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Construction of Partially balanced incomplete block designs based on Latin Square design

KEYWORDS PBIB designs, Latin square design, block designs, treatments, replications

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ABSTRACT *In this paper, we have constructed two and p^{th} associate class (p =5,7,...)is an odd number partially balanced incomplete block (PBIB) designs by establishing a link between PBIB designs and Latin square designs. Illustration of construction of such type of designs are also discussed in detail. Efficiencies of the newly constructed PBIB designs are also computed for the purpose of comparison. During the study, it was found that some designs are new and some newly constructed are more efficient as compared to the existing designs.*

1. Introduction

Latin square designs are normally used in experiments to remove the heterogeneity of experimental material in two directions. These designs require that number of replications equals the number of treatments. In this paper, using Latin square designs Sharma M.K.et.al [2011] who introduced a relatively easy method for constructing application of Latin square designs in CDC system.Mohan.et.al [2006a], Mohan.et.al [2006b] have constructed two and higher associate class PBIB designs. For the literature of PBIB designs Garg (2010) have constructed Pseudo New Modified Latin Square ($NML_m(m)$) type PBIB Designs.Recently, Garg and Gurinder [2011] have obtained three and four associate class PBIB designs using method of Duality. Very, recently Garg and Farooq [2014] constructed some PBIB designs by using Factorial treatment combinations. Here in this paper, we have constructed nth associate class PBIB designs along with new association scheme.

2. Construction method of constructing n^{th} associate class PBIB designs using Latin square design

For the construction of n^{th} associate class PBIB designs, we consider a Latin square design of order 'p' where p = (5,7,...) is an odd number represent treatments arranged in p×p square format. By augmenting first $(p-1)^{th}$ columns to the right of the p × *p* matrix, then move diagonally running from top left to bottom right after this, substituting '1' for odd numbers and '0' for even number. The positions in the matrix so obtained, in which we get 1 are considered as blocks in the respective rows which yields n^{th} PBIB designs with the following parameters:

$$v=p, b=p, r=(p-1)/2, k=(p-1)/2, \lambda_i^s \text{ varies from } 0 \text{ to } (p-3)/2$$

3. Association scheme of n^{th} associate class PBIB designs: In this association scheme, we have v = p treatments and *b* = *'p'* blocks. In these *b* = *p* blocks, every treatment repeats exactly *r* = *(p-1)/2* times. Now we define n^{th} associate class association scheme as follows:

If two treatments occurs together 0, 1 and 2,...,n times in the blocks of the new design, then they are said to be I^{st}, 2^{st}, 3^{rd} ,...,p^{th} associates respectively and n_1=2, n_2=2 and n_3=2,...,n_p=2.

These parameters satisfy all the conditions which are necessary for the existence of an association scheme.

4. Illustration

Example 4.1 As per our construction methodology. Let us consider a Latin square design of order 5×5.

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ 2 & 3 & 4 & 0 & 1 \\ 3 & 4 & 0 & 1 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{pmatrix}$$

By augmenting first 4 columns to the right of the 5 × 5 matrix, then move diagonally running from top left to bottom right after this substituting '1' for odd numbers and '0' for even number we get

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

The positions in the matrix so obtained, in which we get 1 are considered as blocks in the respective rows are given as

1. (2, 4) 2. (1, 2) 3. (3, 4) 4. (1, 5) 5. (2, 3)

These blocks yields two associate class association defined as sec 3 with parameters v=5, b=5, r=2, k=2, λ_1= 0, λ_2= 1

The following table explains the association scheme with two associate classes:

Symbols	*1^{st} associates*	*2^{nd} associates*
1	3, 4	2, 5
2	4, 5	1, 3
3	1, 5	4, 2
4	1, 2	3, 5
5	2, 3	1, 4

The P-matrices of the new association scheme are given by

$$P_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \qquad p_2 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

Note: For p=3, PBIB design is not possible

5. Summary and Discussion:

In this paper, we have constructed nth associate class PBIB designs by establishing a link between PBIB designs and Latin square designs and as a result we get new PBIB designs with nth associate class PBIB designs. Efficiencies of the new designs are also computed for the purpose of comparison. Some newly constructed designs are more efficient, as compared to existing designs.

References

1. 1.Garg, D.K.(2010). Pseudo New Modified Latin Square (NMLm(m)) Type PBIB Designs. Communications in Statistics-Theory and Methods, 39, 3485-3491.

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2. Garg, D.K and Singh, G.P .(2011). Construction of three and four associate class using method of duality.Int.J.Agricult.Stat.Sci., 7(2),579-587.
3. Garg, D.K. & Syed (2014). Construction of PBIB designs using Factorial Treatment combinations. International Journal of Scientific Research,vol.3, No.10,2014. 2014):106-115.
4. Garg, D.K. and Farooq, S.A (2014). Construction of PBIB designs using Modified Magnificent (Mn) Matrices of type-III. Indian Journal of applied Research (IJAR), Vol 4,Issue 2, Feb, 2014
5. Garg, D.K. and Farooq, S.A (2014). Construction of PBIB designs using Magnificent(Mn) matrices. Far East Journal of Theoretical Statistics, vol.48, No.1,2014, pp 29-44,
6. Garg, D.K. and Singh G.P. (2014). Construction of Three associates class Partially Balanced Incomplete Block (PBIB) Designs with three replicates using pairing in triplets system, Paripex- Indian Journal of applied Research (IJAR), 3(3), 37–39
7. Mohan, R.N., & Kageyama, S., Moon Ho Lee, & Yang, G.(2006a). Certain new M Matrices and their properties and applications. Submitted to Linear Algebra and Applications released as e print, arXiv:Math cs/ 0604035 Dt. April 6,2009
8. Mohan, R.N., Kageyama, S., Moon Ho Lee, & Yang,G.(2006b).Certain new M matrices and their properties and applications. Submitted to Linear Algebra and Applications released as e print, arXiv:Math cs/ 0604035 Dt. June 6,2009
9. Sharma M.K, Melesse S.F and Sarial A.K. Application of latin square designs in CDC system. Aligarh Journal of Statistics Vol. 31 (2011), 11-16