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Construction of Super-Saturated Designs Through Cyclic PBIBD(2)'S

KEYWORDS: Cyclic Partially balanced incomplete block design, Super saturated design.

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ABSTRACT *Experimenter has to minimize the number of design points in a design by identifying the effects of active factor combinations for efficient utilization of resources. Super-saturated design is a factorial design having the more number of factors than the number of design points. Several researchers made attempts on the construction of super-saturated designs. In this paper, new series of construction of super-saturated designs using Cyclic partially balanced incomplete block designs was proposed. The construction is also illustrated with suitable example.*

1. INTRODUCTION

In most practical situations of the Design and Analysis of Experiments, it is observed that out of a large number of potential factors, relatively few of the factors are actually effective. Such effects are called the effects of sparsity. The basic approach is to identify these few factors in an efficient way. In this situation, knowledge of each and every main effect is not useful since insignificant factors are not usually of interest.

Hence, the experimenter has to minimize the number of design points to identify the active factors for efficient utilization of resources and minimization of cost and time. If, the number of design points is more, then the problem of reduction of dimensionality occurs. In this aspect, instead of reduction of design points, some optimum design constructions may be proposed. In these situations, a super-saturated design plays a key role which reduces the experimental cost, time significantly.

2. LITERATURE REVIEW ON SUPER-SATURATED DESIGNS

Satterthwaite (1959) initially made an attempt to construct saturated designs randomly and suggested random balance designs. Booth and Cox (1962) initially proposed a systematic method for the construction of super-saturated designs, which are factorial designs in which the number of factors exceeds the number of design points and also computed $E(s^2)$ criterion. After Booth and Cox (1962), no attempts were made till Lin (1993). Later Several researchers made attempts on the construction of super-saturated designs with their $E(s^2)$ optimality. Authors who worked in this direction are: Lin (1993), Nguyen (1996), Tang and Wu (1997), Deng, Lin and Wang (1999), Fang, Lin and Ma (2000), Liu and Zhang (2000), Lu and Sun(2001), Butler, Mead, Eskridge and Gilmour (2001), and Yamada and Lin (2002), Li and Lin (2003), Liu and Dean (2004), Fang, Gennian and Liu (2004), Aggarwal and Gupta (2004), Xu and Wu (2005), Koukouvinov, Mantos and Mylona (2007), Jones, Lin and Nachtsheim (2008) , Nguyen and Cheng (2008), Sun, Lin and Liu (2011) etc.

Lin (1993), Li and Wu (1997) motivated towards the construction of super-saturated designs through column wise, pair wise exchanges. They differ from the k-exchange algorithms in two aspects, one is, they exchange columns instead of rows of the design matrix and another one is, they employ a pair wise adjustment in the search for a better column.

Tang and Wu (1997) proposed the construction procedure for two –level super-saturated designs by using Hadamard matrices with E (s^2) optimality and found the lower bound for E (s^2).

Fang, Lin, and Ma (2000) proposed a construction procedure by embedding a saturated orthogonal design into a uniform design of the same row size. They adopt the collapsing method from Addelman. The basic idea of the construction method is, to collapse a multi-level factor into several low-level factors. Where they collapse U-type uniform designs. They proposed five criteria for comparing multi-level super-saturated designs.

Liu and Zhang (2000) proposed a general algorithm for the construction of $E(s^2)$ optimal super-saturated designs from cyclic BIBD. The general formula for the lower bound of a super-saturated design with 'm' factors with 'n' design points is $[n^2(m-n+1)] / [(n-1)(m-1)]$. Lu and Sun (2001) proposed two criteria denoted by $E(s^2)$ and $\max\{(s^2)\}$ in the construction of multi-level super-saturated designs. Eskridge, Gilmour, Mead, Butler and Travnicek (2001) and Liu and Dean (2002) also considered cyclic generation of E (s^2)-optimal and nearly optimal super-saturated designs.

Fang, Gennian and Liu (2002) proposed a discrete discrepancy as a measure of uniformity for super-saturated designs and a lower bound of this discrepancy is obtained as a benchmark of design uniformity and also proposed construction procedure for uniform super-saturated by using resolvable BIBD along with their properties.

Yamada and Lin (2002) suggested a construction method for mixed-level super-saturated designs consisting of two-level and three-level columns. The chi-square statistics is used for a measure of dependency of the design columns. The dependency properties for the newly constructed designs are derived and discussed.

Liu and Dean (2004) proposed a class of super-saturated designs called k- Circulant super-saturated designs which can be obtained from cyclic development of a generator. This method is a generalization of Plackett-Burman, who introduced the use of cyclic generators for constructing orthogonal saturated designs.

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Fang Kaitai, GE Gennian and Liu Minqian (2004) proposed a combinatorial approach called the packing method. They studied the connection between orthogonal arrays and resolvable packing designs for constructing optimal supersaturated designs and properties of the resulting designs are also proposed.

Xu and Wu (2005) proposed construction methods for multi-level super-saturated designs inspired by Addelman-Kempthorne of orthogonal arrays and also proposed new lower bound for multi-level super-saturated designs.

Koukouvinov, Mantas and Mylona (2007) proposed mixed-level super-saturated designs by using supplementary difference sets with respect to the $E(f_{NOD})$ criterion. Nguyen and Cheng (2008) suggested the construction procedure for super-saturated designs from BIBD and also from regular graph designs when BIBD do not exist.

Sun, Lin and Liu (2011) proposed equivalent conditions for two columns to be fully aliased and consequently proposed methods for constructing $E(f_{NOD})$ and chi-square optimal mixed level super-saturated designs without fully aliased columns via equidistant designs and difference matrices.

Liu and Liu (2012) generalized a method proposed by Liu and Lin to the mixed level case and also they proposed two new practical methods for constructing optimal mixed level supersaturated designs.

Ameen saheb and Bhatracharyulu (2013, 2014) proposed two level supersaturated designs by using cyclic resolvable designs and also they proposed two new methods for constructing supersaturated designs using row-column and cyclic resolvable designs.

In this paper, an attempt is made to construct supersaturated designs using cyclic PBIBD(2).

3. CONSTRUCTION OF SUPERSATURATED DESIGNS FROM CYCLIC PBIBD (2)'S

The step-by-step procedure for the construction of supersaturated designs by using cyclic PBIBD (2) as follows

Step1: Construct a cyclic PBIBD(2) with parameters v=4t+1(prime), b=sv, k=2t, r=sk, n1= n2 = 2t, $p^1_{11}=t-1$, $p^2_{11}=t$ and λ1=s p^1_{11} and λ2=s p^2_{11} using initial blocks and assign +1's to all the treatments in the initial block and assign -1's to all other treatments.

Step2: Transpose the first generating row vector to get a column vector and generate the first (2t-1) columns by cyclic permutation from the first initial block.

Step3: Repeat Step2 with second, third…….etc. generating row vectors.

Step4: Finally, put +1 at the upper end of each of the generating columns.

Step5: The resulting design matrix is called supersaturated design with (2t-1) design points and (8t-4) treatments.

Example 3.1: Consider a cyclic PBIBD (2) with parameters v=13, b=52, r=12, k=3, λ_1=0, λ_2=4, n_1= n_2=6, p^1_{11}=2, p^2_{11}=3 and initial blocks for this design are (1,7,9), (1,3,9), (1,7,9), (2,4,9).Generate the columns 2 to 13 from the first column i.e. first initial block and generate the columns 15 to 26 from the 14th column i.e. second initial block., generate the columns 28 to 51 form the 27 th column etc. Obtain the supersaturated design with 14 design points and 52 treatments by augmenting one row with all +1's.

Table1 about here

Table1: Generating vectors of Cyclic PBIBD (2)-Based Supersaturated Designs

S.NO	V	b	r	k	λ_1	λ_2	n_1	n_2	p^1_{11}	p^2_{11}	Generating vectors
1	13	52	12	3	0	4	6	6	2	3	(1, 7,9), 1,3,9), (1,7,12),(2,4,9)
2	13	65	15	3	0	5	6	6	2	3	(2,3,12), (3,6,7), (0,9,12),(6,9,10), (4,7,8)
3	13	65	15	3	1	4	6	6	2	3	(0,1,6),(3,7,9),(1,3,11),(1,7,12),(0,5,7)
4	13	39	12	4	1	5	6	6	2	3	(0,2,8,10), (0,5,7,11), (4,6,11,12)
5	13	52	16	4	3	5	6	6	2	3	(0,6,11,12), (2,4,7,8), (4,8,10,12),(0,2,5,8)
6	13	65	20	4	3	7	6	6	2	3	(3,4,6,7),(4,8,9,12),(8,9,10,12),(0,6,9,12),(3,8,9,12)
7	13	65	20	4	4	6	6	6	2	3	(1,2,3,10),(7,10,11,12),(0,3,7,11),(6,9,11,12), (1,2,5,11)
8	13	39	15	5	3	7	6	6	2	3	(0,2,4,8,10),(4,5,6,10,12), (1,3,4,9,11)
9	13	39	15	5	4	6	6	6	2	3	(0,1,3,4,12), (0,1,2,5,8), (0,4,6,9,10)
10	13	39	18	6	5	10	6	6	2	3	(1,3,6,8,9,11), (0,2,4,6,8,10),(1,2,3,7,8,9)
11	17	68	12	3	0	3	8	8	3	4	(5,6,7),(6,14,15),(4,6,8),(2,6,10)
12	17	68	12	3	1	2	8	8	3	4	(3,14,16),(5,7,8),(3,11,15),(6,13,14)
13	17	68	16	4	1	5	8	8	3	4	(4,5,10,16),(0,5,7,10),(4,7,10,16),(0,4,7,14)
14	17	34	10	5	2	3	8	8	3	4	(2,4,5,8,14), (4,5,9,11,14)
15	17	68	20	5	4	6	8	8	3	4	(5,8,9,10,11), (1,5,9,14,16),(3,5,9,10,12),(0,1,6,9,10)

16	17	68	24	6	5	10	8	8	3	4	(2,4,7,9,11,14),(0,3,5,6,11,12),(3,9,10,13,15,16),(1,5,10,12,14,15)
17	17	68	24	6	6	9	8	8	3	4	(0,2,6,8,15,16),(5,6,9,10,11,14),(3,4,5,6,14,16),(1,5,10,12,14,15)
18	17	68	24	6	7	8	8	8	3	4	(3,4,5,8,9,16), (3,5,6,10,13,15), (3,4,6,9,10,12),(1,5,7,12,15,16)
19	17	34	16	8	5	9	8	8	3	4	(1,3,6,8,9,12,13,15), (1,3,4,8,9,11,14,15)
20	29	29	7	7	1	2	14	14	6	7	(0, 1,9,12,15,17,19)
21	29	87	21	7	4	5	14	14	6	7	(3,5,7,13,15,18,25),(6,7,11,19,22,25,28),(2,4,18,19,22,23,24)
22	29	145	35	7	7	8	14	14	6	7	(1,3,15,16,23,25,27), (3,6,9,12,14,15,19),(3,7,17,18,22,24,25), (1,5,6,9,18,19,24),(1,3,4,12,14,16,24)
23	29	29	8	8	1	3	14	14	6	7	(2,5,13,14,16,24,26,28)
24	29	58	16	8	3	5	14	14	6	7	(3,4,13,21,25,26,27,28),(3,6,8,9,15,19,24,28)
25	29	116	32	8	7	9	14	14	6	7	(1,7,9,14,15,19,21,27), (0,1,16,20,21,26,27,28),(7,9,13,14,20,23,24,27), 92,6,13,18,22,23,26,28)
26	37	111	12	4	0	2	18	18	8	9	(7,10,14,35),(1,17,26,27),(20,27,30,31)
27	37	111	18	6	2	3	18	18	8	9	(3,5,6,11,19,25), (13,15,24,25,28,32),(4,10,17,19,31,36)
28	37	37	10	10	2	3	18	18	8	9	(2,6,13,14,16,17,18,23,26,32)
29	41	205	20	4	1	2	20	20	9	10	(3,15,21,34),(8,17,23,24),(2,6,13,35),(15,18,20,35),(3,17,20,31)
30	41	205	25	5	1	4	20	20	9	10	(3,4,14,32,38),(8,11,14,27,38),(0,14,22,29,36),(5,18,20,24,31), (9,12,21,33,36)
31	41	205	25	5	2	3	20	20	9	10	(15,27,28,36,39),(12,14,19,25,35),(10,26,28,29,36),(13,15,32,36,40), (18,23,27,32,33)
32	53	53	14	14	3	4	26	26	12	13	(2,10,16,18,20,21,23,27,28,37,40,42,43,51)

4. Concluding Remarks: Our proposed algorithm was used to construct Supersaturated designs using Cyclic PBIBD (2)'s with parameters t=3, 4,7,9,10,13, $3 \le k \le 2t$, $K \le r \le 5k$, $\lambda_1 \le \lambda_2 \le 10$ Such that these designs are not available in the catalogs of experimental designs.

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