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Fuzzy Semi-Connectedness and Fuzzy Pre-Connectedness in Fuzzy Biclosure Space

KEYWORDS: Fuzzy closure space, fuzzy semi-connectedness and fuzzy pre-connectedness in fuzzy closure space, fuzzy biclosure space, fuzzy semi-connectedness and fuzzy Pre-connectedness in fuzzy biclosure space.

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ABSTRACT *Our aim, in the present paper, is to introduce two new types of fuzzy connectedness in fuzzy biclosure spaces namely fuzzy semi-connectedness in fuzzy biclosure space and fuzzy Pre-connectedness in fuzzy biclosure space. We also investigate the fundamental properties of these new types of connectedness.*

1. INTRODUCTION

Fuzzy Closure space was first introduced by A.S. Mash hour and M.H. Ghanim [2]. The notions of closure system and closure operator are very useful tools in several areas of classical mathematics. They play an important role in topological spaces, Boolean algebra, convex sets etc.

Fuzzy Biclosure space was introduced by Tapi U. D. and Navalakhe R. [3]. Such spaces are equipped with two arbitrary fuzzy closure operators. He extended some of the standard results of separation axioms in fuzzy closure space to fuzzy biclosure space. Thereafter a large number of papers have been written to generalize the concept of fuzzy closure space to fuzzy biclosure space. Semi-connectedness and pre-connectedness in biclosure space was introduced by our self [5].We have [4] introduced the fuzzy semi-connectedness and fuzzy pre-connectedness in fuzzy closure space. Here we are generalizing these concepts into fuzzy biclosure space. **In this paper we introduce fuzzy semi-connectedness and fuzzy pre-connectedness in fuzzy biclosure space.**

2. PRELIMINARIES

Definition 2.1 [1]:- Let X is a non empty fuzzy set. A function $k: I^X \to I^X$ is called fuzzy closure operator on X if it satisfies the following conditions

1. $k(\tilde{0}) = \tilde{0}$
2. $\lambda \le k(\lambda)$, for all $\lambda \in I^X$.
3. $k(\lambda_1 \vee \lambda_2) = k(\lambda_1) \vee k(\lambda_2)$ for all $\lambda_1, \lambda_2 \in I^X$.

The pair (X, k) is called fuzzy closure space.

Definition2.2 [4]:- A fuzzy closure space (X, k) is said to be fuzzy semi-connected fuzzy closure space if and only if there exists an Fs-continuous mapping f from X to the fuzzy discrete space $\{\tilde{0}, \tilde{1}\}$ is constant.

Definition2.3 [4]:- A fuzzy closure space (X, k) is called fuzzy pre-connected fuzzy closure space if and only if there exists an Fp-continuous mapping f from X to the fuzzy discrete space $\{\tilde{0}, \tilde{1}\}$ is constant.

Definition2.4 [3]:- A fuzzy biclosure space is a triple (X, u_1, u_2) where X is a nonempty set and u_1, u_2 are two fuzzy closure operators on X which satisfies the following properties:

(i) $u_1\phi = \phi$ and $u_2\phi = \phi$;
(ii) $A \leq u_1A$ and $A \leq u_2A$ for all $A \leq I^X$
(iii) $u_1(A \vee B) = u_1A \vee u_1B$ and $u_2(A \vee B) = u_2A \vee u_2B$ for all $A, B \leq I^X$.

3. **FUZZY SEMI-CONNECTEDNESS IN FUZZY BICLOSURE SPACE**

Definition 3.1:- A fuzzy set χ_λ in a fuzzy biclosure space (X, k_1, k_2) is said to be fuzzy semi-open if $\chi_\lambda \leq k_i(\mathrm{int}_{k_i}(\chi_\lambda))$ for all i= {1, 2}. The complement of fuzzy semi-open set is called fuzzy semi-closed set. The class of all fuzzy semi-open sets of fuzzy biclosure space (X, k_1, k_2) is denoted by FSO (X, k_1, k_2).

Definition 3.2:- Let (X, k_1, k_2) and (Y, c_1, c_2) are fuzzy biclosure spaces and let $i \in \{1, 2\}$. Then an F-mapping $f: (X, k_1, k_2) \rightarrow (Y, c_1, c_2)$ is called:

(i)Fuzzy i-open (respectively, Fuzzy i-closed) if the F-mapping $f: (X, k_i) \rightarrow (Y, c_i)$ is open (respectively, closed).

(ii) Fuzzy open (respectively, Fuzzy closed) if f is fuzzy i-open (respectively, fuzzy i-closed) for all $i \in \{1, 2\}$.

(iii)Fuzzy i-continuous if the F-mapping $f: (X, k_i) \rightarrow (Y, c_i)$ is F-continuous for all $i \in \{1, 2\}$.

(iv) Fuzzy continuous if f is fuzzy i-continuous, for all $i \in \{1, 2\}$.

Definition 3.3:- Let (X, k_1, k_2) and (Y, c_1, c_2) are fuzzy biclosure spaces. An F-mapping $f: (X, k_1, k_2) \rightarrow (Y, c_1, c_2)$ is called Fs-continuous if $f^{-1}(\chi_\lambda)$ is a fuzzy semi-open subset of (X, k_1, k_2) for every fuzzy open subset χ_λ of (Y, c_1, c_2).

Definition 3.4:- A fuzzy biclosure space (X, k_1, k_2) is called fuzzy semi-connected if there exists an Fs-continuous mapping f from X to fuzzy discrete space $\{\tilde{0}, \tilde{1}\}$ is constant.

Example 3.5:- Consider a non empty set X = {a, b, c}, and k_1 and k_2 are two fuzzy closure operators which are defined by $k_1: I^X \rightarrow I^X$ such that

$$k_1(\chi_\lambda)=\begin{cases}\tilde{0}_X; if\ \chi_\lambda = 0\\ \chi_{\{b,c\}};\ \text{ if } 0\neq\chi_\lambda\leq\ \chi_{\{b\}}\\ \chi_{\{b,c\}};\ \text{ if } 0\neq\chi_\lambda\leq\chi_{\{c\}}\\ \chi_{\{b,c\}};\ \text{ if } 0\neq\chi_\lambda\leq\chi_{\{b,c\}}\\ \tilde{1}_X; otherwise.\end{cases}$$

Here (X, k_1) is a fuzzy closure space.

Fuzzy open sets of $(X, k_1) = \{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{a, b\}}, \chi_{\{a, c\}}, \tilde{1}_X, \tilde{0}_X\}$.

$FSO(X, k_1) = \{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{a, b\}}, \chi_{\{a, c\}}, \tilde{1}_X, \tilde{0}_X\}$.

Define another fuzzy closure operator $k_2: I^X \to I^X$ such that

$$k_2(\chi_\lambda)=\begin{cases}\tilde{0}_X; \text{if } \chi_\lambda = 0\\ \chi_{\{a, b\}};\ \text{ if } 0\neq\chi_\lambda\leq\ \chi_{\{a\}}\\ \chi_{\{b, c\}};\ \text{ if } 0\neq\chi_\lambda\leq\chi_{\{b\}}\\ \chi_{\{a, c\}};\ \text{ if } 0\neq\chi_\lambda\leq\chi_{\{c\}}\\ \tilde{1}_X; otherwise.\end{cases}$$

Here (X, k_2) is a fuzzy closure space.

$FSO(X, k_2) = \{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{a, b\}}, \chi_{\{b, c\}}, \chi_{\{a, c\}}, \tilde{1}_X, \tilde{0}_X\}$,

Here (X, k_1, k_2) is a fuzzy biclosure space.

Fuzzy open sets of $(X, k_1, k_2) = \{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{a, b\}}, \chi_{\{a, c\}}, \tilde{1}_X, \tilde{0}_X\}$,

FSO of $(X, k_1, k_2) = \{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{a, b\}}, \chi_{\{a, c\}}, \tilde{1}_X, \tilde{0}_X\}$.

Define a Fs-continuous mapping $f: X \to \{\tilde{0}, \tilde{1}\}$ such that

$f^{-1}\{\tilde{1}\}=\chi_{\{a\}}= \chi_{\{b\}}= \chi_{\{c\}}=\chi_{\{a, b\}}= \chi_{\{b, c\}}=\chi_{\{a, c\}}=\tilde{1}_X$, $f^{-1}\{\tilde{0}\}=\tilde{0}_X$ is constant.

Hence (X, k_1, k_2) is a fuzzy semi-connected fuzzy biclosure space.

Example 3.6:- Let X = {a, b, c, d} is a non empty set and consider a fuzzy closure operator whi

defined by $k_1: I^X \to I^X$ such that

$$k_1(\chi_\lambda) = \begin{cases} \tilde{0}_X ; & \chi_\lambda = 0 \\ \chi_{\{a,\, b\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{a\}} \\ \chi_{\{a,\, b\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{b\}} \\ \chi_{\{b,\, c\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{c\}} \\ \chi_{\{c,\, d\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{d\}} \\ \tilde{1}_X; & \chi_\lambda = 1. \end{cases}$$

For all fuzzy subsets $\chi_\lambda \leq I^X$, let $k_1\ (\chi_\lambda) = \begin{cases} \tilde{0}_X, \text{ if } \chi_\lambda = 0, \\ \vee\{k_1\{\chi_{\{a\}}\}: \chi_{\{a\}} \leq \chi_\lambda\}, \text{ otherwise.} \end{cases}$

Hence (X, k_1) is a fuzzy closure space.

Fuzzy open sets of (X, k_1) $= \{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{d\}}, \chi_{\{b,\, c\}}, \chi_{\{c,\, d\}}, \chi_{\{d,\, a\}}, \chi_{\{b,\, c,\, d\}}, \chi_{\{a,\, d,\, b\}}, \chi_{\{c,\, d,\, a\}}, \tilde{1}_X, \tilde{0}_X\}$.

FSO of (X, k_1) $= \{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{d\}}, \chi_{\{b,\, c\}}, \chi_{\{c,\, d\}}, \chi_{\{d,\, a\}}, \chi_{\{b,\, c,\, d\}}, \chi_{\{a,\, d,\, b\}}, \chi_{\{c,\, d,\, a\}}, \tilde{1}_X, \tilde{0}_X\}$.

Consider another fuzzy closure operator $k_2: I^X \to I^X$ such that

$$k_2(\chi_\lambda) = \begin{cases} \tilde{0}_X ; & \chi_\lambda = 0 \\ \chi_{\{a,\, b,\, c\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{a\}} \\ \chi_{\{b,\, c,\, d\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{b\}} \\ \chi_{\{c,\, a,\, d\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{c\}} \\ \chi_{\{d,\, a,\, b\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{d\}} \\ \tilde{1}_X; & \text{if } \chi_\lambda = 1. \end{cases}$$

For all fuzzy subsets $\chi_\lambda \leq I^X$, let $k_2\ (\chi_\lambda) = \begin{cases} \tilde{0}_X, \text{ if } \chi_\lambda = 0, \\ \vee\{k_2\{\chi_{\{a\}}\}: \chi_{\{a\}} \leq \chi_\lambda\}, \text{ otherwise.} \end{cases}$

Hence (X, k_2) is a fuzzy closure space.

Fuzzy open sets of fuzzy closure space (X, k_2) =

$\{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{d\}}, \chi_{\{a,\, b\}}, \chi_{\{b,\, c\}}, \chi_{\{c,\, d\}}, \chi_{\{d,\, a\}}, \chi_{\{a,\, b,\, c\}}, \chi_{\{b,\, c,\, d\}}, \chi_{\{c,\, d,\, a\}}, \chi_{\{a,\, d,\, b\}}, \tilde{1}_X, \tilde{0}_X\}$.

FSO (X, k_2) = $\{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{d\}}, \chi_{\{a, b\}}, \chi_{\{b, c\}}, \chi_{\{c, d\}}, \chi_{\{d, a\}}, \chi_{\{a, b, c\}}, \chi_{\{b, c, d\}}, \chi_{\{c, d, a\}}, \chi_{\{a, d, b\}}, \tilde{1}_X, \tilde{0}_X\}$.

Here (X, k_1, k_2) is a fuzzy biclosure space.

Fuzzy open sets of fuzzy biclosure space (X, k_1, k_2) are

$\{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{d\}}, \chi_{\{b, c\}}, \chi_{\{c, d\}}, \chi_{\{d, a\}}, \chi_{\{b, c, d\}}, \chi_{\{c, d, a\}}, \chi_{\{a, d, b\}}, \tilde{1}_X, \tilde{0}_X\}$.

FSO (X, k_1, k_2) = $\{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{d\}}, \chi_{\{b, c\}}, \chi_{\{c, d\}}, \chi_{\{d, a\}}, \chi_{\{b, c, d\}}, \chi_{\{c, d, a\}}, \chi_{\{a, d, b\}}, \tilde{1}_X, \tilde{0}_X\}$.

Define a fuzzy semi-continuous mapping $f: X \to \{\tilde{0}, \tilde{1}\}$ such that

$f^{-1}\{\tilde{1}\} = \chi_{\{a\}} = \chi_{\{b\}} = \chi_{\{c\}} = \chi_{\{d\}} = \chi_{\{b, c\}} = \chi_{\{c, d\}} = \chi_{\{d, a\}} = \chi_{\{b, c, d\}} = \chi_{\{c, d, a\}} = \chi_{\{a, d, b\}} = \tilde{1}_X$, $f^{-1}\{\tilde{0}\} = \tilde{0}_X$.

is constant. Hence (X, k_1, k_2) is a **fuzzy semi-connected fuzzy biclosure space.**

Definition 3.7:- A fuzzy biclosure space (X, k_1, k_2) is called fuzzy semi-disconnected if there exists an Fs-continuous mapping f from X to fuzzy discrete space $\{\tilde{0}, \tilde{1}\}$ is surjective.

Theorem 3.8: A fuzzy biclosure space (X, k_1, k_2) is fuzzy semi-connected if and only if every Fs-continuous mapping f from X into a fuzzy discrete space Y= $\{\tilde{0}, \tilde{1}\}$ with at least two fuzzy points is constant.

Proof: Necessary: Let (X, k_1, k_2) is a fuzzy semi-connected fuzzy biclosure space. Then there exists a Fs-continuous mapping f from the X into the fuzzy discrete space Y=$\{\tilde{0}, \tilde{1}\}$, for each $\chi_{\{y\}} \leq \tilde{1}_Y$, $f^{-1}\{\chi_{\{y\}}\} = \tilde{0}_X$ or $\tilde{1}_X$. If $f^{-1}\{\chi_{\{y\}}\} = \tilde{0}_X$ for all $\chi_{\{y\}} \leq \tilde{1}_Y$, then f ceases to be a mapping. Therefore $f^{-1}\chi_{\{y_0\}} = \tilde{1}_X$ for a unique $\chi_{\{y_0\}} \leq \tilde{1}_Y$. This implies that $f(\tilde{1}_X) = \chi_{\{y_0\}}$ and hence f is a constant mapping.

Sufficiency: Let every Fs-continuous mapping f from X into a fuzzy discrete space Y= $\{\tilde{0}, \tilde{1}\}$ is constant. Suppose χ_λ is a fuzzy semi open set in a fuzzy biclosure space (X, k_1, k_2). If $\chi_\lambda = \tilde{0}_X$, we will not fuzzy semi-connected fuzzy biclosure space, which is a contradiction to our initial assumption. Hence the only fuzzy subsets of X both fuzzy semi-open and fuzzy semi-closed are $\tilde{0}_X$ and $\tilde{1}_X$.

[2] ⇒ [3]

Suppose the only fuzzy subsets of X both fuzzy semi-open and fuzzy semi-closed are $\tilde{0}_X$ and $\tilde{1}_X$. Let $f: X \rightarrow \{\tilde{0}, \tilde{1}\}$ is an Fs-continuous surjection. Then $f^{-1}\{\tilde{0}\} \neq \tilde{0}_X$ and $f^{-1}\{\tilde{0}\} \neq \tilde{1}_X$. But $\{\tilde{0}\}$ is both fuzzy open and fuzzy closed in $\{\tilde{0}, \tilde{1}\}$ .Hence $f^{-1}\{\tilde{0}\}$ is fuzzy semi-open and fuzzy semi-closed in X. This is a contradiction to our assumption. Hence no Fs-continuous mapping $f: X \rightarrow \{\tilde{0}, \tilde{1}\}$ is surjective.

[3]⇒ [1]

Let no Fs-continuous mapping $f: X \rightarrow \{\tilde{0}, \tilde{1}\}$ is surjective. If possible let fuzzy biclosure space (X, k_1, k_2) is not fuzzy semi-connected fuzzy biclosure space. So $\tilde{1}_X = \chi_\lambda \vee \chi_\delta$, χ_λ and χ_δ are also fuzzy semi closed sets.

Then $\chi_{\lambda(x)} = \begin{cases} 1; \text{ if } \chi_x \leq \chi_\lambda \\ 0, \text{ if } \chi_x \succ \chi_\lambda. \end{cases}$

is Fs-continuous surjection which is a contradiction to our initial assumption. Hence fuzzy biclosure space (X, k_1, k_2) is fuzzy semi-connected fuzzy biclosure space.

Theorem 3.10: The Fs-continuous image of a fuzzy semi-connected fuzzy biclosure space is fuzzy semi-connected fuzzy biclosure space.

show that $\chi_\lambda = \tilde{1}_X$. Otherwise, choose two fixed points $\chi_{\{y_1\}}$ and $\chi_{\{y_2\}}$ in Y. Define $f: X \to Y$ by

$$f(\chi_{\{x\}}) = \begin{cases} \chi_{\{y_1\}};\ \text{if } \chi_{\{x\}} \le \chi_\lambda \\ \chi_{\{y_2\}};\ \text{otherwise.} \end{cases}$$

Then for any open set χ_δ in Y, $f^{-1}(\chi_\delta) = \begin{cases} \chi_\lambda, \text{ if } \chi_\delta \text{ contains } y_1 \text{ only,} \\ \tilde{1}_X/\chi_\lambda, \text{ if } \chi_\delta \text{ contains } y_2 \text{ only,} \\ \tilde{1}_X, \text{ if } \chi_\delta \text{ contains both } y_1 \text{ and } y_2, \\ \tilde{0}_X, \text{ otherwise.} \end{cases}$

In all the cases $f^{-1}(\chi_\delta)$ is fuzzy semi open in X. Hence f is not constant Fs-continuous mapping. This is a contradiction to our assumption. This proves that the only fuzzy semi–open subsets of X are $\tilde{0}_X$ and $\tilde{1}_X$. Hence (X, k_1, k_2) is fuzzy semi-connected fuzzy biclosure space.

Theorem 3.9: The following assertions are equivalent:

1. (X, k_1, k_2) is fuzzy semi-connected fuzzy biclosure space.

2. The only fuzzy subsets of X both fuzzy semi-open and fuzzy semi-closed are $\tilde{0}_X$ and $\tilde{1}_X$.

3. No Fs-continuous mapping $f: X \to \{\tilde{0}, \tilde{1}\}$ is surjective.

Proof: [1] ⇒ [2]

Let (X, k_1, k_2) is fuzzy semi-connected fuzzy biclosure space. Suppose $\chi_\lambda < \tilde{1}_X$ is both fuzzy semi-open and fuzzy semi-closed such that $\chi_\lambda \ne \tilde{0}_X$ and $\chi_\lambda \ne \tilde{1}_X$, then $\tilde{1}_X = \chi_\lambda \vee \chi_{\lambda^c}$, Where χ_{λ^c} is complement of χ_λ in X. Hence Fs-continuous mapping $f: X \to \{\tilde{0}, \tilde{1}\}$ is not constant i. e. (X, k_1, k_2) is

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Proof: Let fuzzy biclosure space (X, k_1, k_2) is a fuzzy semi-connected fuzzy biclosure space. Consider an Fs-continuous mapping $f: X \rightarrow f(X)$ is surjective. If f(X) is not fuzzy semi-connected fuzzy biclosure space, there would be a Fs-continuous surjection $g: f(X) \rightarrow \{\tilde{0}, \tilde{1}\}$ so that the composite function $g \circ f: X \rightarrow \{\tilde{0}, \tilde{1}\}$ would also be a fuzzy semi-continuous surjection, which is a contradiction to fuzzy semi-connectedness of fuzzy biclosure space (X, k_1, k_2). Hence f(X) is a fuzzy semi-connected fuzzy biclosure space.

4. FUZZY PRE-CONNECTEDNESS IN FUZZY BICLOSURE SPACE

Definition 4.1:- A fuzzy set A in a fuzzy biclosure space (X, k_1, k_2) is said to be fuzzy pre-open if $\chi_\lambda \leq \text{int}_{k_i}(k_i(\chi_\lambda))$ for all i= $\{1, 2\}$. The complement of fuzzy pre-open set is called fuzzy pre-closed set. The class of all fuzzy pre-open sets of fuzzy biclosure space (X, k_1, k_2) is denoted by FPO(X, k_1, k_2).

Definition4.2:- Let (X, k_1, k_2) and (Y, c_1, c_2) are fuzzy biclosure spaces. An F-map

$f: (X, k_1, k_2) \rightarrow (Y, c_1, c_2)$ is called Fp-continuous if $f^{-1}(G)$ is a fuzzy pre-open subset of (X, k_1, k_2) for every fuzzy open subset G of (Y, c_1, c_2).

Definition 4.3:- A fuzzy biclosure space (X, k_1, k_2) is called fuzzy pre-connected if there exists a Fp-continuous mapping f from X to fuzzy discrete space $\{\tilde{0}, \tilde{1}\}$ is constant.

Example 4.4:- Consider a non empty fuzzy set X = {a, b, c}, and k_1 and k_2 are two fuzzy closure operators which are defined by $k_1: I^X \rightarrow I^X$ such that

$$k_1(\chi_\lambda)=\begin{cases}\tilde{0}_X; if\ \chi_\lambda=0\\ \chi_{\{b,c\}};\ \text{if } 0\neq\chi_\lambda\leq\chi_{\{b\}}\\ \chi_{\{b,c\}};\ \text{if } 0\neq\chi_\lambda\leq\chi_{\{c\}}\\ \chi_{\{b,c\}};\ \text{if } 0\neq\chi_\lambda\leq\chi_{\{b,c\}}\\ \tilde{1}_X; otherwise.\end{cases}$$

Then (X, k_1) is called fuzzy closure space.

Fuzzy open sets of $(X, k_1) = \{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{a,b\}}, \chi_{\{a,c\}}, \tilde{1}_X, \tilde{0}_X\}$.

FPO $(X, k_1) = \{\chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{a,c\}}, \tilde{1}_X, \tilde{0}_X\}$.

Define another fuzzy Čech closure operator $k_2: I^X \to I^X$ such that

$$k_2(\chi_\lambda)=\begin{cases}\tilde{0}_X\ ;\ \text{if } \chi_\lambda=0\\ \chi_{\{a,b\}};\ \text{if } 0\neq\chi_\lambda\leq\chi_{\{a\}}\\ \chi_{\{b,c\}};\ \text{if } 0\neq\chi_\lambda\leq\chi_{\{b\}}\\ \chi_{\{c,a\}};\ \text{if } 0\neq\chi_\lambda\leq\chi_{\{c\}}\\ \tilde{1}_X;\ \text{otherwise.}\end{cases}$$

Then (X, k_2) is called fuzzy closure space.

Fuzzy open sets of $(X, k_2) = \{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{a,b\}}, \chi_{\{b,c\}}, \chi_{\{a,c\}}, \tilde{1}_X, \tilde{0}_X\}$,

FPO $(X, k_2) = \{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{a,b\}}, \chi_{\{b,c\}}, \chi_{\{a,c\}}, \tilde{1}_X, \tilde{0}_X\}$,

Here (X, k_1, k_2) is a fuzzy biclosure space.

Fuzzy open sets of $(X, k_1, k_2) = \{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{a,b\}}, \chi_{\{a,c\}}, \tilde{1}_X, \tilde{0}_X\}$,

FPO $(X, k_1, k_2) = \{\chi_{\{a\}}, \chi_{\{a,b\}}, \chi_{\{a,c\}}, \tilde{1}_X, \tilde{0}_X\}$.

Define a Fp-continuous mapping $f: X \to \{\tilde{0}, \tilde{1}\}$ such that

Volume : 6 | Issue : 1 | JANUARY 2016 | ISSN - 2249-555X

$f^{-1}\{\tilde{1}\}=\chi_{\{a\}}=\chi_{\{b\}}=\chi_{\{c\}}=\chi_{\{a,b\}}=\chi_{\{b,c\}}=\chi_{\{a,c\}}=\tilde{1}_X$, $f^{-1}\{\tilde{0}\}=\tilde{0}_X$ is constant.

Hence (X, k_1, k_2) is a **fuzzy pre-connected fuzzy biclosure space**.

Example 4.5:- Let X = {a, b, c, d} is a non empty set and consider a fuzzy closure operator which is defined by $k_1: I^X \to I^X$ such that

$$k_1(\chi_\lambda) = \begin{cases} \tilde{0}_X; & \text{if } \chi_\lambda = 0 \\ \chi_{\{a,b\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{a\}} \\ \chi_{\{a,b\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{b\}} \\ \chi_{\{b,c\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{c\}} \\ \chi_{\{c,d\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{d\}} \\ \tilde{1}_X; & \text{if } \chi_\lambda = 1. \end{cases}$$

For all fuzzy subsets $\lambda \leq I^X$, let $k_1(\chi_\lambda) = \begin{cases} \tilde{0}_X, \text{ if } \chi_\lambda = 0, \\ \vee\{k_1\{\chi_{\{a\}}\}: \chi_{\{a\}} \leq \chi_\lambda\}, \text{ otherwise.} \end{cases}$

Hence (X, k_1) is a fuzzy closure space.

Fuzzy open sets of $(X, k_1) = \{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{d\}}, \chi_{\{b,c\}}, \chi_{\{c,d\}}, \chi_{\{d,a\}}, \chi_{\{b,c,d\}}, \chi_{\{a,d,b\}}, \chi_{\{c,d,a\}}, \tilde{1}_X, \tilde{0}_X\}$.

FPO of $(X, k_1) = \{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{d\}}, \chi_{\{c,d\}}, \chi_{\{d,a\}}, \chi_{\{b,c,d\}}, \chi_{\{a,d,b\}}, \chi_{\{c,d,a\}}, \tilde{1}_X, \tilde{0}_X\}$.

Consider another fuzzy closure operator $k_2: I^X \to I^X$ such that

$$k_2(\chi_\lambda) = \begin{cases} \tilde{0}_X; & \chi_\lambda = 0 \\ \chi_{\{a,b,c\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{a\}} \\ \chi_{\{b,c,d\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{b\}} \\ \chi_{\{c,a,d\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{c\}} \\ \chi_{\{d,a,b\}}; & \text{if } 0 \neq \chi_\lambda \leq \chi_{\{d\}} \\ \tilde{1}_X; & \text{if } \chi_\lambda = 1. \end{cases}$$

For all fuzzy subsets $\lambda \leq I^X$, let $k_2(\chi_\lambda) = \begin{cases} \tilde{0}_X, \text{ if } \chi_\lambda = 0, \\ \vee\{k_2\{\chi_{\{a\}}\}: \chi_{\{a\}} \leq \chi_\lambda\}, \text{ otherwise.} \end{cases}$

Hence (X, k_2) is a fuzzy closure space.

Fuzzy open sets of fuzzy closure space (X, k_2) =

$\{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{d\}}, \chi_{\{a, b\}}, \chi_{\{b, c\}}, \chi_{\{c, d\}}, \chi_{\{d, a\}}, \chi_{\{a, b, c\}}, \chi_{\{b, c, d\}}, \chi_{\{c, d, a\}}, \chi_{\{a, d, b\}}, \tilde{1}_X, \tilde{0}_X\}$,

FPO sets of (X, k_2) =

$\{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{d\}}, \chi_{\{a, b\}}, \chi_{\{b, c\}}, \chi_{\{c, d\}}, \chi_{\{d, a\}}, \chi_{\{a, b, c\}}, \chi_{\{b, c, d\}}, \chi_{\{c, d, a\}}, \chi_{\{a, d, b\}}, \tilde{1}_X, \tilde{0}_X\}$.

Here (X, k_1, k_2) is a fuzzy biclosure space.

Fuzzy open sets of fuzzy biclosure space (X, k_1, k_2) are

$\{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{d\}}, \chi_{\{b, c\}}, \chi_{\{c, d\}}, \chi_{\{d, a\}}, \chi_{\{b, c, d\}}, \chi_{\{a, d, b\}}, \chi_{\{c, d, a\}}, \tilde{1}_X, \tilde{0}_X\}$.

FPO sets of fuzzy biclosure space (X, k_1, k_2) =

$\{\chi_{\{a\}}, \chi_{\{b\}}, \chi_{\{c\}}, \chi_{\{d\}}, \chi_{\{c, d\}}, \chi_{\{d, a\}}, \chi_{\{b, c, d\}}, \chi_{\{a, d, b\}}, \chi_{\{c, d, a\}}, \tilde{1}_X, \tilde{0}_X\}$.

Define a Fp-continuous mapping $f: X \to \{\tilde{0}, \tilde{1}\}$ such that

$f^{-1}\{\tilde{1}\} = \chi_{\{a\}} = \chi_{\{b\}} = \chi_{\{c\}} = \chi_{\{d\}} = \chi_{\{a, b\}} = \chi_{\{b, c\}} = \chi_{\{c, d\}} = \chi_{\{d, a\}} = \chi_{\{a, b, c\}} = \chi_{\{b, c, d\}} = \chi_{\{a, d, b\}} = \chi_{\{c, d, a\}} = \tilde{1}_X$,
$f^{-1}\{\tilde{0}\} = \tilde{0}_X$ is constant.

Hence (X, k_1, k_2) is a **fuzzy pre-connected fuzzy biclosure space.**

Definition 4.6:- A fuzzy biclosure space (X, k_1, k_2) is called **fuzzy pre-disconnected fuzzy biclosure space** if and only if any Fp-continuous mapping f from X to the fuzzy discrete space $\{\tilde{0}, \tilde{1}\}$ is surjective.

Theorem 4.7:- If $\{\chi_{\lambda_i} : i \in \wedge\}$ is a family of fuzzy pre-connected fuzzy biclosure subsets of

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Fuzzy pre-connected fuzzy biclosure space (X, k_1, k_2), then $\vee\chi_{\lambda_i}$ is also a fuzzy pre-connected fuzzy biclosure subset of , where Λ is any index set.

Proof:-Each $\chi_{\lambda_i} : i \in \wedge$ is a fuzzy pre-connected subset of fuzzy pre-connected fuzzy biclosure space (X, k_1, k_2) so there exists Fp-continuous mapping $f_i: \lambda_i \to \{\tilde{0}, \tilde{1}\}$ is constant. Let an Fp-continuous mapping $f: \vee\lambda_i \to \{\tilde{0}, \tilde{1}\}$ is not constant, $f^{-1}\{\tilde{1}\} \neq \chi_{\lambda_i}$ which is a contradiction to each A_i is fuzzy pre-connected subsets of (X, k_1, k_2), i.e. Fp-continuous mapping f is constant. Hence $\vee\chi_{\lambda_i}$ is fuzzy pre-connected fuzzy biclosure space.

Theorem 4.8:- Let (X, k_1, k_2) and (Y, c_1, c_2) are two fuzzy biclosure spaces and $f: X \to Y$ is a fuzzy bijection. Then

1) f is Fp-continuous mapping and X is a fuzzy pre-connected fuzzy biclosure space then Y is fuzzy connected fuzzy biclosure space.

2) f is F-continuous mapping and X is fuzzy pre-connected fuzzy biclosure space then Y is a fuzzy connected fuzzy biclosure space.

3) f is Fp-open mapping and Y is fuzzy pre-connected fuzzy biclosure space then X is fuzzy connected fuzzy biclosure space.

4) f is F-open mapping and X is fuzzy connected fuzzy biclosure space then Y is fuzzy pre-connected fuzzy biclosure space.

Proof: -1. Let (Y, c_1, c_2) is a fuzzy biclosure space and X is a fuzzy pre-connected fuzzy biclosure space then there exists an Fp-continuous mapping $fog: X \rightarrow \{\tilde{0}, \tilde{1}\}$ is constant. Consider an Fp-continuous mapping $g:Y \rightarrow \{\tilde{0}, \tilde{1}\}$, given that $f: X \rightarrow Y$ is Fp-continuous mapping and f is bijection so that g is also a constant mapping. Hence Y is fuzzy connected fuzzy biclosure space.

2. Given that X is a fuzzy pre-connected fuzzy biclosure space, i.e. $g:X \rightarrow \{\tilde{0}, \tilde{1}\}$ Fp-continuous mapping is constant. $f^{-1}:Y \rightarrow X$ is F-continuous bijection, so that $f^{-1}og: Y \rightarrow \{\tilde{0}, \tilde{1}\}$

F-continuous mapping is constant. Hence Y is fuzzy connected fuzzy biclosure space.

3. Given that Y is fuzzy pre-connected fuzzy biclosure space i.e. $g:Y \rightarrow \{\tilde{0}, \tilde{1}\}$ Fp-continuous mapping is constant. Since $f: X \rightarrow Y$ is fuzzy pre-open and F-bijection mapping so that F-continuous mapping $fog: X \rightarrow \{\tilde{0}, \tilde{1}\}$ is constant. Hence X is fuzzy connected fuzzy biclosure space.

4. Given that X is fuzzy connected fuzzy biclosure space i.e. a F-continuous mapping

$g:X \rightarrow \{\tilde{0}, \tilde{1}\}$ is constant and $f^{-1}:Y \rightarrow X$ is F-open mapping so that it is a fuzzy pre-open mapping then $f^{-1}og: Y \rightarrow \{\tilde{0}, \tilde{1}\}$ is a Fp-continuous constant mapping. Hence Y is a fuzzy pre-connected fuzzy biclosure space.

Theorem 4.9:-A fuzzy biclosure space (X, k_1, k_2) is fuzzy pre-disconnected if and only if there exists an Fp-continuous map f from X onto a fuzzy discrete two point space $Y = \{\tilde{0}, \tilde{1}\}$.

Proof: Given that fuzzy biclosure space (X, k_1, k_2) is fuzzy pre-disconnected i.e. there exists a Fp-continuous map $f{:}X \rightarrow \{\tilde{0}, \tilde{1}\}$ is not constant and $f^{-1}\{\tilde{0}\} \neq \phi$. If a Fp-continuous map $f{:}X \rightarrow \{\tilde{0}, \tilde{1}\}$ is onto, so that F-mapping is not constant. Hence (X, k_1, k_2) is fuzzy pre-disconnected fuzzy biclosure space.

Conclusion: - In this paper the idea of fuzzy semi-connectedness and fuzzy pre-connectedness in fuzzy biclosure space were introduced.

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