



Thermo dynamical Aspects of Non-Static Plane Symmetric Universe with $f(R, T)$ Gravity

KEYWORDS

Non-Static Universe; $f(R, T)$ gravity; cosmological constant.

S.R.Bhojar

Department of Mathematics,
Phulsing Naik Mahavidyalaya, Pusa-
445216.India

V. R. Chirde

Department of Mathematics,
G. S. G. Mahavidyalaya,
Umardhed-445206.India

S. H. Shekh

Department of Mathematics, Dr.
B. N. College of Engg. & Tech.,
Yavatmal-445001.India

ABSTRACT In the present study we investigate $f(R, T)$ gravity with cosmological constant term. we obtain the gravitational field equation in the metric formalism, which follow from the covariant divergence of the stress-energy tensor for the specific choice of $f(R, T) = f_1(R) + f_2(T)$ with an individual superior functions $f_1(R) = R$ and $f_2(T) = T$. We find the solution of the field equations by allowing the law of variation of Hubble's parameter which yields a constant value of the deceleration parameter. Moreover, some physical, geometrical and thermodynamical properties of the universe are discussed in detail.

1. Introduction:

Recent theoretical cosmological observations [1-3] suggested that the expansion of the universe is accelerating. In Einstein's theory of general relativity, in order to enclose such acceleration, one needs to introduce a component to the matter distribution of the universe by a large negative pressure. This component is usually referred as Dark Energy (DE). Also same observations indicate that our universe is flat and at present consists of approximately $2/3$ DE and $1/3$ dark matter source. There are many radically different models for DE to fit the current observations such as quintessence [4], phantom [5], tachyon [6], chaplygin gas [7].

One of the most important quantity to describe the features of DE models is the equation of state (EoS) parameter (ω), which is the ratio of pressure (p) to the energy density (ρ) of DE, defined by $\omega = \frac{p}{\rho}$.

Usually EoS parameter is assumed to be a constant with the values -1 , 0 , $-1/3$ and $+1$ for vacuum, dust, radiation and stiff matter dominated universe respectively. However, latest observation [8] indicated that ω is not a constant, either it is a function of time or redshift [9]. In recent years, many authors [10-17] have shown keen interests in studying the DE universe with various contexts. In view of the late time acceleration and existence of the universe of DE and dark matter, several modified theories of gravity have been developed and studied. In particular, $f(R)$ and $f(T)$ to Einstein-Hilbert Lagrangian (where R being Ricci scalar curvature and T be the Torsion). Some authors [18-21] who have investigated numerous aspects of modified $f(R)$ and $f(T)$ gravity models to show the late time acceleration and early inflation. A further generalization of $f(R)$ gravity theory is known as $f(R, T)$ of gravity proposed by Harko et al. [22] where as usual R be the Ricci scalar and T is the trace of the energy-momentum tensor.

Within the framework of $f(R, T)$ gravity Harko et al. [22] have investigated several aspects of this theory including FRW dust universe. At the apparent horizon of FRW universe the laws of thermodynamic in this modified theory of gravity had discussed by Sharif and Zubair [23]. Jamil et al. [24] reconstructed some cosmological model in $f(R, T)$ gravity and it was accomplished that the dust fluid reproduced Λ CDM model, phantom non-phantom era and the phantom cosmology. Chandel and Ram [25] generated new classes of solutions of field equations for an anisotropic Bianchi type-III cosmological model with perfect fluid in $f(R, T)$ gravity. A new class of Bianchi type cosmological models in $f(R, T)$ gravity has obtained by Chaubey et al. [26]. Katore et al. [27] investigated some cosmological model with DE source in $f(R, T)$ gravity. Very recently, Sahoo et al. [28] and Chirde and Shekh [29] investigated an axially symmetric space-time in the presence of a perfect fluid source and non-static plane symmetric space-time

filled with DE within the frame work of same gravity respectively. Shamir [30] have obtained the exact solutions of Bianchi type V metric in modified $f(R, T)$ gravity.

2. Gravitational Field Equations of $f(R, T)$ Gravity:

We assume that the action for modified theory of gravity of the following form Harko et al. [22]

$$s = \frac{1}{16\pi} \int f(R, T) \sqrt{-g} d^4x + \int L_m \sqrt{-g} d^4x, \quad (2.1)$$

where $f(R, T)$ is an arbitrary function of the Ricci scalar R , and T be the trace of the stress energy tensor of the matter.

We define the stress energy tensor of matter as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{\mu\nu}}, \quad (2.2)$$

and its trace given by $T = g^{\mu\nu} T_{\mu\nu}$.

Varying the action (2.1) with respect to the metric tensor components $g^{\mu\nu}$, the gravitational field equation of $f(R, T)$ gravity is obtained as

$$f_R(R, T) R_{ij} - \frac{1}{2} f(R, T) g_{ij} + f_R(R, T) (g_{ij} \nabla^i \nabla_j - \nabla_i \nabla_j) = 8\pi T_{ij} - f_T(R, T) T_{ij} - f_T(R, T) \Theta_{ij}, \quad (2.3)$$

where $f_R = \frac{\delta f(R, T)}{\delta R}$, $f_T = \frac{\delta f(R, T)}{\delta T}$ and $\Theta_{ij} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$. Here ∇_i is the covariant derivative and T_{ij} is usual

matter energy momentum tensor derived from the Lagrangian L_m

The contraction of equation (2.3) yields

$$f_R(R, T) R + 3\Pi f_R(R, T) - 2f(R, T) = (8\pi - f_T(R, T))T - f_T(R, T)\Theta \text{ with } \Theta = g^{\mu\nu} \Theta_{\mu\nu}. \quad (2.4)$$

Combining equation (2.3) and (2.4) and eliminating the $\Pi f_R(R, T)$ term, we get

$$f_R(R, T) \left(R_{\mu\nu} - \frac{1}{3} R g_{\mu\nu} \right) + \frac{1}{6} f(R, T) g_{\mu\nu} = (8\pi - f_T(R, T)) \left(T_{\mu\nu} - \frac{1}{3} T g_{\mu\nu} \right) - f_T(R, T) \left(\Theta_{\mu\nu} - \frac{1}{3} \Theta g_{\mu\nu} \right) + \nabla_\mu \nabla_\nu f_R(R, T), \quad (2.5)$$

where $\Theta_{\mu\nu} = -2T_{\mu\nu} - p g_{\mu\nu}$.

It is mentioned here that these field equation depends on the physical nature of the matter field. Many theoretical models corresponding to different matter contributions for $f(R, T)$ gravity are possible. There are three classes of these models

$$f(R, T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases} \quad (2.6)$$

Bearing in mind all above classes Harko et al. [22] derived the gravitational field equations that may be relevant in explaining some open challenges of cosmology and astrophysics, also he had demonstrated the possibility of reconstruction of arbitrary FRW cosmologies by an proper choice of a stress tensor function $f(T)$. Alvarenga et al. [31] studied the viability of $f(R, T)$ gravity giving to the energy conditions. He presented the general formalism of $f(R, T)$ theory by putting out $f(R, T) = f_1(R) + f_2(T)$, where $f_1(R)$ and $f_2(T)$ be the function of curvature and the trace of the energy momentum tensor. Also, by substituting suitable constraint on the input parameter he obtained $R + 2f(T)$ type model which satisfy the energy condition. In this paper we have focused on the second class, i.e.

$$f(R, T) = f_1(R) + f_2(T). \quad (2.7)$$

The gravitational field equation (2.3) becomes,

$$f_1'(R)R_{\mu\nu} - \frac{1}{2}f_1(R)g_{\mu\nu} + (g_{\mu\nu}\Pi - \nabla_\mu \nabla_\nu)f_1'(R) = 8\pi T_{\mu\nu} + f_2'(R)T_{\mu\nu} + \left(f_2'(T)p + \frac{1}{2}f_2(T)\right)g_{\mu\nu}. \quad (2.8)$$

where the prime denotes the differentiation with respect to the argument.

We consider the particular form of the functions $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T$, where λ_1 and λ_2 are arbitrary constants.

$$\text{Here we take } \lambda_1 = \lambda_2 = \lambda \text{ so that } f(R, T) = \lambda(R + T). \quad (2.9)$$

Above equation (2.8) can be written as

$$\lambda R_{\mu\nu} - \frac{1}{2}\lambda(R, T)g_{\mu\nu} + (g_{\mu\nu}\Pi - \nabla_\mu \nabla_\nu)\lambda = 8\pi T_{\mu\nu} + \lambda T_{\mu\nu} + \lambda(2T_{\mu\nu} + pg_{\mu\nu}), \quad (2.10)$$

setting $(g_{\mu\nu}\Pi - \nabla_\mu \nabla_\nu)\lambda = 0$ we get,

$$\lambda G_{\mu\nu} = 8\pi T_{\mu\nu} + \lambda T_{\mu\nu} + \left(\lambda p + \frac{1}{2}\lambda T\right)g_{\mu\nu}, \quad (2.11)$$

this could be rearranged as

$$G_{\mu\nu} - \left(p + \frac{1}{2}T\right)g_{\mu\nu} = \left(\frac{8\pi + \lambda}{\lambda}\right)T_{\mu\nu}. \quad (2.12)$$

We have the Einstein field equation with cosmological constant

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = -8\pi T_{\mu\nu}. \quad (2.13)$$

We choose a negative small value for the arbitrary λ so that we have the same sign of the *RHS* of equation (2.12) and (2.13), we keep this choice of λ throughout. The term $\left(p + \frac{1}{2}T\right)$ can now be regarded as a cosmological constant. So, in the framework of the $f(R,T)$ gravity, we can get the cosmological constant as a function of the EoS parameter ω , the energy density ρ and the trace of the stress-energy tensor. But since ω and ρ are already included in T so we could just write

$$\Lambda = \Lambda(T) = p + \frac{1}{2}T. \quad (2.14)$$

The dependence of the cosmological constant Λ on the trace of the energy momentum tensor T has been proposed before by Poplawski [32] where the cosmological constant in the gravitational Lagrangian is a function of the trace of the energy-momentum tensor, considering the perfect fluid the trace of our model is $T = -3p + \rho$.

3. Metric and Field equations:

We consider a Riemannian space-time described by the line element

$$dS^2 = e^{2h} \left(dt^2 - dr^2 - r^2 d\theta^2 - s^2 dz^2 \right), \quad (3.1)$$

where r , θ , z are the usual cylindrical polar coordinates and h & s are the metric coefficients which is a functions of time t alone. It is well known that this line element is plane symmetric.

The corresponding Ricci scalar is given by

$$R = e^{-2h} \left\{ 6\ddot{h} + 2\frac{\ddot{s}}{s} + 4\dot{h}^2 + 6\frac{\dot{s}\dot{h}}{s} \right\}. \quad (3.2)$$

The energy momentum tensor T_ν^μ for the perfect fluid distribution is taken as

$$T_\nu^\mu = (p + \rho)u^\mu u_\nu - pg_\nu^\mu, \quad (3.3)$$

satisfying the EoS

$$p = \omega\rho, \quad 0 \leq \omega \leq 1, \quad (3.4)$$

together with comoving co-ordinates

$$u^\mu = (0,0,0,1) \text{ and } u^\mu u_\mu = 1, \quad (3.5)$$

where u^μ is the 4-velocity vector of the cosmic fluid, p and ρ be the anisotropic pressure and energy density of the fluid respectively and ω be the EoS parameter which is not necessarily constant.

In the following we define some physical quantities of the space-time.

We define average scale factor and volume respectively as

$$a = (rse^{4h})^{1/3}, \quad (3.6)$$

$$V = rse^{4h}. \quad (3.7)$$

The Hubble parameter (H) is given in the form

$$H = \frac{1}{3} \frac{\dot{V}}{V}. \quad (3.8)$$

The expansion scalar (θ) is defined as follows

$$\theta = 3 \frac{\dot{a}}{a}, \quad (3.9)$$

The equation of motion (2.12) for non-static plane symmetric universe (3.1) with the fluid of stress energy tensor (3.3) can be written as

$$e^{-2h} \left(2\ddot{h} + \dot{h}^2 + \frac{2\dot{h}\dot{s}}{s} + \frac{\ddot{s}}{s} \right) = \left(\frac{8\pi + \lambda}{\lambda} \right) p - \Lambda, \quad (3.10)$$

$$e^{-2h} (2\ddot{h} + \dot{h}^2) = \left(\frac{8\pi + \lambda}{\lambda} \right) p - \Lambda, \quad (3.11)$$

$$e^{-2h} \left(\frac{2\dot{h}\dot{s}}{s} + 3\dot{h}^2 \right) = \left(\frac{8\pi + \lambda}{\lambda} \right) (-\rho) - \Lambda. \quad (3.12)$$

where the overhead dot denotes the derivative with respect to time t .

4. Thermodynamical Behavior and entropy of Model:

Thermodynamic analysis has become a powerful tool to inspect a gravitational theory. As pivotal events, black hole thermodynamics [33, 34] and recent AdS/CFT (Anti-de Sitter Space/Conformal Field Theory) correspondence [35] show explicit significance and strongly suggest the deep connection between gravity and thermodynamics.

From the thermodynamics, we apply the combination of first and second law of thermodynamics to the system with volume V [36].

$$\tau ds = d(\rho V) + pdV, \quad (4.1)$$

where τ and s represent the temperature and entropy respectively.

Above equation may be written as

$$\tau ds = d[(p + \rho)V] - Vdp. \quad (4.2)$$

The inerrability condition is necessary to define a perfect fluid as a thermodynamical system; it is given by.

$$dp = \left(\frac{p + \rho}{\tau} \right) d\tau. \quad (4.3)$$

Using equations (4.2) and (4.3) we have the differential equation

$$ds = \frac{1}{\tau} d[(p + \rho)V] - (p + \rho)V \frac{d\tau}{\tau^2}. \quad (4.4)$$

Rewriting above equation

$$ds = d\left[\frac{(p + \rho)V}{\tau}\right]. \quad (4.5)$$

Therefore the entropy is defined as

$$s = \left[\frac{(p + \rho)V}{\tau}\right]. \quad (4.6)$$

Let the entropy density be s' , so that

$$s' = \frac{s}{V} = \left(\frac{p + \rho}{\tau}\right) = \frac{(1 + \omega)\rho}{\tau}. \quad (4.7)$$

If we define the entropy density in terms of temperature, then the first law of thermodynamics may be written as

$$d(\rho V) + \gamma \rho dV = (1 + \omega)\tau d\left(\frac{\rho V}{\tau}\right), \quad (4.8)$$

which on integration yields

$$\tau = \rho^{\frac{\omega}{1+\omega}}. \quad (4.9)$$

From equation (4.7), we obtain

$$s' = (1 + \gamma)\rho^{\frac{1}{1+\omega}}. \quad (4.10)$$

Equation (4.6) represents the thermodynamics of the universe (entropy) does not depend on any individual fluids, it depends on the total matter density and isotropic pressure of the fluid.

Chirde and Shekh [21], Yadav et al. [37] and Chawala et al. [38] investigated the actions of thermodynamic parameters, these parameters are directly related to the energy density of the universe, hence our outcomes in equations (4.9) and (4.10) shows the same features with the work prepared by the above authors.

5. Solution of field equations:

The Einstein's field equations (3.10) to (3.12) are a coupled system of non-linear differential equation encloses only three independent equations with five unknowns. In order to solve the field equation, we use the following physically plausible conditions:

i) Among the physical quantities of interest in cosmology, the deceleration parameter q is currently a serious contestant to express the dynamics of the universe. The prediction of the standard cosmology is

that the universe at present is decelerating but the recent observation of the high red shift type Ia supernovae reveal that instead of slowing downward, the growing universe is speeding up. Models with constant deceleration parameter have received considerable attention recently. Berman [39] proposed a variation law for Hubble's parameter for spatially homogeneous and isotropic FRW metric that yields a constant value of deceleration parameter. We extend the same results of Berman [39] to solve the field equations. We assume the variation of the Hubble's parameter of the form

$$H = ka^{-\alpha}, \quad (5.1)$$

where $k > 0$ and $\alpha \geq 0$ are constants. For this choice of Hubble's parameter, the deceleration parameter q comes out to be constant, i.e.

$$q = \alpha - 1 \quad \text{for} \quad \alpha \neq 1 \quad (5.2)$$

$$q = -1 \quad \text{for} \quad \alpha = 1. \quad (5.3)$$

We observe that for $\alpha > 1$ the model represents a decelerating universe and $\alpha < 1$ corresponds to accelerating phase of the universe. But when $\alpha = 1$, we obtain $H = t^{-1}$ and $q = 0$. Since the deceleration parameter q is zero in the model, every galaxy moves with constant speed. For $\alpha = 0$, we get $H = k$ (constant) and $q = -1$. We observe that the Hubble parameter being a large-scale property of the universe is constant in time. Therefore the model may be considered as a steady-state model of the universe. Also deceleration parameter q for this model is negative. Therefore it represents accelerating phase. Integrating (5.1), we obtain

$$a = (\alpha kt)^{\frac{1}{\alpha}}, \quad \text{for} \quad \alpha \neq 0 \quad (5.4)$$

and

$$a = \exp k(t), \quad \text{for} \quad \alpha = 0 \quad (5.5)$$

ii) The observations of the velocity red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropy within ≈ 30 percent [40, 41]. To put more precisely, red-shift studies place the limit $(\sigma/H) \leq 0.3$ on the ratio of shear σ to Hubble constant H in the neighborhood of our galaxy. Collin et al. [42] have pointed out that for spatially homogeneous metric; the normal congruence to the homogeneous expansion satisfies that the condition (σ/θ) is constant i.e. the expansion scalar is proportional to the shear scalar. This gives the relation between metric potentials as

$$e^h = s^n, \quad (5.6)$$

where n is constant and $n > 1$.

5.a Model for $\alpha \neq 0$:

Using equations (3.6), (4.5) and (5.6) the corresponding metric coefficients comes out to be

$$s = (\alpha kt)^{\frac{3}{\alpha(4n+1)}} \quad (5.7)$$

$$e^h = (\alpha kt)^{\frac{3n}{\alpha(4n+1)}} \quad (5.8)$$

Using equations (5.7) and (5.8), non-static plane symmetric universe with time varying Λ in $f(R, T)$ (with proper choice of constants and co-ordinates) becomes

$$ds^2 = (\alpha kt)^{\frac{6n}{\alpha(4n+1)}} \left(dt^2 - dr^2 - r^2 d\theta^2 - (\alpha kt)^{\frac{6}{\alpha(4n+1)}} dz^2 \right). \quad (5.9)$$

Physical parameters of the Model:

We find the physical parameters which are important for describing the physical behavior of the model, solving equations (3.8)-(3.10) and using the equations (5.7) and (5.8). We get the expressions for Cosmological constant,

$$\Lambda = \frac{\alpha_1}{(4n+1)^2 \alpha^2 t^2} (\alpha kt)^{\frac{-6n}{\alpha(4n+1)}} \quad (5.10)$$

Anisotropic pressure,

$$p = \left\{ \frac{(\alpha_2 - \alpha_3)}{(4n+1)^2 \alpha^2 t^2} \right\} (\alpha kt)^{\frac{-6n}{\alpha(4n+1)}}, \quad (5.11)$$

Energy density,

$$\rho = \left\{ \frac{(2\alpha_1 + \alpha_2 - \alpha_3)}{(4n+1)^2 \alpha^2 t^2} \right\} (\alpha kt)^{\frac{-6n}{\alpha(4n+1)}} \quad (5.12)$$

Temperature,

$$\tau = \left\{ \frac{(2\alpha_1 + \alpha_2 - \alpha_3)}{(4n+1)^2 \alpha^2 t^2} \right\}^{\frac{\gamma}{1+\gamma}} (\alpha kt)^{\frac{-\gamma 6n}{\alpha(4n+1)(1+\gamma)}}. \quad (5.13)$$

Entropy density,

$$s' = (1+\gamma) \left\{ \frac{(2\alpha_1 + \alpha_2 - \alpha_3)}{(4n+1)^2 \alpha^2 t^2} \right\}^{\frac{1}{1+\gamma}} (\alpha kt)^{\frac{-6n}{\alpha(4n+1)(1+\gamma)}}. \quad (5.14)$$

$$\alpha_1 = \frac{-\lambda}{(6\pi + 5\lambda)} \left(\begin{array}{c} 72n^2 - 24n^2\alpha - 18n\alpha \\ + 54n - 3\alpha + 9 \end{array} \right), \alpha_2 = \frac{\lambda}{(8\pi + \lambda)} \left(\begin{array}{c} 9n^2 - 24n^2\alpha^2 - 18n\alpha \\ + 9n - 3\alpha + 9 \end{array} \right), \alpha_3 = \frac{\lambda\alpha_1}{(8\pi + \lambda)}$$

The performance of energy density and cosmological constant are shown in the following figures.

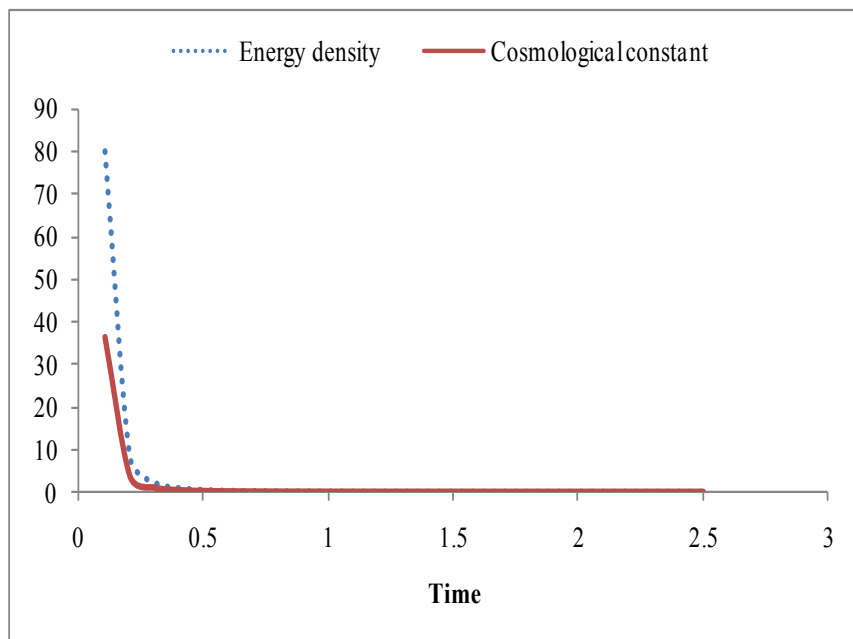


Figure (1): behavior of energy density and cosmological constant verses time t for the model $\alpha \neq 0$ with an appropriate choice of constant.

In this cosmology, it is observed that the energy density and cosmological constant both are positive and decreasing function of time t . At the initial stage $t \rightarrow 0$ the universe has infinitely large energy density $\rho \rightarrow \infty$ but with the expansion of the universe it declines and at large $t \rightarrow \infty$ it is null i.e. $\rho \rightarrow 0$. This behavior of energy density and cosmological constant is clearly shown in Fig. 1, as a representative case with appropriate choice of constants and other parameters using reasonably well known situations.

Kinematical Properties of the model:

The kinematical properties which are important in cosmology for discussing the geometrical behavior of the universe have the following expressions

The spatial volume,

$$V = a^3 = (\alpha kt)^{\frac{3}{\alpha}}$$

The Hubble parameter,

$$H = \frac{1}{\alpha t}$$

The scalar expansion,

$$\theta = \frac{3}{\alpha t}$$

5.b Model for $\alpha = 0$:

Using equations (3.7), (4.5) and (5.6) the corresponding metric coefficients comes out to be

$$s = \exp \left\{ \frac{3kt}{(4n+1)} \right\}, \quad (5.15)$$

$$e^h = \exp \left\{ \frac{3nkt}{(4n+1)} \right\}. \quad (5.16)$$

Using equations (5.15) and (5.16), non-static plane symmetric universe with time varying Λ in $f(R, T)$ (with proper choice of constants and co-ordinates) becomes

$$ds^2 = \exp \left\{ \frac{6nkt}{(4n+1)} \right\} \left(dt^2 - dr^2 - r^2 d\theta^2 - \exp \left\{ \frac{3kt}{(4n+1)} \right\} dz^2 \right). \quad (5.17)$$

5. Physical properties of the model:

We find the physical parameters which are important for describing the physical behavior of the universe, solving equations (3.10)-(3.12) and using the equations (5.15) and (5.16). We get the expressions of Cosmological constant,

$$\Lambda = \frac{\alpha_4}{(4n+1)} \exp \left\{ \frac{-6nkt}{(4n+1)} \right\}. \quad (5.18)$$

Anisotropic pressure,

$$p = \frac{\alpha_5}{(4n+1)} \left(\exp \right)^{\frac{-6nkt}{(4n+1)}}. \quad (5.19)$$

Energy density,

$$\rho = \frac{\alpha_6}{(4n+1)} \exp \left\{ \frac{-6nkt}{(4n+1)} \right\}. \quad (5.20)$$

Temperature,

$$\tau = \left\{ \frac{\alpha_6}{4n+1} \right\}^{\frac{\gamma}{1+\gamma}} \exp \left\{ \frac{-6\gamma nkt}{(4n+1)(1+\gamma)} \right\}. \quad (5.21)$$

Entropy density,

$$s' = (1+\gamma) \left\{ \frac{\alpha_6}{4n+1} \right\}^{\frac{1}{1+\gamma}} \exp \left\{ \frac{-6nkt}{(4n+1)(1+\gamma)} \right\}. \quad (5.22)$$

$$\alpha_4 = \frac{-9\lambda k^2(2n+1)}{2(6\pi+5\lambda)}, \quad \alpha_5 = \frac{9k^2(2n^2+2n+1)}{2(4n+1)} - \frac{9\lambda k^2(2n+1)}{6\pi+5\lambda}, \quad \alpha_6 = \alpha_5 - \frac{9\lambda k^2(2n+1)}{2(6\pi+5\lambda)(4n+1)}$$

The performance of energy density and cosmological constant are shown in the following figures.

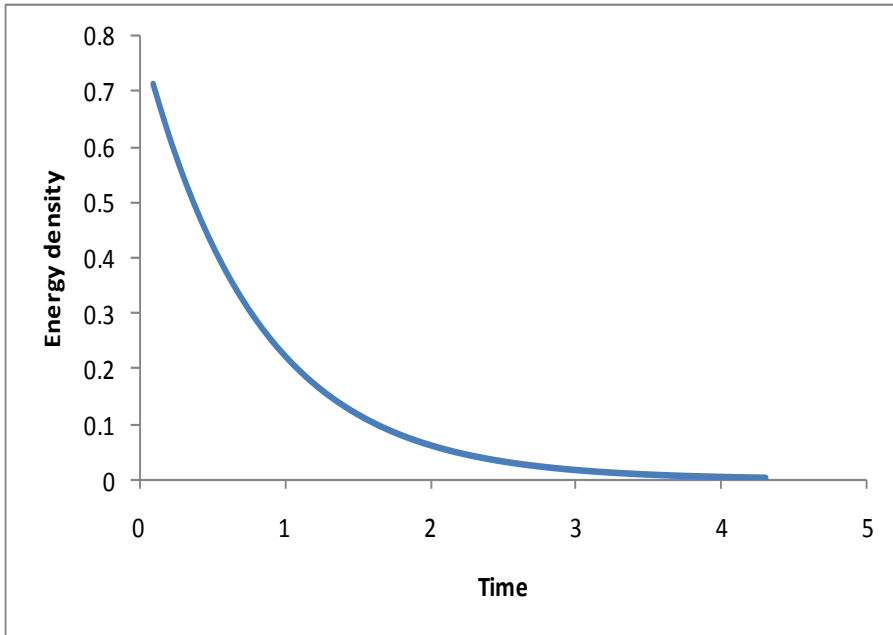


Figure (2): behavior of energy density verses time t for $\alpha = 0$ with an appropriate choice of constant.

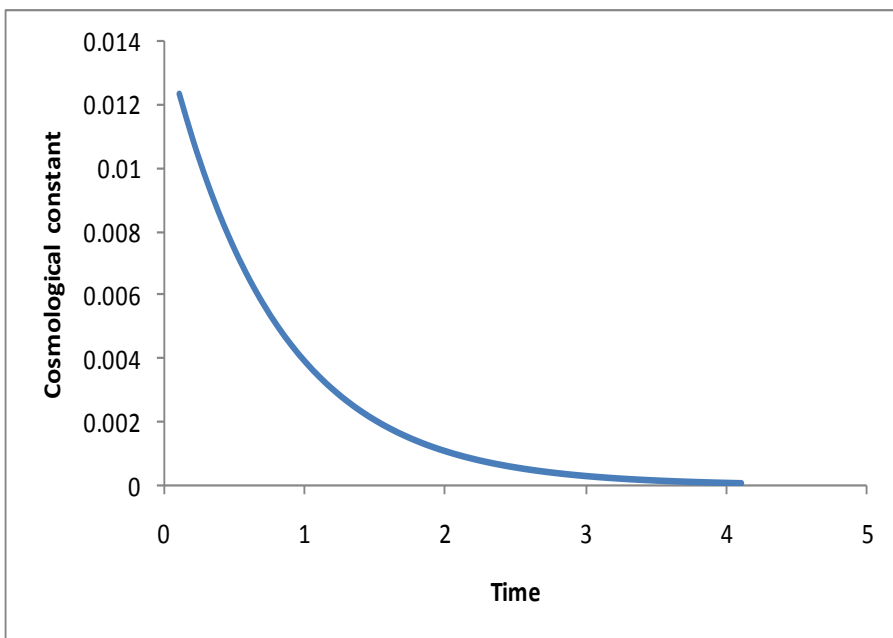


Figure (4): behavior of cosmological constant verses time t for $\alpha = 0$ with an appropriate choice of constant.

In this model, it is observed that the energy density and cosmological constant showing the same behavior as that of above case. Initially the universe has infinitely large energy density. This behavior is clearly shown in Fig. 3 and 4, as a representative case with appropriate choice of constants.

Kinematical Properties of the model:

The kinematical properties which are important in cosmology for discussing the geometrical behavior of the universe that have the following expressions

The spatial volume,

$$V = a^3 = \exp 3k(t)$$

The mean Hubble parameter,

$$H = k$$

The scalar expansion,

$$\theta = 3k$$

5. Conclusions:

This paper is devoted to explore the solutions of non-static plane symmetric space-time in $f(R, T)$ theory of gravity in the background of perfect fluid with cosmological constant term. The assumption of constant deceleration parameter leads to two models of universe, i.e. power and exponential model. Some important cosmic thermodynamical, physical and kinematical parameters for the solutions such as Temperature, entropy density, energy density, pressure, volume, Hubble's parameter and expansion scalar are evaluated.

First we discuss power law model of the universe. This model corresponds to $\alpha \neq 0$ with average scale factor $a = (\alpha kt)^{\frac{1}{\alpha}}$. It has a singularity at $t = 0$. The Kinematical parameters H and θ is infinite at this point but the volume scale factor vanishes. The metric functions s and e^h both are vanish at this point $t = 0$. Thus, it is concluded from these observations that the model starts its expansion with zero volume and it continues to expand.

The exponential model of the universe corresponds to $\alpha = 0$ with average scale factor $a = \exp k(t)$. It is non-singular because exponential function is never zero and hence does not exist any physical singularity for this model. The Kinematical parameters H and θ are constants while metric functions s and e^h do not vanish for this model. The volume scale factor increases exponentially with time which indicates that the universe starts its expansion with constant volume. In both models, it is observed that the thermodynamical parameters such as Temperature and entropy density be also the function of time and related with energy density of the model.

6. References:

- [1] A. G. Riess et al. *AJ*, 116, 1009 (1998)
- [2] S. Perlmutter et al. *Ap J*, 517, 565 (1999)
- [3] J. Tonry et al. *Ap J*, 594, 1 (2003)
- [4] B. Ratra, P. J. E. Peebles *Phys. Rev. D* 37, 321, (1988)
- [5] R. R. Caldwell, *Phys. Lett. B*. 545, 23 (2002)
- [6] T. Padmanabhan, *Phys. Rev. D* 66, 021301 (2002)
- [7] S. K. Srivastav, *Phys. Lett. B* 619, 1 (2005)
- [8] T. Padmanabhan, T. Roy Choudhury, *Phys. Rev. D*, 66, 081301 (2002)
- [9] R. Jimenez, *New astron. Rev.* 47, 761 (2003)
- [10] O. Akarsu, C. Kilinc, *Gen. Relativ. Gravit.* 42, 763 (2010)
- [11] A. Yadav, L. Yadav, *Int. J. Theor. Phys.* 50, 218 (2010)
- [12] S. D. Katore, A. Y. Shaikh, *Prespacetime Journal* 3 (11), (2012)
- [13] S. D. Katore, A. Y. Shaikh, *Astrophysics and Space Science* 357 (1), 1(2015)
- [14] P. K. Sahoo, B. Mishra, *The European Physical Journal Plus*, 129, 196 (2014)
- [15] V. R. Chirde, S. H. Shekh, *The African Review of Physics* 10, (2015)
- [16] V. R. Chirde, S. H. Shekh, P. N. Rahate, *Prespacetime Journal* 5 (9), (2014)
- [17] S. Tade, M. Sambhe, *Astrophys. Space Sci.* 338, 179 (2012)
- [18] T. Chiba, L. Smith, A. L. Erickcek, *Phys. Rev. D* 75, 124014 (2007)
- [19] P. Wu, W. Yu, *Eur. Phys. J. C* 71, 1552 (2011)
- [20] Y. Zhang, H. Li, Y. Gong, Z. H. Zhu, *JCAP* 1107, 015 (2011)
- [21] V. R. Chirde, S. H. Shekh. *Bulgerian journal of physics*, 41, 258 (2014)
- [22] T. Harko, F. Lobo, S. Nojiri, S. Odintsov, *Phys. Rev. D* 84, 024020 (2011)
- [23] M. Sharif and M. Zubair: *JCAP* 03028 (2012)
- [24] M. Jamil, D. Momeni, M. Raza, R. Myrzakulov, *Eur. Phys. J. C* 72, 1999 (2012)
- [25] S. Chandel, S. Ram, *Indian J Phys* DOI 10.1007/s12648-013-0362-9(2013)
- [26] R. Chaubey, A. Shukla, *Astrophys Space Sci* 343, 415 (2013)
- [27] S. Katore, B. Chopde, S. Shekh, *Int. J. Basic and Appl. Res. (Spe. Issue)* 283 (2012)
- [28] P. Sahoo, B. Mishra, G. Chakradhar, D. Reddy, *Eur. Phys. J. Plus* 129, 49 (2014)
- [29] V. R. Chirde, S. H. Shekh, *Astrofisica* 58, 121 (2015)
- [30] M. F. Shamir, *Int. J. Theor. Phys.* 54, 1304 (2015)
- [31] A. Alvarenga et al., *Journal of Modern Physics*, 4, 130 (2013)
- [32] N. Poplawski, arXiv:gr-qc/0608031v2 (2006)
- [33] S. W. Hawking, *Commun. Math. Phys.* 43, 199 (1975)
- [34] J. M. Bardeen, N. Carter and S. W. Hawking, *Commun. Math. Phys.* 31, 161 (1973)
- [35] J. M. Maldacena, *Adv. Theor. Math. Phys.* 2, 231 (1998)
- [36] F. C. Santos, V. Soares, M. L. Bedram.: *Phys. Lett. B* 646, 215 (2007)
- [37] M. K. Yadav, A. Rai, A. Pradhan, *Int. J. Theor. Phys.* 46, 2677 (2007)
- [38] C. Chawla, R. K. Mishra, *Rom.Jou.Phys.* 558 (8) 75–85 (2014)
- [39] M. Berman, *Nuovo Cimonto B* 74 182 (1983)
- [40] R. Kantowski, R. Sachs, *J. Math. Phys.* 7, 433 (1966)
- [41] J. Kristian, S. Sachs, *Astrophys. J.* 143, 379 (1966)
- [42] C. Collins, E. Glass, D. Wilkisons, *Gen. Rel. Grav.* 12, 805 (1980)