RESEARCH PAPER | Statistics | Volume : 6 | Issue : 3 | March 2016 | ISSN - 2249-555X | IF : 3.919 | IC Value : 74.50

Dominating Directions – Free Disposable Hull

KEYWORDS: Data Envelopment Analysis, Free Disposable Hull, Directional Distance Function, Dominated Directions.

H. Ravi Sankar	C. Subbarami Reddy
Research Scholar, Department of Statistics, S.V.University, Tirupati	(Rtd) Professor, Department of Statistics, S.V.University, Tirupati

ABSTRACT *Data Envelopment Analysis (DEA) is a linear programming technique, that provides efficiencies for Decision Making Units (DMUs). For inefficient firms it provides virtual DMU's production plan as bench mark. These bench marks are either linear or convex combinations of more than one relatively extremely efficient DMU. Non-convex technology based on Free Disposable Hull provides shorter input and output targets in addition to one and only one efficient peer. In this paper to assess input and output oriented efficiency the directional distance function is implemented, and the inefficient production plans are moved in the direction of dominating decision making units. The method is capable of grouping inefficient firms such that for each group there exists one and only one efficient peer.*

INTRODUCTION:

Data Envelopment Analysis is a Linear programme based deterministic approach that estimates efficiencies of decision making units. The production possibility set that promoted DEA is built on axioms such as, inclusion, free disposability, convexity and minimum extrapolation. This technique was launched by Charnes, Cooper and Rhodes (1978)[1] and improved by Bankar, Charnes and Cooper (1984)[2]. Following these fundamental contributions voluminous theoretical developments have taken place to suit empirical research needs, that suitably tailored the primitive DEA constraints.

Farrell (1957)[3] was the first one who suggested a practical approach to measure radial efficiencies of production units. Due to the advent of high power computers DEA and its extensions have become powerful tools to assess efficiencies of production units which combine similar (multiple) inputs to produce similar (multiple) outputs.

DEA optimal solutions provide input / output efficiencies, input / output losses, shadow prices of inputs / outputs whenever actual prices are not available, economic efficiency, in addition to setting targets for inefficient decision making units. DEA handles environmental variables, undesirable outputs, categorical variables, and exogenous inputs with comfortable ease.

Tulkens (1993)[4] removed the convexity axiom, with inclusion and free disposability and the minimum extrapolation axioms as maintained hypothesis, a non-convex production possibility set was introduced, resulting in Free Disposable Hull (FDH). Projections on to the non-convex frontier (FDH) provide not only shorter targets, but a single efficient peer for an inefficient decision making unit[5].

DISTNACE FUNCTIONS:

For measuring efficiency and setting targets for inefficient firms, the chief tool is distance function. Following Farrell and Shephard[6], numerous distance functions were introduced and combined with DEA constraint inequalities, by researchers. These distance functions constitute two subsets (i) radial distance functions and (ii) non-radial distance functions[7]. To implement distance functions an orientation may to be selected. Selection of orientation further depends upon rigidity of technology. In Economic Theory two familiar concepts are 'ex ante production' and 'ex post production'[8]. Suppose a technology has to be selected out of the technologies available either before the factory is constructed or at the time the available technology is obsolete which has to be replaced, the entrepreneur has the opportunity to mix inputs and or outputs (ex ante production). But, once the technology is selected (ex post production), input mix and / or output mix are more or less fixed. In addition, suppose outputs cannot be expanded but inputs can be contracted, without change in input mix. The orientation suggested in this case is input perspective, and the suitable distance functions are radial. If inputs cannot be contracted, but output can be expanded without change in the mix, the output orientation, coupled with radial expansion serve the purpose. The CCR and BCC measures are radial efficiency measures. These are called technical efficiency measures.

DIRECTIONAL DISTANCE FUNCTION:

An important class of distance functions, called the Directional Distance Functions (DDF) were formulated by Chambers et.al (1996, 1998)[9,10], for which the Farrell's (Shaphard's) distance functions can be obtained as special cases. For any suitably structured production possibility set T, the directional distance function is formulated as,

$$\vec{D}_T\left(x_0, y_0; g_x, g_y\right) = Max\ \beta$$

such that $\left(x_0 - \beta g_x, y_0 + \beta g_y\right) \in T$ ...... (1)

g_x and g_y are the directional vectors along which inputs are contracted and outputs are expanded simultaneously.

The directional distance function is finite in value and for efficient firms it vanishes. It is homogeneous of degree -1, non-decreasing in inputs, non-increasing in outputs, in addition satisfies the translation property.

The directional distance function is sensitive to the choice of the direction choosen for input contraction and / or output expansion. The directions are exogeneous for policy

Volume : 6 | Issue : 3 | March 2016 | ISSN - 2249-555X | IF : 3.919 | IC Value : 74.50

and regulatory applications, but endogeneous for internal performance evaluation.

Fare et.al (2013)[11] produced an endogeneous directional vector that is exogeneously normalized.

PRESENT STUDY:
The present study explores smallest targets in the frame work of directional distance functions. These distance functions are extremely flexible, since several popular distance functions can be derived as their particular cases.

INPUT AND OUTPUT ORIENTATION:
Let $x_0, x_k \in R_m^+;\ y_0, y_k \in R_s^+$ and $T = T_{FDH}$

where $k \in R(x_0, y_0, \delta = 1)$ R is the index set of input and output vectors which dominate (x_0, y_0)

$$x_0 \geq x_k$$

$\delta = 1$ allows returns to scale to vary

(i) $$x_0 - \beta_k x_k \geq x_k, k \in R(x_0, y_0, \delta = 1)$$

$$\Rightarrow x_{i0} - \beta_k x_{ik} \geq x_{ik},\ i \in M$$

$$\Rightarrow \beta_k x_{ik} \leq x_{i0} - x_{ik}$$

$$\beta_k \leq \frac{x_{i0}}{x_{ik}} - 1$$

$$\beta_k \leq \underset{i}{Min} \frac{x_{i0}}{x_{ik}} - 1 \qquad \ldots\ldots (2)$$

(ii) $$y_0 + \beta_k y_k \leq y_k,\ k \in R(x_0, y_0, \delta = 1)$$

$$\beta_k y_{rk} \leq y_{rk} - y_{r0}$$

$$\beta_k \leq 1 - \frac{y_{r0}}{y_{rk}},\ r \in S$$

$$\beta_k \leq \underset{r}{Min}\left(1 - \frac{y_{r0}}{y_{rk}}\right)$$

$$= 1 + \underset{r}{Min}\left(-\frac{y_{r0}}{y_{rk}}\right)$$

$$= 1 - (-1)\underset{r}{Min}\left(-\frac{y_{r0}}{y_{rk}}\right)$$

$$\beta_k \leq 1 - \underset{r}{Max} \frac{y_{r0}}{y_{rk}} \qquad \ldots\ldots (3)$$

Combine (2) and (3) to obtain the following:

$$\beta_k \leq Min\left\{\underset{i}{Min}\frac{x_{i0}}{x_{ik}} - 1,\ 1 - \underset{r}{Max}\frac{y_{r0}}{y_{rk}}\right\} \qquad \ldots\ldots (4)$$

$$\overrightarrow{DDF_k}\left(x_0, y_0; g_x = x_k, g_y = y_k\right) = Max\,\beta_k$$

$$= Min\left\{\underset{i}{Min}\frac{x_{i0}}{x_{ik}} - 1,\ 1 - \underset{r}{Max}\frac{y_{r0}}{y_{rk}}\right\},\ K \in R(x_0, y_0; \delta = 1) \qquad \ldots\ldots (5)$$

$$\overrightarrow{DDF}\left(x_0, y_0; g_x, g_y\right) = \underset{k}{Max}\left[\overrightarrow{DDF_k}\left(x_0, y_0; g_x, g_y\right)\right]$$

$$= \underset{k \in R(x_0, y_0; \delta = 1)}{Max}\ Min\left\{\underset{i}{Min}\frac{x_{i0}}{x_{ik}} - 1,\ 1 - \underset{r}{Max}\frac{y_{r0}}{y_{rk}}\right\}$$

$$= Min\left\{\underset{i}{Min}\left(\frac{x_{i0}}{x_{ik*}}\right) - 1,\ 1 - \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk*}}\right)\right\},\ k^* \in R(x_0, y_0; \delta = 1)$$

The endogeneous directional vector for DMU_0 is provided by the input and output vector of DMU_{k^*}. That is,

$$g_{x_0} = x_{k^*},\ k^* \in R(x_0, y_0; \delta = 1)$$

$$g_{y_0} = y_{k^*},\ k^* \in R(x_0, y_0; \delta = 1)$$

$$\overrightarrow{DDF_F} = \underset{k}{Max}\, Min\left\{\underset{i}{Min}\left(\frac{x_{i0}}{x_{ik*}}\right) - 1,\ 1 - \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk*}}\right)\right\}$$

5. DIRECTIONAL EFFICIENCY – INPUT ORIENTATION:

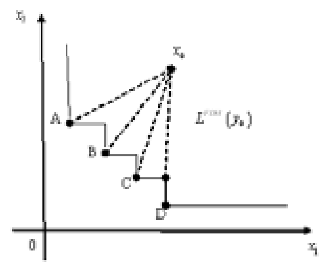

Figure (1): FDH INPUT SET

In the above diagram the inputs x_1 and x_2 are measured along horizontal and vertical axes respectively. $L^{FDH}(y_0)$ is the free disposable hull based input level set that consists all input vectors which can produce output y_0

$$L^{FDH}(y_0) = \{x : y_k \leq y_0, x \geq x_k\}$$

The directional distance problem under input orientation may be expressed as,

$$\beta_k = Max\,\beta$$

Volume : 6 | Issue : 3 | March 2016 | ISSN - 2249-555X | IF : 3.919 | IC Value : 74.50

s.t $x_0 - \beta x_k \geq x_k$

$$y_0 \leq y_k,\ k \in R(x_0, y_0; \delta = 1)$$

$$\beta x_{ik} \leq x_{i0} - x_{ik},\ i \in M$$

$$\beta \leq \frac{x_{i0}}{x_{ik}} - 1$$

$$\beta \leq \underset{i}{Min} \frac{x_{i0}}{x_{ik}} - 1$$

$$\beta \leq \underset{i}{Min} \frac{x_{i0}}{x_{ik}} - 1,\ k \in R(x_0, y_0; \delta = 1)$$

$$DDF_i\left(y_0, x_0;\ g_{y_0} = 0\ g_{x_0} = x_k\right) = \underset{k}{Max}\, \beta_k^i$$

$$= \underset{k}{Max}\, \underset{i}{Min} \frac{x_{i0}}{x_{ik}} - 1 \qquad \ldots\ldots (6)$$

6. DIRECTIONAL EFFICIENCY – OUTPUT ORIENTATION:

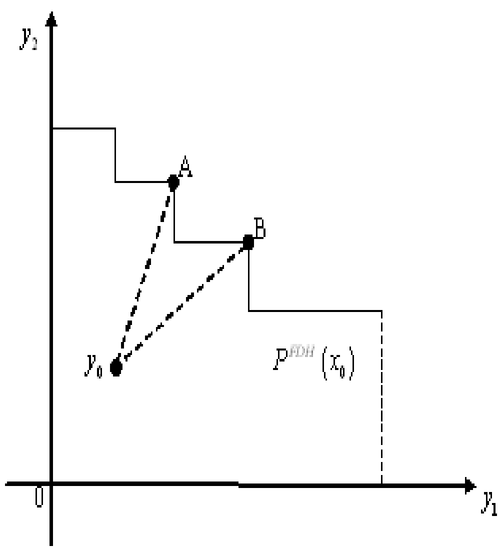

Figure (2): FDH OUTPUT SET

$P^{FDH}(x_0)$ is output level set that contains all output vectors, capable of being produced by x_0

$$P^{FDH}(x_0) = \{y : x_k \leq x_0, y_k \geq y\}$$

$k \in R(x_0, y_0; \delta = 1)$ implies,

$x_k \leq x_0$ and $y_k \geq y_0$

The directional distance problem under output orientation may be expressed as,

$$\beta_k = Max\, \beta$$

s.t $x_0 \geq x_k$

$$y_0 + \beta y_k \leq y_k$$

$$k \in R(x_0, y_0; \delta = 1)$$

$$\beta y_k \leq y_k - y_0$$

$$\beta y_{rk} \leq y_{rk} - y_{r0}$$

$$\beta \leq 1 - \frac{y_{r0}}{y_{rk}}$$

$$\beta \leq Min\left[1 - \frac{y_{r0}}{y_{rk}}\right]$$

$$= 1 - \underset{r}{Max} \frac{y_{r0}}{y_{rk}} \qquad \ldots\ldots (7)$$

$$\beta_k^0 = 1 - \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right),\ k \in R(x_0, y_0; \delta = 1)$$

$$\underset{k}{Max}\, \beta_k^0 = \underset{k}{Max}\left[1 - \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)\right]$$

$$\overline{DDF}_0\left(x_0, y_0; g_{x_0} = 0,\ g_{y_0}\right) = 1 - \underset{k}{Min}\, \underset{r}{Max} \frac{y_{r0}}{y_{rk}} \qquad \ldots\ldots (8)$$

EMPIRICAL ANALYSIS:

To assess directional efficiency of inefficient firms inputs are contracted and outputs are expanded in the direction of the input and output of the dominating firms. The best of these input contractions and output expansions provide efficient bench marks for the inefficient decision making unit whose directional efficiency is under evaluation. The efficiency measure (5) is implemented to estimate the directional efficiency of total manufacturing sectors of 28 Indian States. Fixed capital and number of employees are inputs; value added is the only output. The input and output oriented efficiency measurements are furnished in the following table:

Volume : 6 | Issue : 3 | March 2016 | ISSN - 2249-555X | IF : 3.919 | IC Value : 74.50

DIRECTIONAL EFFICIENCY SCORES FOR INPUT CONTRACTION AND OUTPUT EXPANSION

S.No	Total Manufacturing Sector	Directional Efficiency score	Efficient Peer
1	Maharastra (MH)	0	-
2	Gujarat (GUJ)	0	-
3	Tamilnadu (TN)	0	-
4	Karnataka (KA)	0	-
5	Uttar Pradesh (UP)	0	-
6	Haryana (HA)	0	-
7	Uttarakhund (UK)	0	-
8	Rajasthan (RA)	0.1589	UK
9	Telangana (TEL)	0.1033	UK
10	Andhra Pradesh (AP)	0.3437	UK
11	West Bengal (WB)	0.3862	UK
12	Himachal Pradesh (HP)	0	-
13	Madhya Pradesh (MP)	0.0353	HP
14	Jharkhand (JHA)	0.0174	HP
15	Punjab (PUN)	0	-
16	Odisha (ODI)	0.1883	HP
17	Chettisgarh	0	-
18	Daman and Diu (DD)	0	-
19	Kerala	0.0619	DD
20	Goa	0	-
21	Dadra Nagar Haveli	0.0875	GOA
22	Delhi	0.2565	GOA
23	Assam	0.3067	DD
24	Jammu & Kashmere (JK)	0	-
25	Sikkim (SIK)	0	-
26	Bhihar	0.6427	SIK
27	Puducheri	0.5328	SIK
28	Meghalaya	0.1662	SIK

In the table above directional distance efficiency scores are evaluated for 28 total manufacturing sectors of Indian states which simultaneously expand outputs and contract inputs at the same proportional rate. Directional efficiency score, β is such that, $0 \le \beta < \infty$. $\beta = 0$ implies that the decision making unit is directional efficient. It performs to its potential, neither its inputs nor outputs are freely disposed off. 14 out of 28 total manufacturing sectors are directional distance efficient. If β departs from zero $(\beta > 0)$ the decision making unit is inefficient, signifying the possibility of input contraction and output expansion at the rate of β in the direction of its dominating efficient peer that serves as leader of the inefficient DMU.

The directional distance measurement is not radial, the inefficient decision making unit should be capable of input substitution and output transformation simultaneously, the act of which is possible only in long run when the producer will be in search of ex ante production techniques.

The directional efficient states are Maharastra, Gujarat, Tamilnadu, Karnataka, Uttar Pradesh, Haryana, Uttarakhund, Himachal Pradesh, Punjab, Chettisgarh, Daman and Diu, Goa, Jammu & Kashmere and Sikkim.

The total manufacturing sectors of Jharkhand, Madhya Pradesh and Odisha experienced input and output losses 1.7%, 3.55% and 18.8% of inputs and outputs of Himachal Pradesh respectively. Jharkhand performed better than Madhya Pradesh which is followed by Odisha and Kerala. The input and output losses of the total manufacturing sector of Rajasthan, Telangana, Andhra Pradesh and West Bengal are 16%, 10%, 34%, 39% of the inputs and outputs of the Total Manufacturing sector of Uttarakhund respectively. Of these four decision making units the best performer is Telangana which is followed by Rajasthan. The total manufacturing sector of Andhra Pradesh better performed than West Bengal but worse when compared with Rajasthan.

The total manufacturing sectors of Kerala and Assam are FDH directional distance inefficient. The input and output losses due to free disposability experienced by these states are 6% and 31% of the inputs and outputs of Daman& Diu respectively. For these states the referent DMU is Daman & Diu.

For Dadra & Nagar Haveli and Delhi the efficient peer is the total manufacturing sector of Goa in whose direction inputs are contracted and outputs are expanded simultaneously. The input and output losses experienced by these states are 9% and 26% of the inputs and outputs of the total manufacturing sector of Goa.

For the total manufacturing sectors of Meghalaya, Puducheri and Bihar the referent DMU is Sikkim. These states suffer from input and output losses of 17%, 53% and 64% of inputs and outputs of the total manufacturing sector of Sikkim.

CONCLUSIONS:

In a similar manner to assess input dominated directional and output dominated directional efficiencies one can implement equations (6) and (7). Under input orientation output directional vector is null vector. Output directional efficiency measurement requires the input directional vector to be set to zero. Input orientation helps to assess input losses, while output orientation provides potential output losses. Further, the FDH input and output targets are closer than the convex input and output targets to the observed input and output vector. The methodology developed can group the inefficient firms such that for each group a dominating production unit is the efficient peer.

REFERENCES:

[1] Charnes, A., Cooper, W.W., Rhodes, E. (1978), 'Measuring the efficiency of Decision Making Units', European Journal of Operations Research, 2(6), pp:429-444.

[2] Banker, Charnes and Cooper (1984), 'Some models for estimating technical and scale inefficiencies in data envelopment analysis', European Journal of Operations Research, 30(9), pp: 1078-1092.

[3] Farell, M.J.(1957), 'The Measurement of Productive Efficiency', Journal of Royal Statistical Society, Series – A, 120, pp:253-290.

[4] Henry Tulkens (1993), 'On FDH Efficiency Analysis, Some methodological Issues and Applications to Retail Banking, Courts and Urban Transit', The Journal of Productivity Analysis, 4, pp:183-210.

[5] Briec, W., Kerstens and Eeckaut, P (2004), 'Non-convex Technologies and cost functions: definitions, duality and non-parametric tests of convexity', Journal of Economics, 81(2), pp: 155-192. - Peypoch, N. (2011), 'Exact relations between four definitions of Productivity Indices and Indicators', Bulletin of Economic Research.

[6] Shephard, R.W (1970), 'Theory of Cost and Production Functions' Princeton University Press, Princeton, New – Jersey.

[7] Russell, R.R. (1985), 'Measures of Technical Efficiency', Journal of Economic theory, 35, pp:109-126.

[8] Murray Brown (1966), 'On the theory and Measurement of Technological change', Oxford University Press.

[9] Chambers, R., R. Fare and S. Grosskopf (1996), 'Productivity Growth in APEC countries', Pasith Economic Review 1(3), pp: 181-190.

[10] Chambers, R., Chung, Y., and Fare, R. (1998), 'Profit, Directional Distance functions and Nerlovian Efficiency', Journal of Optimization Theory and Applications, 98, pp:351-364.

[11] Fare, S. Grosskopf and G. Whittakar (2013), 'Directional output Distance Functions: endogeneous directions based on exogeneous normalization constraints', Journal of Productivity analysis, 40, pp: 267-269