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Soft Alexandroff Spaces in Soft Ideal Topological Spaces

KEYWORDS: soft $I_{\pi g}$ - open set, soft $I_{\pi g}$ - closed set, soft Alexandroff space, soft I - Alexandroff space, soft $I_{\pi g}$ - Alexandroff space

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ABSTRACT *In this paper, we present some new spaces namely Alexandroff spaces in soft topological spaces. Several characterizations of this concept are established.*

1. Introduction

Molodtsov[13] introduced soft set theory in 1999. This theory has been applied to many fields such as optimization theory, basic mathematical analysis etc. Shabir and Naz[17] appilied this theory to topological structure and studied about soft topological spaces. R. Sahin and A. kucuk[15] initiated the concept of soft ideal. Then Mustafa and Sleim[14] defined a different version of soft ideal. Using this definition Kandil et al[11] presented a soft $*$-topology finer than soft topology. Alexandroff spaces were first studied by Alexandroff[1]. It is a topological space in which arbitrary intersection of open sets is open. Equivalently each singleton set has a minimal neighbourhood base. Arenas et al[2] studied some weaker separation aioms related with Alexandroff topological spaces. The notion of I-Alexandroff and I_g-Aleandroff ideal topological spaces are derived by Erdal Ekici[4]. In this paper we derive the notion of soft Alexandroff ideal spaces and examine some of their Properties.

2. Preliminaries

Throughout this paper, X will be a nonempty initial universal set and E will be a set of parameters. Let P(X) denote the power set of X and S(X) denote the set of all soft sets of X.

Definition: 2.1[13]

Let X be an initial universe and E be a set of parameters. Let P(X) denotes the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X, where F is a mapping given by F: A → P(X).

Definition: 2.2[3]

A subset (A, E) of a topological space X is called soft regular closed, if cl(int(A,E)) = (A,E). The complement of soft regular closed set is soft regular open set.

Definition: 2.3[3]

The finite union of soft regular open sets is said to be soft π-open. The complement of soft π-open is said to be soft π-closed.

Definition: 2.4[3]

A subset (A, E) of a topological space X is called soft πg-closed in a soft topological space (X,τ, E), if cl(A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft π-open in X. The complement of soft πg-closed set is soft πg-open set.

Definition: 2.5[17]

Let (X, τ, E) be a soft topological space over X and $x, y \in$ X such that $x \neq y$. If there exist soft open sets (F, E) and (G, E) such that $x \in$ (F, E) and $y \notin$ (F, E) and $y \in$ (G, E) and $x \notin$ (G, E), then (X, τ, E) is called a soft T_1-space.

Definition: 2.6[11]

Let I be a non-null collection of soft sets over a universe X with the same set of parameters E. Then I ∈ $SS(X)_E$ is called a soft ideal on X with the same set E, if

(1) (F, E) ∈ I and (G, E) ⊆ (F, E) implies (G, E) ∈ I

(2) (F, E) ∈ I and (G, E) ∈ I implies (F, E) ∪ (G, E) ∈ I

Definition: 2.7[11]

Let X be a universe set. Then I_n = {(G, E) ⊆ $SS(X)_E$: int(cl(G,E)) = ϕ} is called soft ideal of nowhere dense soft sets in (X, τ ,E).

Definition: 2.8[11]

Let (X, τ, E) be a soft topological space and I be a soft ideal over X with the same set of parameters E. Then $(F,E)^*(I, \tau) = \cup\{x_e \in X : O_{x_e} \cap (F,E) \notin I \ \forall\ O_{x_e} \in \tau\}$ is called the soft local function of (F,E) with respect to I and τ , where O_{x_e} is a τ -open soft set containing x_e.

Definition: 2.9[16]

A subset (A, E) of a soft ideal space (X, τ, E, I) is said to be soft $I_{\pi g}$ - closed, if $(A, E)^* \subseteq$(U, E) whenever (A, E) ⊆ (U, E) and (U, E) is soft π-open. The complement of soft $I_{\pi g}$ - closed set is soft $I_{\pi g}$ - open set.

Theorem: 2.10[16]

A subset (A, E) of a soft ideal space (X, τ, E, I) is soft $I_{\pi g}$ - open if and only if (F, E) ⊆ int^*(A, E) whenever (F, E) is soft π- closed and (F, E) ⊆ (A, E).

Theorem: 2.11[16]

Let (X, τ, E, I) be a soft ideal space. Then every subset of (X, τ, E, I) is soft $I_{\pi g}$ - closed set if and only if every soft π-open set is soft $*$- closed set.

3. Soft I-Alexandroff spaces

Definition: 3.1

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A soft ideal space (X, τ, E) is said to be a soft Alexandroff space, if any intersection of soft open sets is soft open.

Definition: 3.2

A soft ideal space (X, τ, E, I) is said to be a soft I- Alexandroff space, if any intersection of soft open sets is soft ∗- open.

Theorem: 3.3

Let (X, τ, E, I) be a soft ideal topological space. Then the following properties are equivalent:

(1) (X, τ, E, I) is soft I-Alexandroff space

(2) Any union of soft closed sets in (X, τ, E, I) is soft ∗-closed

Proof:

It follows from the fact that the complement of a soft ∗-open set is soft ∗-closed.

Definition: 3.4

A function f: (X, τ, E, I) ⟶ (Y, σ, E, J) is said to be soft ∗- closed , if f(A, E) is soft ∗- closed in (Y, σ, E, J) for every soft ∗-closed subset (A, E) of (X, τ, E, I).

Theorem: 3.5

Let f: (X, τ, E, I) ⟶ (Y, σ, E, J) be a soft continuous and soft ∗- closed surjective function. If (X, τ, E, I) is a soft I - Alexandroff space then (Y, σ, E, J) is a soft I - Alexandroff space.

Proof:

Suppose that f: (X, τ, E, I) ⟶ (Y, σ, E, J) is a soft continuous and soft ∗- closed function. Let (X, τ, E, I) be a soft I - Alexandroff space. Suppose that {(M_i, E) : i ∈ I} is a family of soft closed sets in (Y, σ, E, J). Since f: (X, τ, E, I) ⟶ (Y, σ, E, J) is a soft continuous, then (N, E) = $\bigcup_{i\in I} f^{-1}$(M_i, E) is a soft ∗-closed set in X. We take (M, E) = $\bigcup_{i\in I}(M_i, E)$. Since f: (X, τ, E, I) ⟶ (Y, σ, E, J) is a soft∗- closed function, then f(N, E) = f($\bigcup_{i\in I} f^{-1}$(M_i, E)) = (M, E) is soft ∗-closed. It follows that Y is a soft I - Alexandroff space.

Theorem: 3.6

Let (X, τ, E, I) be a soft ideal topological space and I = {ϕ}, then the following properties are equivalent:

(1) (X, τ, E, I) is a soft Alexandroff space

(2) (X, τ, E, I) is a soft I - Alexandroff space

Proof:

Since I = {ϕ} then we have $\tau = \tau^*$, it follows that (X, τ, E, I) is a soft Alexandroff space if and only if X is a soft I- Alexandroff space.

Theorem: 3.7

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For a soft ideal topological space (X, τ, E, I) the following conditions are equivalent:

(1) X is soft T_1 and soft I-Alexandroff space

(2) X is soft discrete space

Proof:

(1) ⟹ (2)

For each $x_e \in$ X and $y_e \neq x_e$ there exists a soft open set (U, E) containing x_e, such that $y_e \notin$ (U, E). Since X is soft I- Alexandroff and since $\{x_e\} = \bigcap_{y_e \neq x_e}(U, E)$ then $\{x_e\}$ is soft open. Thus X is soft discrete.

(2) ⟹ (1)

Let X be a soft discrete space then every singleton set is soft open. Therefore for any two soft points x_e and y_e we can find two soft open sets such that $x_e \in$ (U, E) and $y_e \notin$ (U, E). Similarly $y_e \in$ (V, E) and $x_e \notin$ (V, E). Hence it is soft T_1- space. Also since the underlying space is soft discrete, arbitrary intersection of soft open sets is soft ∗-open set. Therefore X is soft I-Alexandroff space.

Theorem: 3.8

If X and Y are soft I-Alexandroff spaces then X × Y is soft I- Alexandroff space.

Proof:

Let { (W_i, E) : i ∈ I } be a collection of soft open sets in X × Y and let (W, E) be its intersection. Suppose (W, E) is not a soft ∗-open set. Then there exists (x_e , y_e) ∈ (W, E) and soft open sets (U, E) ⊆ X and (V, E) ⊆ Y with (x_e , y_e) ∈ U × V and (U × V) ∩ int(W, E) = ϕ. For each i ∈ I there exists soft open sets (F_i, E) ⊆ X and (G_i, E) ⊆ Y with (x_e , y_e) ∈ ($F_i \times G_i$, E) ⊆ (W_i , E). Since X and Y are soft I- Alexandroff spaces, there exist a non-empty soft ∗-open sets (H, E) ⊆ X and (K, E) ⊆ Y with (H, E) ⊆ (U, E) ∩ ($\bigcap_{i \in I}(F_i$, E)) and (K, E) ⊆ (V, E) ∩ ($\bigcap_{i \in I}(G_i$, E)). Clearly ($H_i \times K_i$, E) ⊆ (U × V) ∩ int(W, E) which is a contradiction. Hence X × Y is soft I- Alexandroff space.

4. Soft $I_{\pi g}$ -Alexandroff spaces

Definition: 4.1

A soft ideal space (X, τ, E) is said to be a soft πg - Alexandroff space, if any intersection of soft open sets is soft πg - open.

Definition: 4.2

A soft ideal space (X, τ, E, I) is said to be a soft $I_{\pi g}$ - Alexandroff space, if any intersection of soft open sets is soft $I_{\pi g}$ - open.

Theorem: 4.3

Let (X, τ, E, I) be a soft ideal topological space. If there exists a soft point $x_e \in X$ such that x_e has only soft ∗- neighbourhood which is X itself, then (X, τ, E, I) is soft $I_{\pi g}$ - Alexandroff space.

Proof:

Suppose that there exists a soft point $x_e \in X$ such that x_e has only soft ∗- neighbourhood which is X itself. Let {(K_i , E): i ∈ I} is a family of soft open sets in (X, τ, E, I) for each i ∈ I. We take (K, E) =$\bigcap_{i\in I}(K_i$, E). Let (M, E) = (K, E) and (M, E) be soft closed set. Suppose (M, E) = ϕ, then (M, E) ⊆ int^*(K, E). Suppose (M, E) ≠ ϕ. If (M, E) = X then (M, E) ⊆ (K, E) = X. Hence (M, E) ⊆ int^*(K, E). If (M, E) ≠ X then X – (M, E) is soft open set. It follows that $x_e \notin$ X – (M, E) and then $x_e \in$ (M, E). Since (M, E) = (K, E) then $x_e \in$ (K_i, E) for i ∈ I. Since x_e has only soft ∗- neighbourhood which is X itself then (K_i, E) = X for i ∈ I. Moreover we have (K_i, E) = X and then (M, E) ⊆ int^*(K, E). Hence (K, E) is soft $I_{\pi g}$ – open. Thus (X, τ, E, I) is soft $I_{\pi g}$ - Alexandroff space.

Theorem: 4.4

Let (X, τ, E, I) be a soft ideal topological space. If X is soft I - Alexandroff space then it is soft $I_{\pi g}$ - Alexandroff space.

Proof:

The proof follows from the fact that any soft ∗-open set is soft $I_{\pi g}$ - open.

Remark: 4.5

The reverse implication of the above theorem need not be true as shown in the following example.

Example: 4.6

X = {a, b, c} and E = {e_1, e_2}.

(F_1, E) = {(e_1, {a, b}), (e_2, {a, b})}

(F_2, E) = {(e_1, {b}), (e_2, {a, c})}

(F_3, E) = {(e_1, {b, c}), (e_2, {a})}

(F_4, E) = {(e_1, {b}), (e_2, {a})}

(F_5, E) = {(e_1, {a, b}), (e_2, X)}

(F_6, E) = {(e_1, X) , (e_2, {a, b})}

(F_7, E) = {(e_1, {b, c}), (e_2, {a, c})}

where (F_1, E) , (F_2, E) , (F_3, E) , (F_4, E) , (F_5, E) , (F_6, E) , (F_7, E) are soft sets over X and $\tau = \{\tilde{X}, \tilde{\phi}$, ($F_1$, E) , ($F_2$, E) , ($F_3$, E) , ($F_4$, E) , ($F_5$, E) , ($F_6$, E) , ($F_7$, E) } is a soft topology over

X. Let I = {$\tilde{\phi}$, (I_1, E)} be a soft ideal over X, where (I_1, E) = {(e_1, {b}), (e_2, b)}. Take (F, E) = {(e_1, {a, b}), (e_2, {b, c})} and (G, E) = {(e_1, {a}), (e_2, {a, c})} which are soft sets in X. Then (F, E) ∩ (G, E) = {(e_1, {a}), (e_2, {c})} is soft $I_{\pi g}$ - open set but not soft ∗-open set. Hence (X, τ, E, I) is soft $I_{\pi g}$ - Alexandroff space but not soft I - Alexandroff space.

Theorem: 4.7

Let (X, τ, E, I) be a soft ideal topological space and (M, E) ⊆ X. If X is a soft $I_{\pi g}$ - Alexandroff space and (M, E) is soft closed then (M, E) is a soft $I_{\pi g}$ - Alexandroff space.

Proof:

Let (X, τ, E, I) be a soft ideal topological space and (M, E) ⊆ X be a soft closed set. Suppose that {{(S_i , E): i ∈ I} is a family of soft open sets in ((M, E), τ_M) .We take (S, E) =$\bigcap_{i\in I}(S_i$, E). It follows that (S_i , E) = (M, E) ∩ (K_i , E) where (K_i , E) is a soft open set in (X, τ, E, I) for each i ∈ I. Let (N, E) ⊆ (M, E) be a soft closed set in ((M, E), τ_M) and (N, E) ⊆ (S, E). This shows that (N, E) is a soft closed set in (X, τ, E, I) and (N, E) ⊆ $\bigcap_{i\in I}(K_i$, E). Since X is $I_{\pi g}$ - Alexandroff space then (N, E) ⊆ int^*($\bigcap_{i\in I}(K_i$, E)). Also we have (M, E) ∩ int^*($\bigcap_{i\in I}(K_i$, E)) ⊆ (S, E).Since (M, E) ∩ int^*($\bigcap_{i\in I}(K_i$, E)) is a soft ∗-open set in (M, E) then (N, E) ⊆ int_M^*(S, E). This implies that (S, E) is soft $I_{\pi g}$ - open in (M, E). Hence (M, E) is a soft $I_{\pi g}$ - Alexandroff space.

Theorem: 4.8

Let (X, τ, E, I) be a soft ideal topological space and (S, E) ⊆ X be soft $I_{\pi g}$ - closed. If f: (X, τ, E, I) ⟶ (Y, σ, E, J) is a soft continuous and soft ∗- closed function then f(S, E) is a soft $I_{\pi g}$ - closed set in Y.

Proof:

Suppose that (S, E) ⊆ X is a soft $I_{\pi g}$ - closed set and f: (X, τ, E, I) ⟶ (Y, σ, E, J) is a soft continuous and soft ∗-closed function. Let f(S, E) ⊆ (K, E) where (K, E) is soft open in Y. It follows that (S, E) ⊆ f^{-1}(K, E). Since f is a soft continuous function and (S, E) is a soft $I_{\pi g}$ - closed set, then we have cl^*(S, E) ⊆ f^{-1}(K, E). Moreover f(cl^*(S, E)) ⊆ f(f^{-1}(K, E)) ⊆ (K, E). Since f is a soft ∗-closed function, cl^*(f(S, E)) ⊆ cl^*f(cl^*(S, E)) = f(cl^*(S, E)) ⊆ (K, E). This implies that cl^*(f(S, E)) ⊆ (K, E). Hence f(S, E) is a soft $I_{\pi g}$ - closed set in Y.

Theorem: 4.9

Let f: (X, τ, E, I) ⟶ (Y, σ, E, J) be a soft continuous and soft ∗- closed surjective function. If (X, τ, E, I) is a soft $I_{\pi g}$ - Alexandroff space then (Y, σ, E, J) is a soft $I_{\pi g}$ - Alexandroff space.

Proof:

Suppose that f: (X, τ, E, I) ⟶ (Y, σ, E, J) is a soft continuous and soft ∗- closed surjective function. Let (X, τ, E, I) be a soft $I_{\pi g}$ - Alexandroff space and {(M_i, E) : i ∈ I} be a family of soft closed sets in (Y, σ, E, J). Since f is a soft continuous function, then (K, E) = $\bigcup_{i\in I} f^{-1}$(M_i, E) is a soft $I_{\pi g}$ - closed set in X. We take (M, E) = $\bigcup_{i\in I}(M_i\,,E)$. This implies that f (K, E) = f($\bigcup_{i\in I} f^{-1}$(M_i, E)) = (M, E) is soft $I_{\pi g}$ -closed. Hence Y is a soft $I_{\pi g}$ - Alexandroff space.

Theorem: 4.10

Let (X, τ, E, I) be a soft ideal topological space and (A, E) ⊆ X. If (A, E) is soft ∗-dense in itself and soft $I_{\pi g}$ - closed in X then (A, E) is soft πg- closed.

Proof:

Suppose (A, E) is soft ∗-dense in itself and soft $I_{\pi g}$ - closed in X. If (U, E) be any soft π-open set containing (A, E), then cl^*(A, E) ⊆ (U, E).Since (A, E) is soft ∗-dense in itself, cl(A, E) ⊆ (U, E). Hence (A, E) is soft πg- closed.

Corollary: 4.11

If (X, τ, E, I) be a soft ideal topological space where I = {ϕ} then (A, E) is soft $I_{\pi g}$ - closed set if and only if (A, E) is soft πg- closed.

Theorem: 4.12

Let (X, τ, E, I) be a soft ideal topological space. Suppose that every subset of X is soft ∗-dense in itself. Then the following properties are equivalent:

(1) (X, τ, E, I) is a soft $I_{\pi g}$ - Alexandroff space

(2) (X, τ, E, I) is a soft πg - Alexandroff space

Proof:

Since every subset is soft ∗-dense in itself, then by theorem: 4.10, X is a soft $I_{\pi g}$ - Alexandroff space if and only if X is a soft πg - Alexandroff space.

Theorem: 4.13

For a soft ideal topological space (X, τ, E, I) where I = {ϕ}, the following properties are equivalent:

(1) (X, τ, E, I) is a soft $I_{\pi g}$ - Alexandroff space

(2) (X, τ, E, I) is a soft πg - Alexandroff space

Proof:

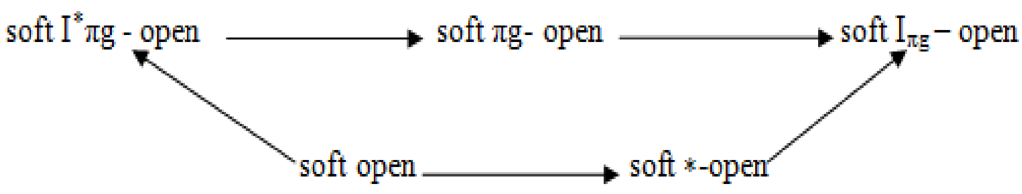
Let (X, τ, E, I) be a soft ideal topological space where I = {ϕ} and (S, E) ⊆ X. Since (S, E) is soft $I_{\pi g}$ - closed set if and only if X is a soft πg -closed. Then X is soft $I_{\pi g}$ - Alexandroff space if and only if X is a soft πg - Alexandroff space.

Definition: 4.14

A subset (S, E) of a soft ideal space (X, τ, E, I) is said to be soft $I^*\pi g$ - closed, if cl(A, E) ⊆(U, E) whenever (S, E) ⊆ (U, E) and (U, E) is soft ∗π-open. The complement of (S, E) is soft $I^*\pi g$ -open set.

Remark: 4.15

Let (X, τ, E, I) be a soft ideal topological space. Then the following diagram holds for a subset (S,E) of X.



Example: 4.16

X = {a, b, c, d} and E = {e_1, e_2}.

(F_1, E) = {(e_1, {c}), (e_2, {a})}

(F_2, E) = {(e_1, {d}), (e_2, {b})}

(F_3, E) = {(e_1, {c, d}), (e_2, {a, b})}

(F_4, E) = {(e_1, {a, d}), (e_2, {b, d})}

(F_5, E) = {(e_1, {b, c, d}), (e_2, {a, b,c})}

(F_6, E) = {(e_1, {a, c, d}), (e_2, {a, b, d})}

where (F_1, E) , (F_2, E) , (F_3, E) , (F_4, E) , (F_5, E) , (F_6, E) are soft sets over X and $\tau = \{\tilde{X}, \tilde{\phi}$, ($F_1$, E) , ($F_2$, E) , ($F_3$, E) , ($F_4$, E) , ($F_5$, E) , ($F_6$, E) } is a soft topology over X.

Let I = {$\tilde{\phi}$, (I_1, E), (I_2, E), (I_3, E)} be a soft ideal over X, where

(I_1, E) = {(e_1, {a}), (e_2, ϕ)}

(I_2, E) = {(e_1, {b}), (e_2, ϕ)}

(I_3, E) = {(e_1, {a, b}), (e_2, ϕ)}

Take (G, E) = {(e_1, {a}), (e_2, {d})} which is soft $I_{\pi g}$ - open but not soft ∗-open.

Example: 4.17

X = {a, b, c, d} and E = {e_1, e_2}.

(F_1, E) = {(e_1, {a}), (e_2, {c})}

(F_2, E) = {(e_1, {c}), (e_2, {a})}

(F_3, E) = {(e_1, {a, c}), (e_2, {a, c})} where (F_1, E), (F_2, E), (F_3, E) are soft sets over X and $\tau = \{\tilde{X}, \tilde{\phi}$, ($F_1$, E) , ($F_2$, E) , ($F_3$, E) $\}$ is a soft topology over X.

Let I = {$\tilde{\phi}$, (I_1, E), (I_2, E), (I_3, E)} be a soft ideal over X, where

(I_1, E) = {(e_1, ϕ), (e_2, {c})}

(I_2, E) = {(e_1, ϕ), (e_2, {a})}

(I_3, E) = {(e_1, ϕ), (e_2, {a,c})}

Take (H, E) = {(e_1, {a, c}), (e_2, {a, c})} which is soft $I^*\pi g$ - open but not soft open.

Theorem: 4.18

For a subset (S, E) of a soft ideal topological space (X, τ, E, I), (S, E) is soft $I^*\pi g$- open if and only if (N, E) ⊆ int (S, E) whenever (N, E) ⊆ (S, E) and (N, E) is soft ∗π- closed in X.

Proof:

Let (S, E) be a soft $I^*\pi g$- open set in (X, τ, E, I). Suppose that (N, E) ⊆ (S, E) and (N, E) is soft ∗π- closed in X. It follows that X − (S, E) ⊆ X − (N, E) and X − (N, E) is soft ∗π- open set in X.

Since X − (S, E) is soft $I^*\pi g$- closed, then cl(X − (S, E)) ⊆ X − (N, E). We have cl(X − (S, E)) = X − int(S, E) ⊆ X − (N, E). Thus (N, E) ⊆ int(S, E). The converse is similar.

Theorem: 4.19

Let (X, τ, E, I) be a soft ideal topological space. The following properties are equivalent:

(1) (X, τ, E, I) is a soft $I_{\pi g}$ - Alexandroff space

(2) Any intersection of soft $I^*\pi g$- open sets in (X, τ, E, I) is soft $I_{\pi g}$ - open.

Proof:

(1) ⟹ (2)

Let (X, τ, E, I) be a soft $I_{\pi g}$ - Alexandroff space. Suppose that {(S_i , E): i ∈ I} is a family of soft $I^*\pi g$- open sets. We take (S, E) =$\bigcap_{i\in I}(S_i$, E). Let (K, E) ⊆ X be a soft closed set and (K, E) ⊆ (S, E). We have (K, E) ⊆ (S_i, E) for each i ∈ I. Since (S_i, E) is soft $I^*\pi g$- open set for each i ∈ I, (K, E) ⊆ int^*(S_i, E) for each i ∈ I. We take (M, E) =$\bigcap_{i\in I} int(M_i$, E). Since X is soft $I_{\pi g}$ - Alexandroff space then (M, E) = $\bigcap_{i\in I} int(M_i$, E) is soft $I_{\pi g}$ - open. Since (M, E) = $\bigcap_{i\in I} int(M_i$, E) is soft soft $I_{\pi g}$ - open and (K, E) ⊆ (M, E) then (K, E) ⊆ int^*(S, E). It follows that (S, E) is soft $I_{\pi g}$ - open.

(2) ⟹ (1)

Suppose that any intersection of soft $I^*\pi g$- open sets in X is soft $I_{\pi g}$ - open. Since every soft open set is soft $I^*\pi g$- open, any intersection of soft open sets in X is soft $I_{\pi g}$ - open.

Thus X is a soft $I_{\pi g}$ - Alexandroff space.

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Theorem: 4.20

Let (X, τ, E, I) be a soft ideal topological space. The following properties are equivalent:

(1) (X, τ, E, I) is a soft $I_{\pi g}$ - Alexandroff space

(2) Any union of soft $I^*\pi g$- closed sets in (X, τ, E, I) is soft πg-closed.

Proof:

The proof follows from Theorem: 4.19.

Theorem: 4.21

The product of two soft $I_{\pi g}$ -Alexandroff spaces is soft $I_{\pi g}$ - Alexandroff space.

Proof:

Let { (W_i, E) : i ∈ I } be a collection of soft open sets in X × Y and let (W, E) be its intersection. Suppose (W, E) is not a soft $I_{\pi g}$ - open set. Then there exists (x_e , y_e) ∈ (W, E) and soft open sets (U, E) ⊆ X and (V, E) ⊆ Y with (x_e , y_e) ∈ U × V and (U × V) ∩ int(W, E) = ϕ. For each i ∈ I there exists soft open sets (F_i, E) ⊆ X and (G_i, E) ⊆ Y with (x_e , y_e) ∈ (F_i × G_i, E) ⊆ (W_i , E). Since X and Y are soft $I_{\pi g}$ - Alexandroff spaces, there exist a non-empty soft $I_{\pi g}$ -open sets (H, E) ⊆ X and (K, E) ⊆ Y with (H, E) ⊆ (U, E) ∩ ($\bigcap_{i\in I}(F_i$, E)) and (K, E) ⊆ (V, E) ∩ ($\bigcap_{i\in I}(G_i$, E)). Clearly (H_i × K_i, E) ⊆ (U × V) ∩ int(W, E) which is a contradiction. Hence X × Y is soft $I_{\pi g}$ - Alexandroff space.

Definition: 4.22

Let (X, τ, E, I) be a soft ideal topological space and it is said to be a soft F^*- space if every soft open subset of X is soft ∗-closed.

Theorem: 4.23

Let (X, τ, E, I) be a soft ideal topological space. If X is a soft T_1- space and soft F^*- space then X is a soft discrete ideal space with respect to τ^*.

Proof:

Suppose that (X, τ, E, I) is a soft T_1- space and soft F^*- space. Since X is a soft T_1- space, then $\{x_e\}$ is a soft closed set for every x_e ∈ X. Since X is a soft F^*- space, then $\{x_e\}$ is a soft ∗-open set for every x_e ∈ X. It follows that X is a soft discrete ideal space with respect to τ^*.

Theorem: 4.24

Let (X, τ, E, I) be a soft ideal topological space. Then the following properties are equivalent:

(1) (X, τ, E, I) is a soft F^*- space

(2) Every soft subset of X is a soft $I_{\pi g}$ - closed set

Proof:

The proof follows from theorem: 2.13

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Theorem: 4.25

Let (X, τ, E, I) be a soft ideal topological space. If X is a soft F^*- space, then X is a soft $I_{\pi g}$ -Alexandroff space.

Proof:

Suppose that (X, τ, E, I) is a soft F^*- space. By theorem: 4.24, every subset of X is soft $I_{\pi g}$ -closed set. This implies that X is a soft $I_{\pi g}$ -Alexandroff space.

Definition: 4.26

A soft topological space (X, τ, E) is said to be a soft R_0-space, if $cl(\{x_e\}) \subseteq (U, E)$ for each $x_e \in X$ and each soft open set (U, E) with $x_e \in (U, E)$.

Theorem: 4.27

Let (X, τ, E, I) be a soft ideal topological space. If X is a soft R_0-space and soft $I_{\pi g}$ -Alexandroff space then X is a soft F^*- space.

Proof:

Let (X, τ, E, I) be a soft R_0-space and soft $I_{\pi g}$ -Alexandroff space. Suppose that (S, E) ⊆ X is a soft open set. Since X is a soft R_0-space, then we have $cl(\{x_e\}) \subseteq (S, E)$ for each $x_e \in (S, E)$. This implies that $(S, E) = \bigcup_{x \in S} cl(\{x_e\})$. Since X is a soft $I_{\pi g}$ -Alexandroff space, then (S, E) is a soft $I_{\pi g}$ - closed set. Since (S, E) ⊆ (S, E) and (S, E) is a soft $I_{\pi g}$ - closed set, then $cl^*(S, E) \subseteq (S, E)$. This implies that (S, E) is soft ∗π-closed set. We have every soft ∗π-closed set is soft ∗-closed set. Hence X is a soft F^*- space.

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