rules $k= (2n)^{0.333}$ (Lolla and Hoberock, 2011), n - the number of epicenters in cluster. The negative correlation $r=-0.28772$ between magnitude of earthquakes and removal of epicenters from middle point

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of first cluster is significant at the significance level α=0.01. Let the null hypothesis H_0 implies that the dataset of epicenters in clusters has unimodal distribution and the alternative H_1 forms that the distribution is multimodal. For evaluation, the cluster structure of a dataset based on testing the empirical density distribution of the data set for unimodality was used the bimodality coefficient:

$$BC = \frac{m_3^2 + 1}{m_4 + \frac{3(n-1)^2}{(n-2)(n-3)}} \qquad (11)$$

where m_3 is skewness and m_4 is kurtosis. Values of b greater than 0.555 (the value for a uniform population) may indicate bimodal or multimodal marginal distributions (Knapp, 2007).

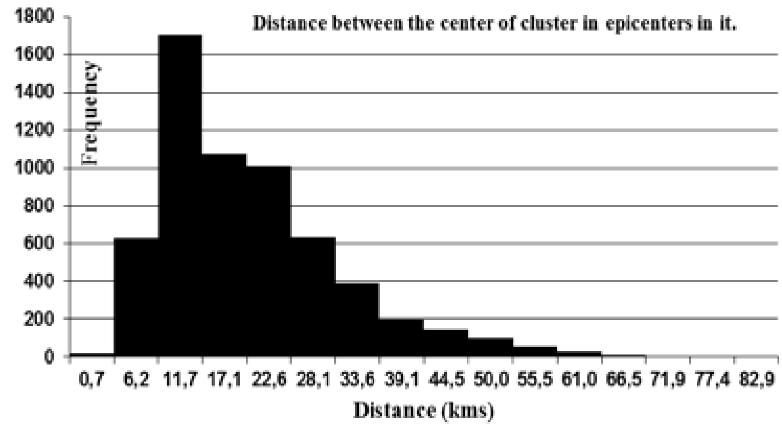

Fig. 2. Graphical representation of pairwise distance (similarities) distribution between the center of first cluster (Vrancea seismic zone) and the epicenters in it. On horizontal axe represented bin boundaries and on vertical axe the number of epicenters in bins.

Sign (+) in Table 1 means the hypothesis H_0 that $F_n(x)$ – the empirical distribution of distance is unimodal and should be retain (H_0 is plausible) – the bimodality coefficient (*BC*) is less that threshold value 0.555. The sign (-) indicates that the hypothesis H_0 must be rejected in favor of H_1.

Table 1.

The value of bimodality coefficient

Number of cluster	1	2	3	4	5	6	7	8	9	10	11	12	13
BC H_0	0.532 +	0.504 +	0.397 +	0.403 +	0.426 +	0.588 -	0.609 -	0.677 -	0.440 +	0.543 +	0.38 +	0.69 -	0.539 +

If the hypothesis H_0 is rejected it means that the dataset in cluster is heterogeneity and contains multiple cluster structure. The Markov model of seismic events sequence is useful by solving the problems with heterogeneity in seismic studies.

2. PROBABILISTIC MODEL OF SEISMICITY

The seismic zone Z^s occupies some space in earth environment $(F^s x \Psi^s x H^s)$. The *n*-th earthquake with its origin time t_n can be considered as a position in the four-dimensional space Ξ^s of the *s*-th seismic zone Z^s, which is the Cartesian product $\Xi^s=(F^s x \Psi^s x H^s x M^s)$ of diapasons of possible values of earthquake parameters. The parameters used are the geographical coordinates of hypocenters φ, ψ, h (latitude, longitude, depth) and the earthquakes magnitude *m*. The probability of Eq. (1) can be evaluated by predicting the four-dimensional interval that contains the earthquakes' parameters. In the catalogues where the values of earthquake parameters are given with step up 0.001 to 0.1, then the state space Ξ^s of parameters is countable and the variables φ, ψ, h, m must be considered as continuous, therefore for define the probabilistic distribution of parameters their values need to be grouped. After partition of diapasons F^s, Ψ^s, H^s, M^s on R_φ, R_ψ, R_h, R_m parts respectively, the space Ξ^s will consists from four-dimensional and non-overlapping intervals $\{F_i \times \Psi_j \times H_k \times M_n$, $i=1,\ldots,R_\varphi; j=1,\ldots,R_\Psi; k=1,\ldots,R_h; n=1,\ldots,R_m\}$ having the forms of four-dimensional parallelepipeds.

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Using the lexicographical order the four-dimensional indices (i, j,k,n) can be converted to the following one-dimensional indices that is more convenient for practical applications:

$$r=(i-1)R_\Psi R_h R_m+(j-1)R_h R_m+(k-1)R_m+n,\quad R_s=R_\varphi R_\Psi R_h R_m, \tag{12}$$

Therefore, the four-dimensional state space Ξ^s subdivided $\Xi^s=\bigcup_{r=1}^{R_s}\Sigma_r$ into R_s non-intersecting subspaces named elementary seismic sources (ESS). The occurrence on the time t of earthquake in the *η-th* elementary seismic zone Σ_η will mean observation of the *η-th* random seismic event denoted by the same symbol $\Sigma_\eta(t)$.

The seismic zone Z^s is considered as a physical system with state space Ξ^s, which at each instant of time can be in one of the states: $\{\Sigma_r,\ r\epsilon(1,R_s)\}$ changing the states at random time moments. If at time t_k in ESS Σ_η occur earthquake, then we will say at time t_k the physical system be find in *η-th* state. If at time t_k is observed earthquake in *η-th* ESS Σ_η, and at moment t_{k+1} *in μ-th* ESS Σ_μ, it means that during the time interval $\tau_k=t_{k+1}-t_k$ the transition of system from state Σ_η into state Σ_μ took place. To demonstrate development of Markov model as the example is used the sequence of 705 earthquakes, occurred between 1900-2014 years in the seismic zone Vrancea, included in the catalogue ROMPLUS - first cluster in Figure 1 identified by cluster analysis:

$$\Sigma_{r_1}(t_1),\ldots,\Sigma_{r_{705}}(t_{705}),\ r\in[1,R_s] \tag{13}$$

Relative to the sequence (6) the hypothesis H_0 proposed, stating that this trajectory presents a realization of the four-dimensional continuous Markov chain on the state space:

$$\Xi^s=\bigcup_{r=1}^{R_s}\Sigma_r \tag{14}$$

Sequence of events $\Sigma_1, \Sigma_2,\ldots,\Sigma_{705}$ forms the embedded Markov chain of continuous Markov chain. The Markov model of the seismic events sequence denotes the following: at initial time $t=t_0$ the physical system is found in one of R_s possible states Σ_η, and remains in this state during the random time τ_0, exponentially distributed with parameter $u_\eta>0$. At time $t_1=t_0+\tau_0$ the system, with probability $\pi_{\eta\mu}$ instantly transits to a certain state Σ_μ. In this state the system remains, as well during a random time interval τ_1, exponentially distributed with parameter $u_\mu>0$, etc. The tendency of the system's transitions from one state into another is described by the matrix of transitional probabilities of embedded Markov chains: determine the probability of transition from state Σ_η into state Σ_μ. The maximum likelihood estimator of probability is the proportion (Huisinga, Meerbach, 2005):

$$\pi_{\eta\mu}=\frac{N_{\eta\mu}}{\sum_{\mu=1}^{R_s}N_{\eta\mu}}\ ;\quad \Pi_s=(\pi_{\eta\mu})\ ;\ \eta,\mu\in[1,R_s] \tag{15}$$

where $N_{\eta\mu}$ are observed number of one-step transitions from state Σ_η into state Σ_μ. The probabilities $\pi_{\eta\mu}$, η, $\mu\epsilon[1,R_s]$ are the elements of the transitional probabilities matrix Π_s Eq. (15). The continuous Markov chain with R_s states is completely determined by setting $(R_s)^2$ number of values $u_{\eta\mu}$, satisfying the condition:

$$\sum_{\mu=1}^{R_s}u_{\eta\mu}=0\,;\ u_{\eta\mu}>0;\ \eta\neq\mu;\quad u_{\mu\mu}=-\sum_{\eta\neq\mu}u_{\eta\mu}\ ;\ U=(u_{\eta\mu});\ \eta,\ \mu\epsilon[1,R_s] \tag{16}$$

The values $u_{\eta\mu}$ tied by the relationship Eq. (16), constitute the infinitesimal matrix of the continuous Markov chain.

The investigation of the transitional probabilities matrix Π_s Eq. (15) of embedded and infinitesimal matrix U Eq. (16) of continuous Markov chains demonstrated that the Markov model is ergodic. The stationary extreme distributions of states in the embedded ergodic Markov chains with any degree of accuracy can be determined from one its sufficiently large realization (Langrock and Jahn, 1979):

$$\pi_{r,s}=P\{(\varphi_i,\psi_j,h_k,m_n)\in\Sigma_r\},\quad \pi_{r,s}=\frac{N_{r,s}}{N_s} \tag{17}$$

where $N_{r,s}$ is the observed number of earthquakes in ESS Σ_r per N_s events in seismic zone Z^s. Hence, we can define the probability $\pi_{r,s}$ of occurrence of earthquakes in ESS Σ_r, $r=1,\ldots,R_s$.

The main purpose of the application of Markov model is determine the distribution of earthquake parameters, i.e. the probability of occurrence of an earthquake in the ESS Σ_r, $r=1,\ldots,R_s$. Let the random value $N_r(t)$ is counting the number of earthquakes in Σ_r up to time t. It is assumed that the variables $\{N_r(t),\ r=1,\ldots,R_s\}$, are Poisson distributed with parameter λ_r. The superposition of R_s Poisson processes is also a Poisson process with rate equal to the sum of the rates of the individual processes (Huissinga, 2005, Soong, 2004):

$$N_s(t)=\sum_{r=1}^{R_s}N_r(t),\quad \lambda_s=\sum_{r=1}^{R_s}\lambda_r \tag{18}$$

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Therefore, the counting process $N_s(t)$ in the seismic zone Z^s associated with the time period t is a homogeneous Poisson process, which distribution is given by the Poisson distribution with parameter $\lambda_s t$:

$$p(t,n_s)=\frac{e^{-\lambda_s t}(\lambda_s t)^{n_s}}{n_s!} \tag{19}$$

For sample applying the criterion χ^2 for evaluation of the deviation of the observed frequencies n_r from the theoretical $n_r(t)=\lambda_s t\pi_r$, $r=1,\dots,R_s$, for seismic zone Vrancea we obtain $\chi_q=8.69$. The probability that a chi-square statistic having 11 degrees of freedom is more than 8.69, is more than the significance level 0.05 and the observed frequencies of ESS are in good agreement with the theoretical frequencies.

If in the zone Z^s N_s seismic events observed, then the probability of occurrence N_1 earthquakes in ESS Σ_1, N_2 earthquakes in Σ_2,… and so on will correspond to polynomial scheme Eq. (20):

$$p(t,n_1,n_2,\dots,n_{R_s})=p(t,n_s)\frac{n_s!}{n_1!\dots n_{R_s}!}\left(\pi_1^s\right)^{n_1}\left(\pi_2^s\right)^{n_2}\dots\left(\pi_{R_s}^s\right)^{n_{R_s}};\; n_s=\sum_{r=1}^{R_s}n_r \tag{20}$$

The probability β_s of earthquake occurrence in a seismic zone Z^s, $s=1\dots S$, define the contribution of its seismicity in summary seismic hazard. The maximum likelihood estimator of probability β_s is the formula Eq. (21) (Soong, 2004):

$$\beta_s=\frac{n_s}{n} \tag{21}$$

This formula means that the probability of earthquake occurrence in a seismic zone Z^s, s=1,…S is the ratio of the number n_s of events in mentioned seismic zone to the total number of events over all seismic zones n ($n=n_1+n_2+\dots+n_S$) observed during the given time period.

3. ATTENUATION EQUATION OF SEISMIC ACTIONS

Numerous seismologists offered various variants attenuation law in terms of MSK intensity scale for investigated region (Demetrescu, 1941; Jianu, 1992; Radu, 1963; Drumea, Shebalin, 1986). However, the calculated value of the intensity of the seismic actions deviated from the observed values is large, the total standard deviation of residuals is statistical significantly. This circumstance provoked to carry out own study of the intensity attenuation. The main source of data for this study is the macroseismic database includes 8646 macroseismic observations made at 4088 different sites (Kronrod et al, 2012). All of macroseismic observations belong to the intermediate depth earthquakes occurred on 11.10.1940, 07.04.1977, 31.08.1986, 30.05.1990 and 31.05.1990 in Vrancea area expressed in terms of MSK-64 scale. Detailed macroseismic data provide estimate the irregular character of spatial attenuation of the Vrancea earthquakes. Test criteria indicate that the ellipse smoothed the observed macroseismic field (Burtiev 2014).

Macroseismic observations should be processed so that whenever possible precisely to reflect the general tendency of attenuation depending of earthquake parameters and to smooth the casual deviations provoked by inevitable errors of observations and definition of macroseismic intensity. It does not exist a standard criterion for determination of minimal and maximal epicenter distances in the process of macroseismic data processing. In our case, the maximal epicenter the distance was defined taking into account the volume of the data set. The largest length of radius is the value out of which there is an insignificant number of intensity data points (IDPs). For investigated earthquakes, values of R_{max} are equal to 845, 805, 685, 385, 445 km for above enumerated earthquakes respectively.

Macroseismic field of Vrancea intermediate earthquakes is extended from the southwest on the northeast. The configuration, having the shape of an ellipse with major axis turned on some angle γ_0 concerning the positive direction of abscissa axis. The relationships between seismic intensity and earthquake characteristics in the Vrancea zone are represented by empirical attenuation equations (Alkaz 2007; Alvarez-Rubio Sonia et al. 2010; Ardeleanu et al. 2007; Maaz 1985):

$$I=aM_W-b\log D+c\,,\; D=\log\sqrt{R^2+h^2} \tag{22}$$

were: I - the intensity at the site located at the hypocentral distance D, M_W-moment magnitude; h- focal depth; R- epicentral distance, a, b, c – attenuation coefficients.

The distance is a computed parameter that depends on the earthquake location and rupture dimension, which in turn has some uncertainty. For consideration of azimuthally dependence of coefficient b from earthquakes' parameters in attenuation law the value of factor b is defined by the formula:

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$$b = \frac{b_{max} b_{min}}{\sqrt{b_{min}^2 \cos^2(\gamma - \gamma_0) + b_{max}^2 \sin^2(\gamma - \gamma_0)}} \quad (23)$$

where b_{max}, b_{min} the big and small axes of ellipse, γ angle between a direction on the $Q_i(\varphi_i,\psi_i)$ point and a positive direction of an abscissa axis, γ-γ_0 the angle between the major axis of the ellipse and the direction in which we are interested in attenuation (Fig. 2).

Thus, the general problem is given functions M_w, distance D, and values of coefficients a, b, c are found those that the linear combination Eq. 23 is the best approximation of the data. The parameters in the equation Eq. 23, be are evaluated using the least squares method (MLS).

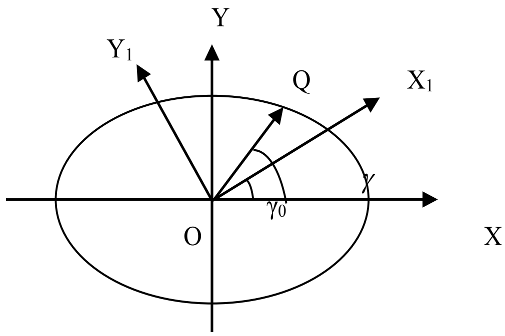

Fig. 3. The plan of definition of elliptic dependence of intensity attenuation of seismic actions.

The quality, what mean by "best fit" is the sum of quadrates of deviations of IDP's values from their theoretical values (Pöschko 2003):

$$S = \sum_{n=1}^{N} \sum_{k=1}^{K_n} (I_{nk} - aM_W^n + b\log\sqrt{\Delta_{nk}^2 + h_n} - c)^2 \rightarrow \min \quad (24)$$

where I_{nk} intensity in k-th IDP $Q_k(\varphi_k,\psi_k)$, $k=1,\ldots,K_n$; at n-th earthquake $n=1, \ldots,N$; Δ_{nk} the distance between epicenter and geographical position of site Q_k, h_n depth of nth earthquake, K_n number of IDPs of n-th earthquake. The objective is to estimate the values of a, b_{max}, b_{min}, γ_0, c that minimize the error Eq. 25.

Target function Eq. 25, is differentiable, but derivatives of numerical parameters a, b_{max}, b_{min}, c, γ_0 are not linearly. The values of attenuation coefficients lie in a limited multiple dimensions intervals: $[a_i; b_i]$, i=1, 2... 5. In this case the grid search method to find a minimum of target function can be applied (Pöschko 2009), do a completely search over that intervals for the parameters a, b_{max}, b_{min}, c, γ_0, that produces the largest likelihood. Each interval has a range of values, divided into a set of equal-value intervals: $h_i=(b_i–a_i)/n_i$. The grid search method is a good way to demonstrate that we can find the maximum of the likelihood function by repeated approximation and iteration. Optimal values, on minimum of criterion Eq.25, coefficients are: a=1.6; c=7.2; the major b_{max}=5.6 and the minor b_{min}=4.9 intensity attenuations law. The angle between major axis of the attenuation ellipse and the positive direction of abscissa axis constitute $\gamma_0=51^0$ degree. So, the attenuation of seismic intensity for intermediate Vrancea earthquakes should be evaluated by formula:

$$\bar{I} = 1.6M_W - b\lg\sqrt{h^2 + r^2} + 7.2, \quad b = \frac{27.44}{\sqrt{24.01\cos^2(\gamma - 51^0) + 31.36\sin^2(\gamma - 51^0)}} \quad (25)$$

where b – is the azimuth-dependent coefficient.

There is not a set of the macroseismic observations to develop an attenuation law of crustal earthquakes. Therefore, in the formula for the attenuation function for crustal earthquakes are used average for the eastern European region values of attenuation coefficients: a=1.5, b=3.5, c=3.0 (Shebalin 2003). In the catalogue of the Romanian earthquakes, ROMPLUS (http://www.infp.ro), for several hundred seismic events is given epicenter intensity I_0 in terms of MSK scale. Assuming a linear dependence epicenter intensity I_0 from magnitude M_w and depth h of earthquakes is evaluated the regression relation:

$$I_0 = 1.49M_W - 0.01h - 0.93 \quad (26)$$

4. THE SEISMIC HAZARD ASSESSMENT METHOD

The events $B_k=\{I^k \leq I < 12\}$, can occur together with one and only one of the R_s mutually exclusive events $\{\Sigma_r, r \in [1,R_s]\}$.

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$$B^k = \sum_{r=1}^{R_s} B^k \Sigma_r \qquad (27)$$

where the events $B_k\Sigma_i$ and $B_k\Sigma_j$ with different subscripts i and j are mutually exclusive. Conditional probability in degree of MSK intensity in the site with coordinates (φ_l,ψ_l), $l=1,\ldots,L$, is given by formula:

$$p_r^k = P\left(I \ge I^k / \Sigma_r\right) = \frac{1}{\sigma\sqrt{2\pi}} \int_{I_k}^{12} e^{-\frac{(I-\bar{I})^2}{2\sigma^2}} dI \qquad (28)$$

The formula for unconditional probability is obtained using the formula of total probability:

$$P(B^k) = \sum_{r=1}^{R_s} P(B^k / \Sigma_r) P(\Sigma_r) = \sum_{r=1}^{R_s} p_r^k \pi_r \qquad (29)$$

The conditional probability of shakes intensity in the sites influenced by earthquakes of seismic zone Z^s are the components of matrix:

$$P^s = \begin{pmatrix} p_1^1 \ldots p_1^{12} \\ \ldots\ldots\ldots\ldots \\ p_{R_s}^1 \ldots p_{R_s}^{12} \end{pmatrix} \qquad (30)$$

For determination of distribution of shakes intensity in matrix form, we must multiply the matrix Eq. 31, on the right to vector of distribution of elementary seismic zones π_r^s Eq. 18:

$$\vec{p}_s = \vec{\pi} P^s,\ \vec{\pi} = \left(\pi_1, \pi_2, \ldots, \pi_{R_s}\right),\ \vec{p}_s = \left(p_s^1, p_s^2, \ldots, p^{12}\right) \qquad (31)$$

The earthquake can occur, with probability γ_s in some seismic zone Z^s. If the site $Q(\varphi,\psi)$ is affected by seismic actions from several seismic zones Z^s, $s=1,\ldots,S,$ then the total probability of intensity of shakes β_k can be estimates by formula:

$$\vec{\beta} = \left(\beta_1, \beta_2, \ldots, \beta_{12}\right),\ \beta_k = \sum_{s=1}^{S} p_s^k \gamma_s \qquad (32)$$

Thus, we have all needed for PSHA earthquake parameters:

- the probability γ_s, $s=1,\ldots,S$ that the earthquake will occur in some seismic zone Z^s, which is contribution of seismic zones in the summary seismic hazard.
- the probability, π_r^s $r=1,\ldots,R_s$, $s=1,\ldots,S$ that the parameters of earthquake will contain in four-dimensional interval Σ_r – the earthquake will occur in the elementary zone Σ_r of seismic zone
- the attenuation function of intensity for intermediate Vrancea earthquakes
- the distribution β_k, $k=1,\ldots,12$ of seismic shakes of intensity, taking in account the contribution of earthquakes from all seismic zones to total seismic hazard.

The annual rate of exceeding $\lambda_k= \lambda(I^k)$, is as the number of exceedances per year of intensity level I^k at the site under consideration. The probability of occurrence of n_k impacts with intensity more as I^k, during the period t has Poisson distribution:

$$P(t, n_k) = \frac{(\lambda_k t)^{n_k} e^{-\lambda_k t}}{n_k!} \qquad (33)$$

where λ_k is the annual rate of exceedance of level I^k degree of MSK scale. This value will be estimate from relation $\lambda_k=\lambda t\beta_k$, λ – the rate of earthquakes Eq. 22.

According to EUROCOD 8, the PSHA indicate maximum horizontal ground acceleration with 10 % chance of exceeded in the next 50 years - equivalent to 475 years return period.

For PSHA mapping of the investigated territory, influenced by regional earthquakes, a system of grid points with 0.2°×0.2° spacing was used. The PSHA was performed in each grid point in terms of seismic intensity (MSK) using the probability of exceedance of 10% in 50 years (Solomos, et al.2008):

$$1 - P(50,0) = 1 - e^{-\lambda_k 50} \qquad (34)$$

Seismic hazard was computed over the area bounded by 41º - 49º N and 20º - 31º E. Map of seismic hazard (Fig. 3) consists of points in which the probability defined by Eq. 35, is equal to 0.1. Joiner-Boor distance was used for near zone and hypocentral distance for far zone. The map (Fig. 3) was built on the base of attenuation relation with parameters: $a=1.6$; $c=7.2$; $b_{max}=5.6$, $b_{min}=4.9$ and $\gamma_0=51^0$ and Eq. 27, for crustal earthquakes.

Extreme values of magnitude in Vrancea seismic zone $M_{W,0.999}=8.2$, expected once every 1000 years (Burtiev 2003), define the extreme scenario of seismic hazard in Romania and Moldova (Fig. 4).

Denote by n the number of seismic events occurrences at the site $Q(\varphi,\psi)$ and by m_k the number of shakes with degree I^k of MSK intensity scale, which can occur with probability β_k Eq. 33. The conditional probability of occurrence shakes of intensity I^1, m_1 times, I^2, m_2 times, and I^k m_k times ($m_1+m_2+...+m_{12}=n$) is:

$$p_n(m_1,...,m_{12})=\frac{n!}{m_1!....m_{12}!}\beta_1^{m_1}\beta_2^{m2}...\beta_{12}^{m_{21}} \qquad (35)$$

The probability $P(t,n)$ that during time interval t at the site $Q(\varphi,\psi)$ will occur n seismic events has the Poisson distribution with parameter λ. Hence, the probability $P(t,n,m_k)$ of occurrences n seismic shakes during time t and m_k of them with intensity I^k may be assessing by formula:

$$p(t,n,m_k,I^k)=\frac{(\lambda t)^n e^{-\lambda t}}{m_k!(n-m_k)!}\beta_k^{m_k}(1-\beta_k)^{n-m_k} \qquad (36)$$

The probability of events, that at the site Q(φ,ψ) during the time interval t will occur N seismic shakes and at least one time of intensity I^k, may be estimate by formula:

$$p\{t,n,m_k\geq 0\}=1-\sum_{n=1}^{N}\frac{(\lambda t)^n e^{-\lambda t}}{n!}(1-\beta_k)^n \qquad (37)$$

The probability of occurrences n earthquakes and the number m_k of them with degree I^k of MSK scale will be between m_1 and m_2 is assessed by DeMoivre-Laplace formula (Gnedenko 1969):

$$\mathrm{P}\{\mathrm{m}_1\leq \mathrm{m}_k\leq \mathrm{m}_2\}=\frac{1}{\sqrt{2\pi}}\int_{x_1}^{x_2}e^{-\frac{x^2}{2}}dx \qquad (38)$$

where

$$x=\frac{m-n\beta_k}{\sqrt{n\beta_k(1-n\beta_k)}} \qquad (39)$$

For example, the probability that in Chisinau (capital city Republic of Moldova) during 10 years will occur 185 seismic events and the number of shakes with degree of intensity 7 will be between 0 and 10 is $P=0.0288$.

Table 2. The characteristics of seismic situation in Chisinau

	I^k	4	5	6	7	8
1	β_k	0.0128	0.00517	0.00178	0.000841	0.000201
2	$P(475,I^k,m\geq 0)$	0.99	0.99	0.99	0.99	0.98
3	$n\beta_k$ (1000 years	352	157	43	29	9
4	Return period (years)	2.8	6.4	23	34	114
5	n	2312	2312	2312	2312	2312
6	Most probable value $-m_k$	31	13	6	3	2
7	$p(50,n,m,I^k)$	0.000591	0.000916	0.00106	0.00178	0.0004
8	$P(n,m_1\leq m\leq m_2,I^k)$	0.004	.0041	.0027	.004	.0018
9	m_1,m_2	$30\leq m\leq 229$	$12\leq m\leq 211$	$5\leq m\leq 204$	$2\leq m\leq 201$	$1\leq m\leq 200$
10	$1-P(50,0,\lambda)$	1.0	1.0	0.984	0.857	0.372
11	$P(50,m\geq 0,\lambda)$	1.0	0.999	0.964	0.855	0.377

Let the Poisson distributed random variable N, means the number of earthquakes in Chisinau for 50 years. Theoretically, the set of possible values of number of seismic events is computable, but practically possible range of values determined by the interval [N_{min}, N_{max}], the values of which have a positive probability.

That is, the number of seismic shocks can take values from the interval [N_{min},N_{max}]. For assessing the seismic hazard in norms of EUROCOD-8 is applicable the formula:

$$P(t, m \geq 0, I^k) = \sum_{n=N_{min}}^{N_{max}} \left(\sum_{m=1}^{n} C_n^m \beta_k^m (1-\beta_k)^{n-m} \right) \frac{e^{-\lambda t} (\lambda t)^n}{n!} \qquad (40)$$

There are two most probable values of binomial probability distribution (35): $m=np_k-(1-p_k)$, $m_k=np_k+p_k$ by occurrence of n earthquakes, m_k is the number of seismic shakes with intensity I^k, which can occure with probability p_k. The values of the seismic hazard characteristics in Chisinau are given in Table 2: first row contains the probability distribution of seismic shakes intensity; second row – the probability of occurrence in 475 years at least one shakes with intensity I^k; third rows – the average number of shakes with intensity I^k; fourth row - return period; 5-th row the average number of all earthquakes in 50 years; 7-th row the probabilty of occurence 2312 seismic events in 50 years, and m_k of them will have degree of intensity I^k of MSK-64 scale; 10-th and 11-th rows the probability of occurence at least one seismic shake of intensity I^k evaluated using the formulas (37) and (40) respectivaly.

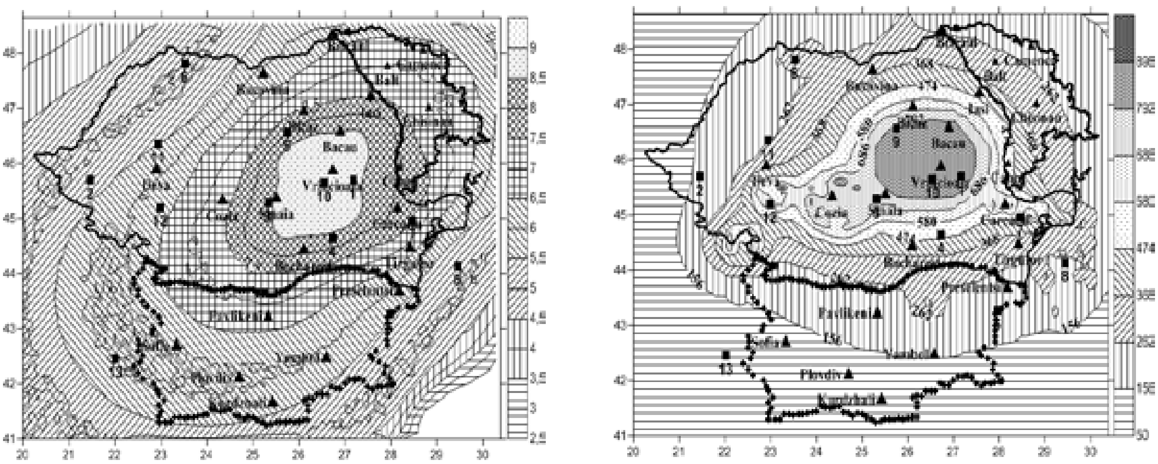

Fig. 4. Seismic hazard in MSK-64 intensity terms for recurrence period 475 years - (left figure) and in terms of EUROCOD-8, (right figure). Symbols: (+) - the national border; (▲) - the cities, with name below; (■) - the center of clusters (seismic zones) with number right; right column – the intervals of intensity values. The numbers on the isolines (2,5; 3;3,5;…) represents the intensities in MSK-64 scale and numbers: (262, 368,…) represents the values of PGA, cm/s^2. The k-th zone of intensity I^k contains the points, where the intensity takes values from interval [I^k-0.5, I^k+0.5] of scale MSK-64.

5. ASSESSING OF SEISMIC RISK

On the base of the developed seismic hazard, assessment method is also carried out the seismic risk estimation method. For example, let random value D is the amount of economical damage, in a conventional money unit, which can cause seismic shake of intensity *I* some localities, such as (*town не есть city*) ("Seismotown" with the coordinates (45N, 28E). If *F(d/I)* is the conditional distribution function for a random variable *D*, then the integral

$$M\left(D/I\right) = \int d\,dF\left(d/I\right) \qquad (41)$$

will estimate the conditional expectation of the variable D with respect to the event ($I=I^k$) – occurrence of seismic shake with intensity of I^k. Assume that the value of D with probability p (d_l/I^k) takes one of the L possible values d_l, (l=1,2,...,L), in case of occurrence of seismic shake wit intensity I^k. The probabilities $p(d_l/I^k)$ form (K, L) seismic risk matrix D:

Using the relation (41) it is unable to obtain estimate the conditional expectation of the variable D with respect to the event ($I=I^k$) for seismic risk assessing in matrix form:

$$M\left(D/I^k\right) = \sum_{l=1}^{L} d_l\, p\left(d_l/I^k\right) = D_r \vec{d}, \quad \vec{d} = (d_1, d_2, ..., d_L) \qquad (42)$$

$$D_r = \begin{pmatrix} p\left(d_1/I^1\right).. & p\left(d_2/I^1\right) & . & . & p\left(d_L/I^1\right) \\ p\left(d_1/I^2\right)... & p\left(d_2/I^2\right) & . & . & p\left(d_L/I^2\right) \\ . & . & . & . & . \\ . & . & . & . & . \\ p\left(d_1/I^{12}\right)... & p\left(d_2/I^{12}\right) & . & . & p\left(d_L/I^{12}\right) \end{pmatrix} \quad (43)$$

The random variable I get one of values I^k, (k=1,...,12), then the events $\{I=I^1\},\{I=I^2\},...,\{I=I^{12}\}$ forms the complete group of mutually exclusive events and $F(d/(I=I^1))$, $F(d/(I=I^2))$,…, $F(d/(I=I^{12}))$ the conditional distribution functions of the variable D corresponding to the events. Let F(d) denote the unconditional distribution function of D; using the formula of total probabilty we get (Gnedenco, 1969):

$$F(d) = \sum_{k=1}^{12} P(I = I^k)\, F\left(d/(I = I^k)\right) \quad (44)$$

This equation enables us, together with formulas (32) and (42), to obtain the formula for summary seismic risk from one earthquake assessment in matrix form:

$$MD = \sum_{k=1}^{12} P(I = I^k)\, M\left(d/I^k\right) = \sum_{k=1}^{12} \beta_k\, M\left(d/I^k\right) = \vec{\beta} D_r \vec{d} \quad (45)$$

For assessing the seismic risk in time *t*, taking in account that the average number of earthquakes in time is λt, may be used the relation: $MD_t=\lambda tMD$.

Assume that, for each value of I^k the damage D receives only one value d_k, that is $P(D=d_k)=1$, (k=1,...,12). Therefore, when occurs event I^k can occur only one event D_k, i.e. I^k event leading to implementation events ($D=d_k$) and conditional probability is equal to unity, $p\ (D_k/I^k)=1$. From the equality to one of conditional probability implies that the joint probability of the fact that as a result of the intensity of the earthquake will shake the value I^k, and with it the amount of the damage will take the value of D_k, determined by the probability of occurrence of seismic shakes of intensity I^k:

$$p(d_l, I^k) = P(D = d_l, I = I^k) = P\left(D = d_l / I = I^k\right) P(I = I^k) = p(I^k) \quad (46)$$

Table 3. Assessing of seismic risk

N	I^k	4	5	6	7	8	9
1	β_k	0.0128	0.00517	0.00178	0.000841	0.000201	0
2	d_k	25600	62500	129600	240100	409600	656100
3	$d_k\beta_k$	327	323	231	202	82	7.7
4	$\lambda\beta_k 475 d_k$	718.10^4	729.10^4	506.10^4	443.10^4	181.10^4	169.10^3
5	$\overline{D} = \sum_{k=1}^{12} \beta_k d_k = 1462$		$\overline{D_{475}} = \sum_{k=1}^{12} \lambda t \beta_k d_k = 321.10^5$				

In this case the conditional expectation of the variable D with respect to the event ($I=I^k$) is:

$$M\left(d/I^k\right) = \sum_{j=1} d_j P\left(d_j/I^k\right) = d_k \quad (47)$$

The first line (Tab. 3) shows the probability of occurrence of a seismic shock of the intensity I^k throughout the year. The second line (Tab. 3) contain the amount of damage D_k, which can cause the seismic shake of intensity I^k; and in the third line of the possible extent of damage within one year. The volume average damage over a period of 475 years is given in the fourth row. Possible total damage in the event of earthquakes;

possible over a period of 475 years are shown in the fifth row. That is, in a similar way, the algorithm can be used to calculate seismic risk in settlements.

CONCLUSIONS

The new measure of seismic hazard and PSHA method was proposed. The proposed measure completely describes the seismic situation in points of Earth surface. It was carried out the seismic hazard analysis of Moldavian and Romanian territories. The method of cluster analysis based on ROMPLUS catalogue has identified 13 seismic zones. For quick primary identification of seismic zones, the cluster analysis is quite useful.

The seismic zone has been considered as a physical system and Markov model of it was elaborated. Studies based of the macroseismic data of intermediate earthquakes, which have occurred, on 10.11.1940, 7.04.1977, 31.08.1986, 30.05.1990 and 31.05.1990 in Vrancea show that a comprehensible smoothing line of observed data is the ellipse. Optimal coefficients based on minimum criterion are a=1.6; c = 7.2; the major b_{max}=5.6 and the minor b_{min}=4.9 intensity attenuations of seismic action; the angle between major axis of ellipse of attenuation and the positive direction of abscissa axis is $\gamma_0=51^0$.

On the base of the developed seismic hazard assessment method the seismic risk estimation was performed. For demonstration of performance is assessed the seismic in some phantom city "Seismotown".

The proposed method of seismic hazard evaluation allows the consideration of own law of attenuation of shakes intensity for each seismic zone and territories.

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